



# **EEL 5245 POWER ELECTRONICS I**

## **Lecture #9: Chapter 3**

### **Power System Basics and Sinusoidal Systems**

# Objectives

**EEL 5245**

- Review of Basic Power Concepts
  - Efficiency-Practical Example
  - Average and RMS Calculations
    - Commonly observed waveform
- Sinusoidal Systems
  - Load Types
  - Average, Reactive, and Apparent Power
  - Power Factor
  - Power Factor Correction
  - 3 Phase, 4 Wire Neutral Currents

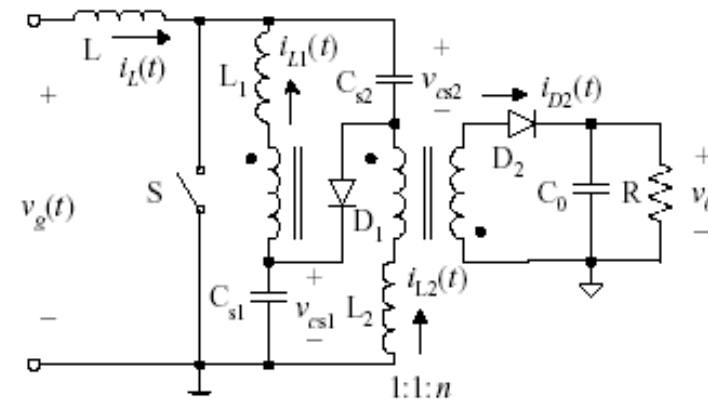
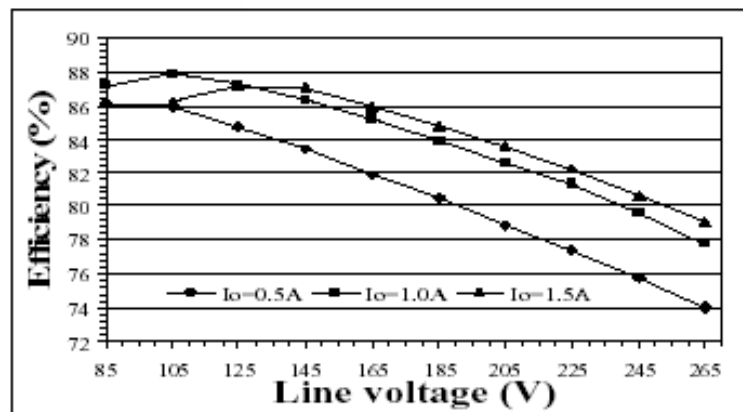
# Efficiency in Power Electronic Converters

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \quad \eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}}$$

- Efficiency is one of the most significant figures of merit
  - Can be used to judge overall performance of topology in many cases
- Modern Power Electronic Converters have efficiencies approaching 100%
- For in-class analysis, if power electronic circuit consists of ideal switches, transformers, and energy elements (L&C) efficiency is 100%
- In Real World Circuits, losses play a significant role:
  - Dominate Loss Mechanisms
    - Semiconductor Switch Loss: Conduction and Switching
  - Losses are a function of operating point

# Efficiency in “Real World” Power Electronics Circuits

## Example – Power Factor Connections



| Single-Stage, Single-Switch Topology Loss Analysis-50W |       |       |       |       |       |       |
|--------------------------------------------------------|-------|-------|-------|-------|-------|-------|
| Input Voltage (VRMS)                                   | 90    | 100   | 110   | 120   | 130   | 140   |
| Total Loss (W) Experimental                            | 9.8   | 8.6   | 8.4   | 8.3   | 8.3   | 8.4   |
| Efficiency (%) Experimental                            | 84.3% | 86.0% | 86.3% | 86.4% | 86.4% | 86.3% |
| Conduction Loss Main Switch (W) Experimental           | 1.25  | 0.94  | 0.82  | 0.74  | 0.70  | 0.65  |
| Total Loss Main Switch (W) Experimental                | 2.44  | 1.51  | 1.31  | 1.24  | 1.14  | 1.14  |
| RDSon (Ω) Experimental                                 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 | 0.748 |
| RDSon (Ω) Simulated                                    | 0.688 | 0.688 | 0.688 | 0.688 | 0.688 | 0.688 |

# Average and RMS Calculations

Given Periodic Function  $f(t)$ :

Average of  $f(t)$ :

$$F_{avg} = \frac{1}{T_s} \cdot \int_{t_0}^{t_0+T_s} f(t) dt$$

Rms of  $f(t)$ :

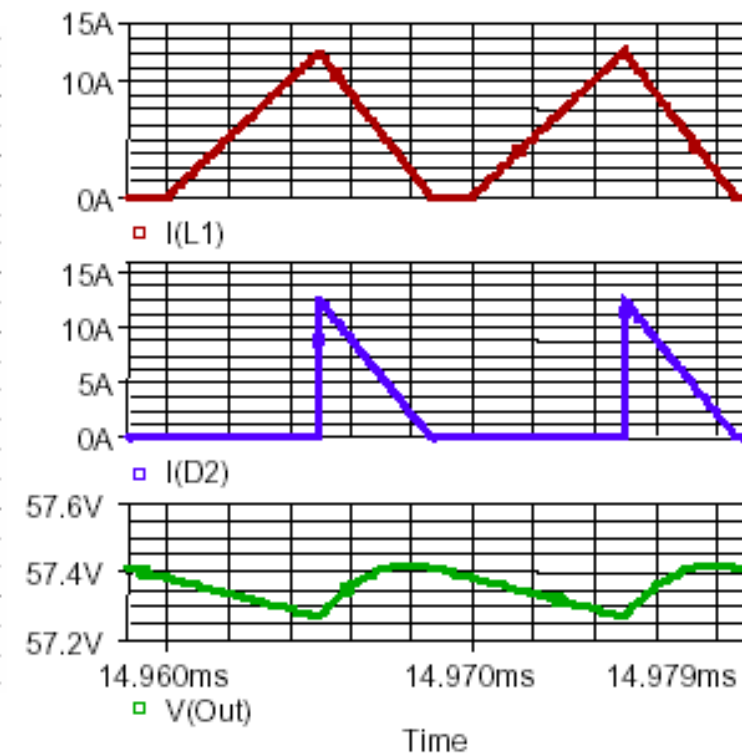
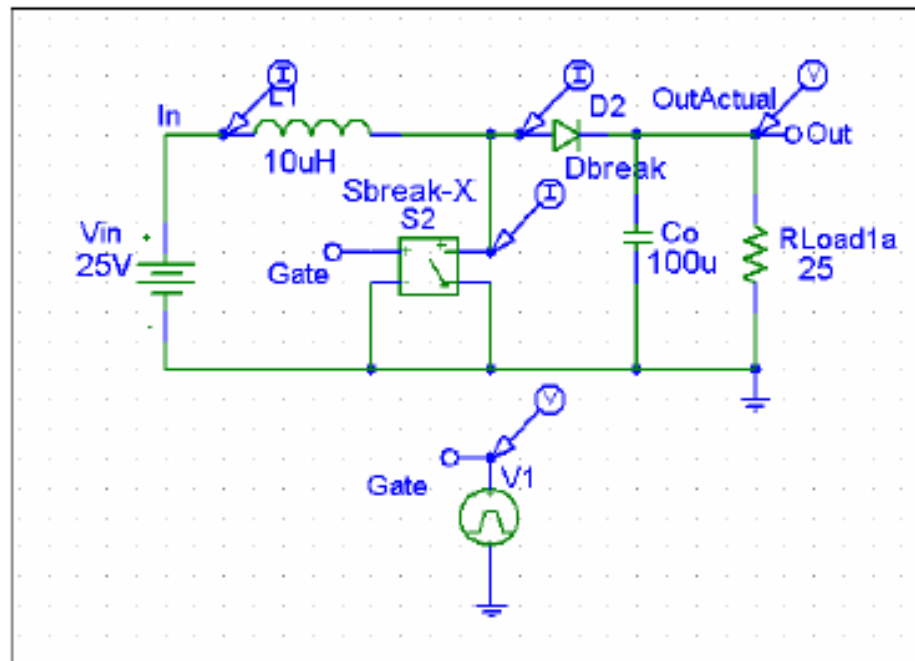
$$F_{RMS} = \sqrt{\frac{1}{T_s} \cdot \int_{t_0}^{t_0+T_s} f(t)^2 dt}$$

- In modern power electronics converters, electrical parameters have periodic and nonlinear waveforms (steady state)
  - Switch Current, Diode Voltage, Inductor Current, etc.
- Both average and RMS values of these highly, nonlinear wave shapes are often needed for design
  - Average is area under curve divided by period
  - RMS used for power loss calculation

# Average and RMS Calculations

## Nonlinear/Piecewise Wave shapes

### Example of Boost DC-DC Converter



# Instantaneous and Average Power

$$p(t) = v(t) \cdot i(t)$$

$$P_{avg} = \frac{1}{T_s} \cdot \int_{t_0}^{t_0+T_s} p(t) dt$$

$$P_{app} = V_{RMS} \cdot I_{RMS}$$

- Instantaneous power is point for point product of voltage and current
- “Useful” power is average of Instantaneous,  $P_{avg}$  (aka Real Power)
- RMS definition can be derived from instantaneous power average
- Apparent Power is product of RMS values of voltage and current

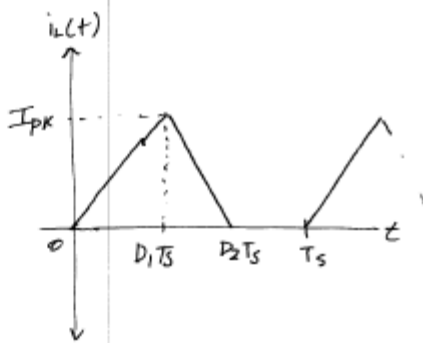
# RMS & Average Value Calculation

- RMS & Average Value Calculation
- Examples of Commonly observed P.E. waveforms
- DCM Induced Current

DCM INDUCTOR CURRENT

$$I_{LAVG} = \frac{AREA}{T_S}$$

$$= \frac{\frac{1}{2} (D_2 T_S) I_{PK}}{T_S}$$

$$= \frac{1}{2} \cdot D_2 \cdot I_{PK}$$


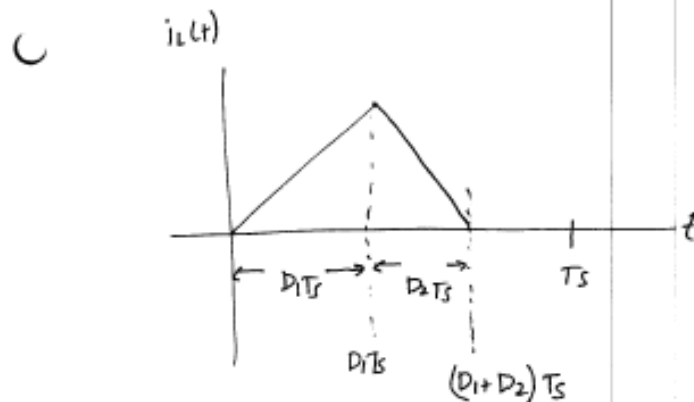
$$I_{L_{RMS}} = \sqrt{\frac{1}{T_S} \int_0^{T_S} i_L^2(t) dt} =$$

$$= \sqrt{\frac{1}{T_S} \cdot \left[ \int_0^{D_1 T_S} \left( \frac{I_{PK}}{D_1 T_S} t \right)^2 dt + \int_{D_1 T_S}^{D_2 T_S} \left( \frac{-I_{PK}}{(D_2 - D_1) T_S} (t - D_1 T_S) \right)^2 dt \right]}$$

$$= \sqrt{\frac{1}{T_S} \cdot \left[ \frac{I_{PK}^2}{D_1^2 T_S^2} \cdot \frac{t^3}{3} \Big|_0^{D_1 T_S} + \frac{I_{PK}^2}{(D_2 - D_1)^2 T_S^2} \cdot \frac{(t - D_1 T_S)^3}{3} \Big|_{D_1 T_S}^{D_2 T_S} \right]}$$

$$= \sqrt{I_{PK}^2 \cdot \left[ \frac{D_1}{3} + \frac{(D_2 - D_1)}{3} \right]} = I_{PK} \cdot \sqrt{\frac{D_2}{3}}$$

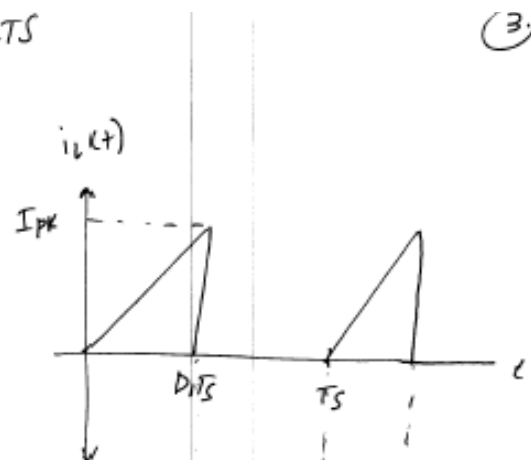
IF WE REDEFINE INTERVALS AS  $\Rightarrow$



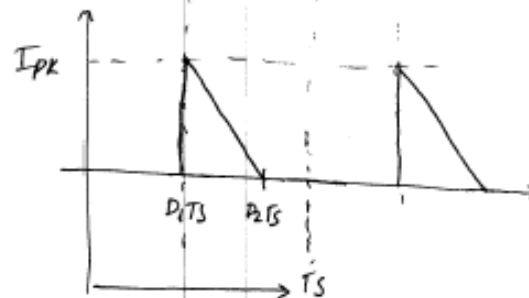
$$\begin{aligned}
 I_{L_{RMS}} &= \sqrt{\frac{1}{T_s} \left[ \int_0^{D_1 T_s} \frac{I_{pk}^2}{D_1^2 T_s^2} t^2 dt + \int_{D_1 T_s}^{(D_1+D_2) T_s} \frac{I_{pk}^2}{D_2^2 T_s^2} (t - D_1 T_s)^2 dt \right]} \\
 &= \sqrt{I_{pk}^2 \left[ \frac{1}{D_1^2 T_s^3} \cdot \frac{t^3}{3} \Big|_0^{D_1 T_s} + \frac{1}{D_2^2 T_s^3} \cdot \frac{(t - D_1 T_s)^3}{3} \Big|_{D_1 T_s}^{(D_1+D_2) T_s} \right]} \\
 &= I_{pk} \sqrt{\left[ \frac{D_1}{3} + \frac{(D_1+D_2-D_1)^3}{D_2^3 \cdot 3} \right]} \\
 &= I_{pk} \cdot \sqrt{\frac{D_1 + D_2}{3}}
 \end{aligned}$$

EXTENDING THESE RESULTS  
 ○ OTHER CASES  $\Rightarrow$

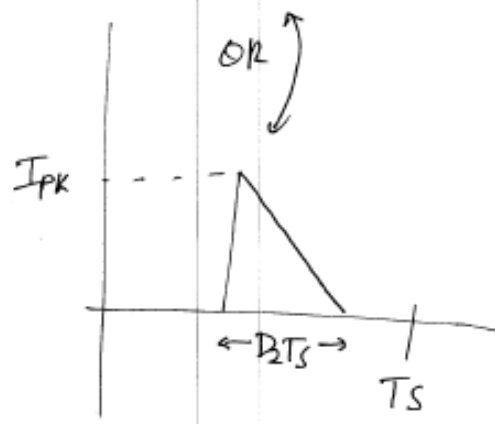
$$I_{L_{RMS}} = I_{PK} \cdot \sqrt{\frac{D_1}{3}}$$



$$I_{L_{RMS}} = I_{PK} \cdot \sqrt{\frac{D_2 - D_1}{3}}$$

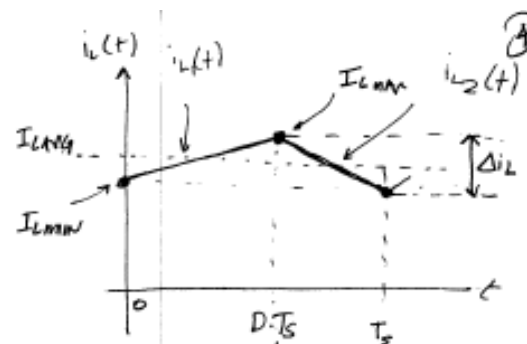


$$I_{L_{RMS}} = I_{PK} \cdot \sqrt{\frac{D_2}{3}}$$



DC + LINEAR RIPPLE

$$\begin{aligned} I_{LAVG} &= \frac{I_{LMAX} + I_{LMIN}}{2} \\ &= I_{LMIN} + \frac{\Delta i_L}{2} \end{aligned}$$



$$RMS = \sqrt{\frac{1}{T_s} \int_0^{T_s} i_L(t)^2 dt}$$

$$i_{L1}(t) = \frac{\Delta i_L}{D \cdot T_s} \cdot t + I_{LMIN} \quad i_{L2}(t) = \frac{-\Delta i_L}{(1-D) \cdot T_s} \cdot (t - T_s) + I_{LMIN}$$

$$i_{L1}(t)^2 = \frac{\Delta i_L^2}{D^2 \cdot T_s^2} \cdot t^2 + \frac{2 \cdot I_{LMIN} \cdot \Delta i_L}{D \cdot T_s} \cdot t + I_{LMIN}^2$$

$$i_{L2}(t)^2 = \frac{\Delta i_L^2}{(1-D)^2 \cdot T_s^2} \cdot (t - T_s)^2 - \frac{2 \cdot \Delta i_L}{(1-D) \cdot T_s} \cdot (t - T_s) \cdot I_{LMIN} + I_{LMIN}^2$$

$$RMS = \sqrt{\frac{1}{T_s} \left[ \int_0^{D T_s} i_{L1}(t)^2 dt + \int_{D T_s}^{T_s} i_{L2}(t)^2 dt \right]}$$

(1)                      (2)

$$\begin{aligned}
 \int \textcircled{1} \Rightarrow & \int_0^{DT_S} i_{L1}(t)^2 dt = \int_0^{DT_S} \frac{\Delta i_L^2}{D^2 T_S^2} t^2 + 2 \frac{I_{Lmin} \cdot \Delta i_L}{DT_S} t + I_{Lmin}^2 dt \\
 & = \frac{\Delta i_L^2}{D^2 T_S^2} \cdot \frac{t^3}{3} \bigg|_0^{DT_S} + 2 \frac{I_{Lmin} \cdot \Delta i_L}{DT_S} \cdot \frac{t^2}{2} \bigg|_0^{DT_S} + I_{Lmin}^2 \cdot t \bigg|_0^{DT_S} = \\
 & = \frac{\Delta i_L^2 \cdot D \cdot T_S}{3} + I_{Lmin} \cdot DT_S \cdot \Delta i_L + I_{Lmin}^2 \cdot DT_S \\
 & \int \left[ \frac{(I_{Lmax} - I_{Lmin})^2}{3} + I_{Lmin} \cdot (I_{Lmax} - I_{Lmin}) + I_{Lmin}^2 \right] \cdot DT_S \\
 & = \frac{1}{3} (I_{Lmax}^2 + I_{Lmin} \cdot I_{Lmax} + I_{Lmin}^2) \cdot DT_S
 \end{aligned}$$

$$\begin{aligned}
 \int \textcircled{2} \Rightarrow & \int_{DT_S}^{T_S} \frac{\Delta i_L^2}{(1-D)^2 T_S^2} (t - T_S)^2 - \frac{2 \Delta i_L (t - T_S) I_{Lmin}}{(1-D) T_S} + I_{Lmin}^2 dt
 \end{aligned}$$

EXPECT  $\int \Theta$  HAS SAME FORM AS  $\int \textcircled{1} \therefore \Rightarrow \textcircled{6}$

$$\int \textcircled{2} \Rightarrow \frac{1}{3} (I_{LMAX}^2 + I_{LMIN} \cdot I_{LMAX} + I_{LMIN}^2) (1-D) T_S$$

$$\begin{aligned} \therefore R_{MS} &= \sqrt{\frac{1}{T_S} \cdot [\int \textcircled{1} + \int \textcircled{2}]} \\ &= \sqrt{\frac{1}{3} \cdot (I_{LMAX}^2 + I_{LMIN} \cdot I_{LMAX} + I_{LMIN}^2)} \end{aligned}$$

WRITE IN TERMS OF  $\Delta I_L$  &  $I_{LAVG}$

$$\Delta I_L = (I_{LMAX} - I_{LMIN}) \quad I_{LAVG} = \frac{I_{LMAX} + I_{LMIN}}{2}$$

$$\Delta I_L^2 = I_{LMAX}^2 - 2 \cdot I_{LMAX} \cdot I_{LMIN} + I_{LMIN}^2$$

$$I_{LAVG}^2 = \frac{1}{4} \cdot (I_{LMAX}^2 + 2 \cdot I_{LMAX} \cdot I_{LMIN} + I_{LMIN}^2)$$

$$R_{MS} = \sqrt{\left[ 3 \left( \frac{4 \cdot I_{LAVG}^2 - \Delta I_L^2}{4} \right) + \Delta I_L^2 \right] \cdot \frac{1}{3}}$$

$$I_{RMS} \Rightarrow$$

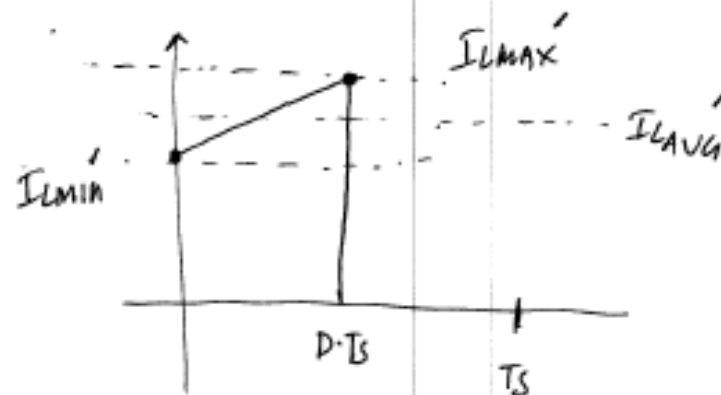
$$= \sqrt{I_{LNG}^2 - \frac{\Delta i_L^2}{4} + \frac{\Delta i_L^2}{3}}$$

$$= \sqrt{I_{LNG}^2 - \frac{3\Delta i_L^2}{12} + \frac{4\Delta i_L^2}{12}}$$

$$= \sqrt{I_{LNG}^2 + \frac{1}{12} \Delta i_L^2} = i_{L_{RMS}}$$

OTHERS FOR YOU TO DO  $\Rightarrow$

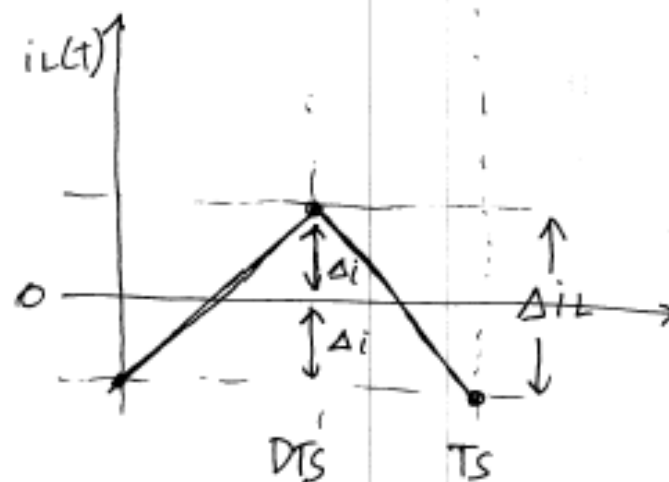
SWITCH  
CURRENT  
RMS = ?

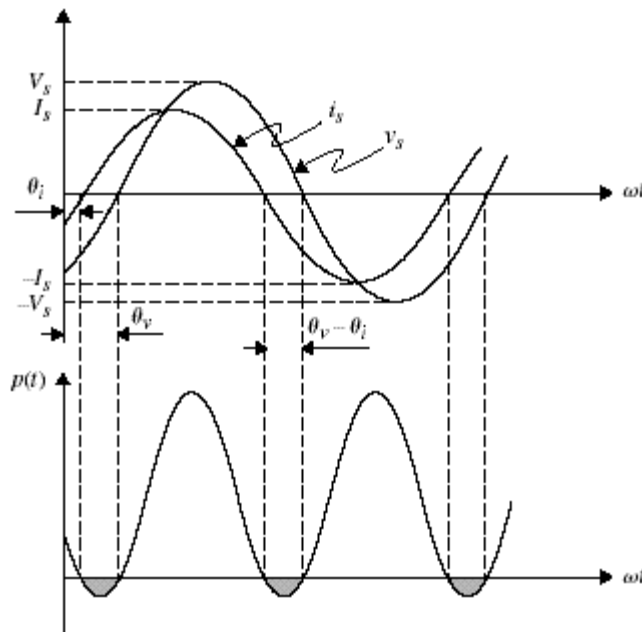


NO DC  
COMPONENT

$$\int_0^T i_L(t) dt = 0$$

RMS = ?





$$V_{rms} = \frac{V_s}{\sqrt{2}} \quad V_{avg} = 0$$

$$I_{rms} = \frac{I_s}{\sqrt{2}} \quad I_{avg} = 0$$

$$P_{apparent} = V_{RMS} \cdot I_{RMS} = \frac{V_s \cdot I_s}{2}$$

$$\phi = \theta_v - \theta_i \text{ phaseshift}$$

- Recall that the RMS value of any pure sinusoidal is peak divided by root 2
- Phase shift in this diagram a result of reactive elements
  - In this case, a capacitive load as I leads V
- Real power is maximized when v(t) and i(t) are in phase since overlap is maximized
  - Any relative shift between the two means real power is less than maximum while RMS is unaffected

# Sinusoidal Systems

## Purely Resistive Load

$$V_{\text{rms}} := 120 \quad f := 60 \quad \theta_V := 0$$

$$I_{\text{rms}} := 50 \quad \omega := 2 \cdot \pi \cdot f \quad \theta_I := 0$$

$$v_S(t) := V_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_V)$$

$$i_S(t) := I_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_I)$$

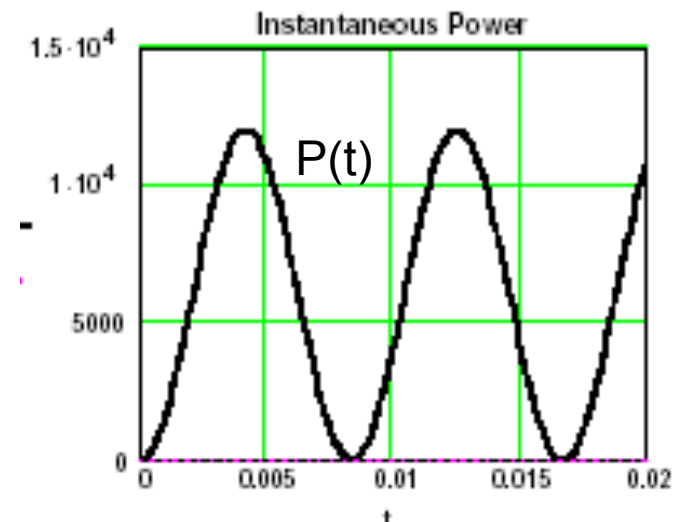
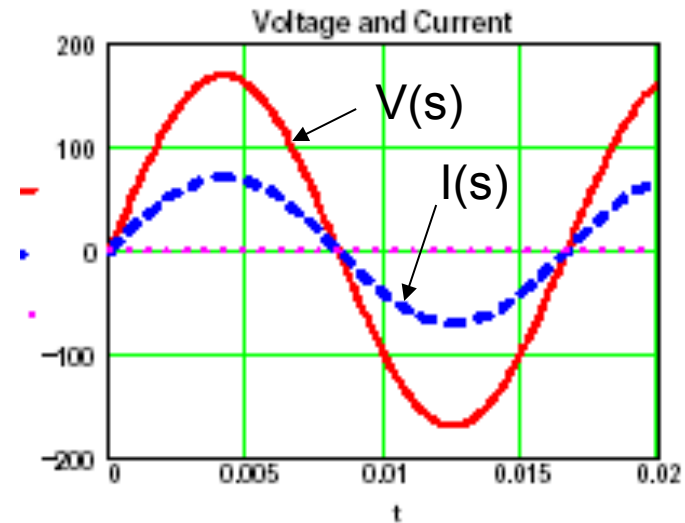
$$p(t) := v_S(t) \cdot i_S(t)$$

$$t := 0, 10^{-6} \dots 20 \cdot 10^{-3}$$

$$P_{\text{AVG}} := \frac{1}{T} \cdot \int_0^T p(t) \, dt \quad P_{\text{apparent}} = V_{\text{rms}} \cdot I_{\text{RMS}}$$

$$P_{\text{AVG}} = 6 \times 10^3$$

$$P_{\text{apparent}} = 6000$$



# Sinusoidal Systems

## Resistive-Inductive Load

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$$V_{\text{rms}} := 120 \quad f := 60$$

$$\theta_V := 0$$

$$I_{\text{rms}} := 50 \quad \omega := 2 \cdot \pi \cdot f$$

$$\theta_I := \frac{\pi}{6}$$

$$v_S(t) := V_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_V)$$

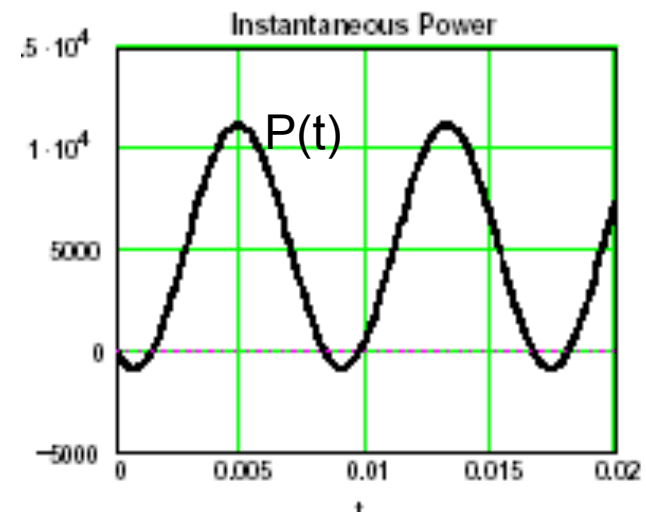
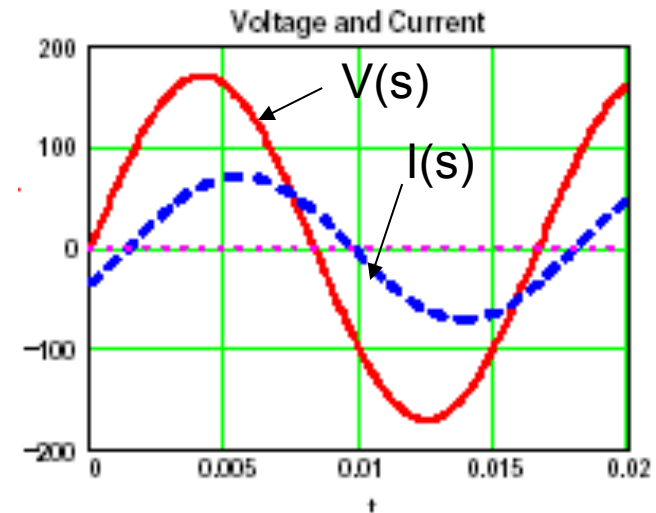
$$i_S(t) := I_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_I)$$

$$p(t) := v_S(t) \cdot i_S(t)$$

$$P_{\text{AVG}} := \frac{1}{T} \cdot \int_0^T p(t) \, dt \quad P_{\text{apparent}} = V_{\text{rms}} \cdot I_{\text{RMS}}$$

$$P_{\text{AVG}} = 5.196 \times 10^3$$

$$P_{\text{apparent}} = 6000$$



# Sinusoidal Systems

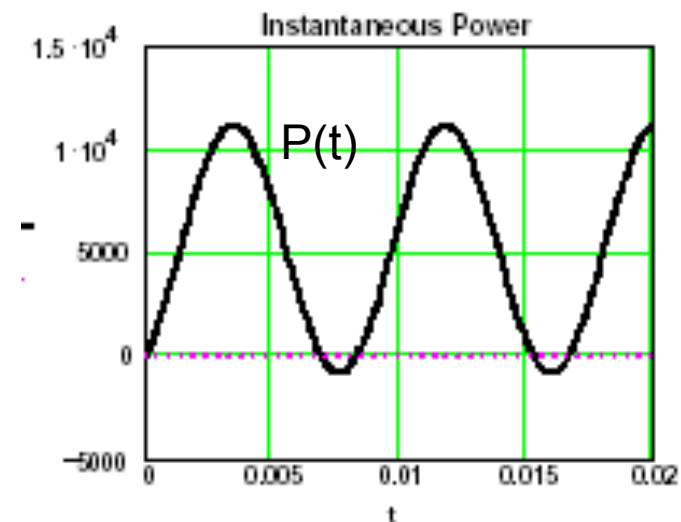
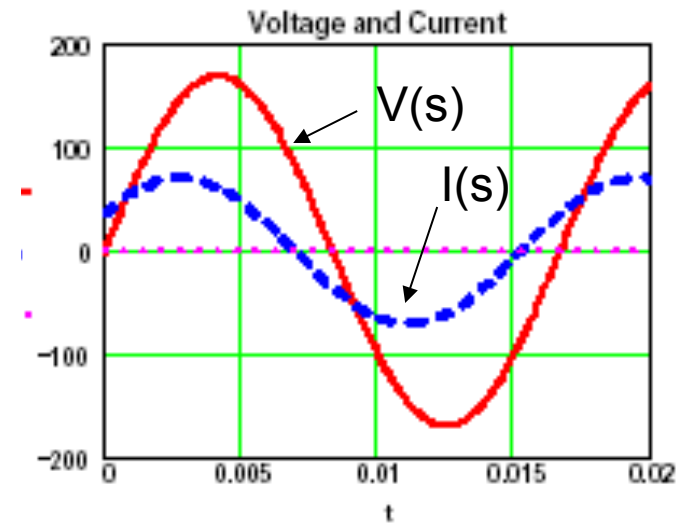
## Resistive-Capacitance Load

$$\begin{aligned}
 V_{\text{rms}} &:= 120 & f &:= 60 & \theta_V &:= 0 \\
 I_{\text{rms}} &:= 50 & \omega &:= 2 \cdot \pi \cdot f & \theta_I &:= \frac{\pi}{6} \\
 v_S(t) &:= V_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_V) \\
 i_S(t) &:= I_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_I) \\
 p(t) &:= v_S(t) \cdot i_S(t)
 \end{aligned}$$

$$P_{\text{AVG}} := \frac{1}{T} \cdot \int_0^T p(t) \, dt \quad P_{\text{apparent}} = V_{\text{rms}} \cdot I_{\text{RMS}}$$

$$P_{\text{AVG}} = 5.196 \times 10^3$$

$$P_{\text{apparent}} = 6000$$



# Sinusoidal Systems

## Purely Inductive Load

$$V_{\text{rms}} := 120 \quad f := 60 \quad \theta_v := 0$$

$$I_{\text{rms}} := 50 \quad \omega := 2 \cdot \pi \cdot f \quad \theta_i := \frac{\pi}{2}$$

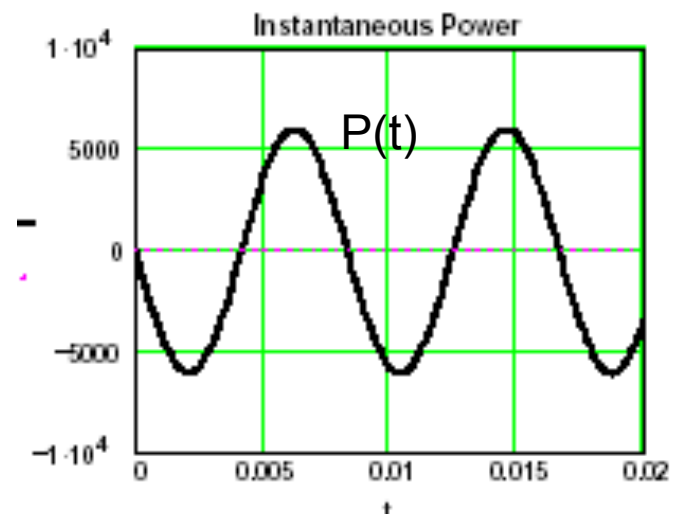
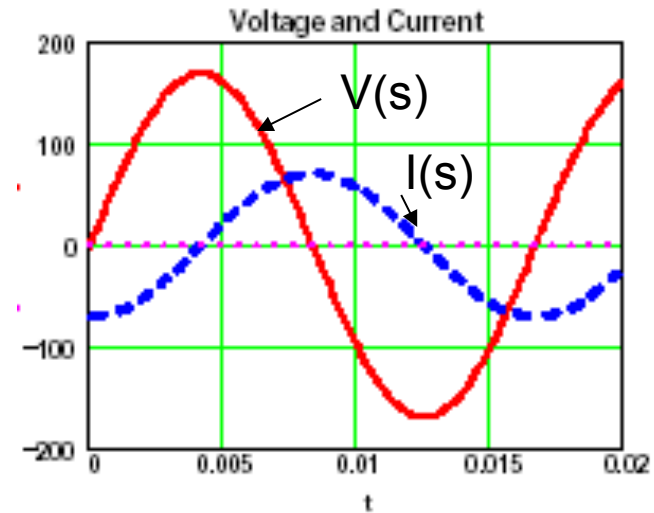
$$v_s(t) := V_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_v)$$

$$i_s(t) := I_{\text{rms}} \cdot \sqrt{2} \cdot \sin(\omega \cdot t - \theta_i)$$

$$p(t) := v_s(t) \cdot i_s(t)$$

$$P_{\text{AVG}} := \frac{1}{T} \cdot \int_0^T p(t) \, dt \quad P_{\text{apparent}} = V_{\text{rms}} \cdot I_{\text{RMS}}$$

$$P_{\text{AVG}} = -2.387 \times 10^{-13} \quad P_{\text{apparent}} = 6000$$



# Sinusoidal Systems-Power Factor

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$$\theta = \theta_v - \theta_i$$

$$P_{avg} = \frac{1}{T} \int_0^T p(t) dt = V_{rms} \cdot I_{rms} \cdot \cos(\theta)$$

$$\text{PowerFactor} = \frac{P_{avg}}{P_{apparent}} = \frac{V_{rms} \cdot I_{rms} \cdot \cos(\theta)}{V_{rms} \cdot I_{rms}}$$

**Definition of  
power factor  
(always true)**

**Only true for  
sinusoidal  
systems**

$$\text{PowerFactor} = \cos(\theta)$$

**Commonly called Displacement  
Power Factor**

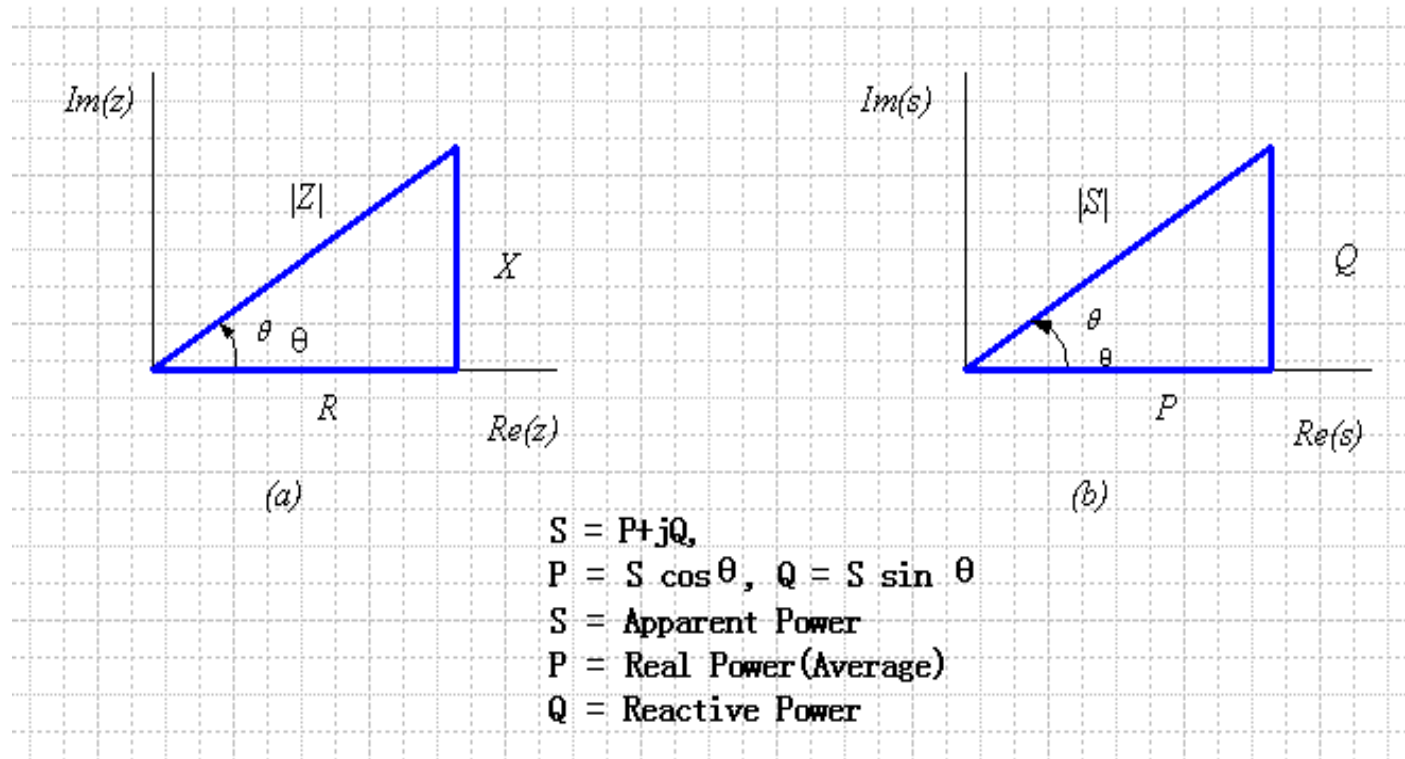
- For the resistive load ( $\theta=0$ ), both real and apparent power are  $V_{rms} \cdot I_{rms}$
- $I_{rms}$  is current load and power distribution system
- For resistive load, power system capacity is most efficiently utilized
- For any relative phase shift (i.e.  $\theta$  Not equal to 0), power system capacity more stressed than what is strictly necessary to deliver required real power to load
- Power Factor is measure of how effectively power system is being utilized

# Sinusoidal Systems-Reactive Power

- What makes up difference between apparent power and real power when  $\text{pf} < 1$
- For any relative phase shift (i.e.  $\Theta$  not equal to 0), additional circulating energy due to charging and discharging of system reactive elements
- Reactive Power passed back and forth from source to load using system capacity but provides no real power to load (i.e. performs no work)

# Sinusoidal Systems

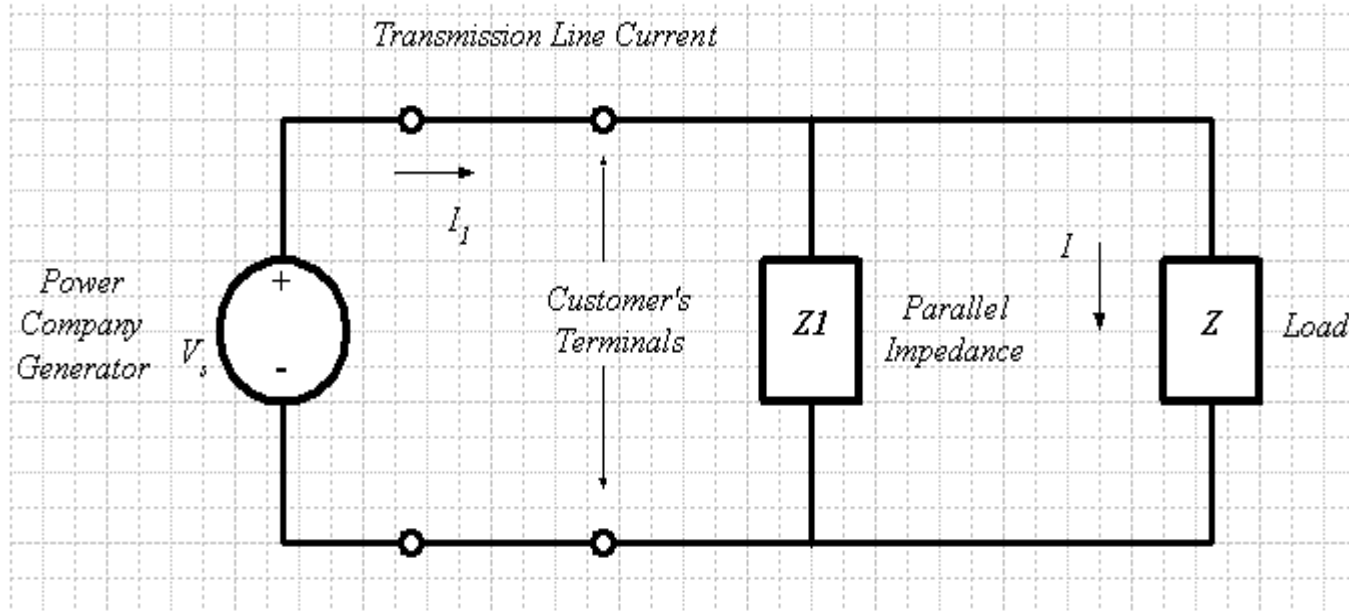
## Real, Reactive, and Apparent Power Related



- Power Triangle above drawn for R-L load so if  $0 < \theta < 90$  we say power factor is “lagging”, since  $I$  lags  $V$  with an inductive load

# Sinusoidal Systems

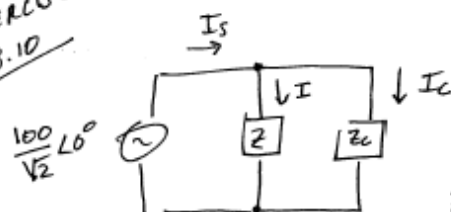
## Power Factor Correction



- Utility wants to provide minimum current necessary to meet customer Real (Average) power requirements
  - Transmission lines have maximum capacity
  - Higher current means more  $I^2 \cdot R$  losses
- Utilities will place capacitor banks in parallel with user loads at the remote location to make load look resistive to their generators

## Exercise 3.10

EXERCISE  
3.10



BOOK SOLUTION

$$Z_L = 2.12 \angle -62.24^\circ$$

$$Z = .5 \angle 41.41^\circ$$

$$I = \frac{\left(\frac{100}{\sqrt{2}}\right)^2}{Z} = 142.42 \angle 41.41^\circ$$

⌋

$$I_S = I_L + I$$

$$= 142.42 \angle 41.41^\circ + \frac{\frac{100}{\sqrt{2}} \angle 0^\circ}{2.12 \angle -62.24^\circ}$$

$$= 142.42 \angle 41.41^\circ + 33.35 \angle 62.24^\circ$$

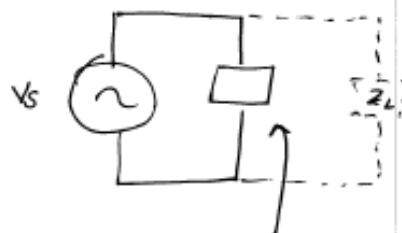
$$= 138.4 \angle -27.9^\circ$$

⌋

$$S = V \cdot I^* = \frac{100}{\sqrt{2}} \angle 0^\circ \cdot (138.4 \angle -27.9^\circ)^*$$

$$= 9786.2 \angle 27.86^\circ \rightarrow \cos \theta = .88$$

$$V_s = 100 \cdot \sin \omega t$$



10 kVA

pf = .75 lagging

$$\theta = \cos^{-1}.75 = \underline{41.41^\circ}$$

$$S = 10 \text{ kVA} \angle 41.41^\circ$$

$$P = 7500 \text{ W} \quad Q = 6614.4 \text{ VARs}$$

$$= |S| \cdot \cos \theta \quad = |S| \sin \theta$$

$$\frac{V^2}{X_c} = Q = 6614 = \frac{\left(\frac{100}{\sqrt{2}}\right)^2}{X_c}$$

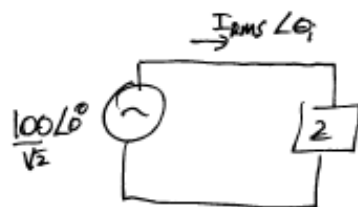
$$X_c = \frac{\left(\frac{100}{\sqrt{2}}\right)^2}{6614.4} = .7559$$

↑  
for CORRECTION TO  
w/ the pf

ADD PARALLEL  
CONNECTED LOAD  
 $Z_L$  TO CORRECT  
pf = .97

$$S = 10000 \angle 41.41^\circ$$

$$= 7500 + j 6614.4 \text{ VA}$$



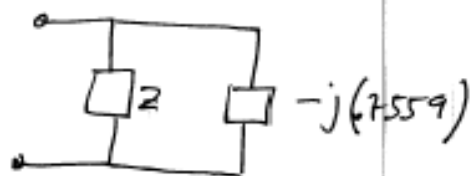
$$V \cdot I_{RMS}^* = 10000 \angle 41.41^\circ$$

$$I_{RMS}^* = \frac{10000}{\frac{100}{\sqrt{2}}} \angle 41.41^\circ$$

$$I_{RMS} = 141.42 \angle -41.41^\circ$$

$$Z = \frac{V}{I} = \frac{70.7 \angle 0^\circ}{142.42 \angle -41.41^\circ}$$

$$Z = .5 \angle 41.41^\circ$$



$$|Z| = \frac{V^2}{|S|} = \frac{(100)^2}{\frac{10000}{V_2}} = .5$$

$$Z = .5 \angle +41.41^\circ = .37499 + j.33072$$

$$Z_T = Z \parallel Z_C = .5 \angle +41.41^\circ \parallel 7.559 \angle -90^\circ$$

$$.5 \angle +41.41^\circ + .7559 \angle -90^\circ$$

$$= .667 \angle 0^\circ$$

$$\text{ASSUME } \omega = 2\pi \cdot 60 = 377$$

$$X_C = .7559 = \frac{1}{377 \cdot C} \Rightarrow C = 3.51 \mu\text{F}$$

$$R = .375$$

$$X_L = .331 = \omega L = 377 \cdot L$$

$$L = .878 \text{ mH}$$

for correction to .97 pf

✓

$$7500 + j(Q) = S$$

$$\cos \theta = .97 \quad \theta = 14^\circ$$

$$\tan 14^\circ = \frac{Q}{7500} \quad Q = 1879.7 \text{ VARs}$$

$$S = 7731.9 \angle 14^\circ$$

$$= 7500 + j1879.7$$

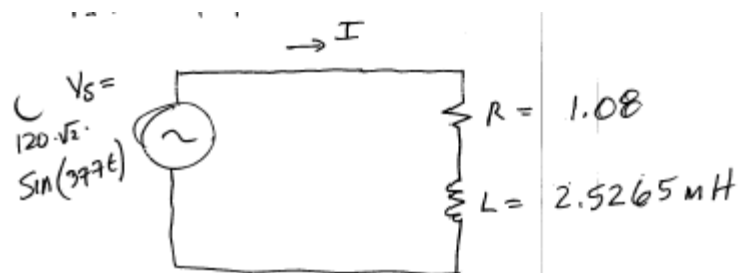
so C must  
supply  
 $6614.4 - 1879.7$   
 $= 4734.4$

$$X_C = \frac{\left(\frac{100}{\sqrt{2}}\right)^2}{4734.4} = 1.056$$

$$Z_C = -j 1.056$$

$$C \Rightarrow X_C = \frac{1}{\omega C} \quad C = \frac{1}{\omega X_C} = 2.512 \text{ mF}$$

# P.F. Example

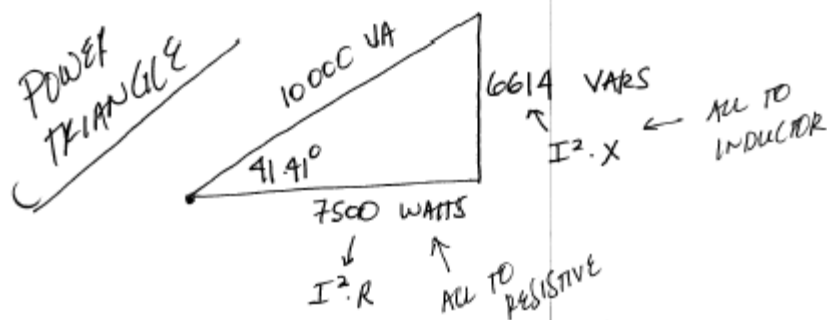
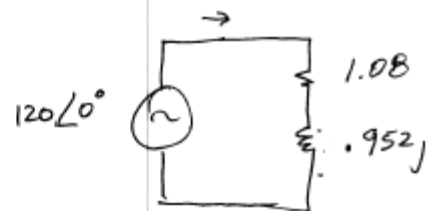


$\Rightarrow$  CONVERT TO FREQUENCY DOMAIN  $\Rightarrow$

$$I = \frac{120 \angle 0^\circ}{1.08 + j1.952}$$

$$= 83.33 \angle -41.41^\circ \leftarrow \text{ARMS}$$

$$S = V \cdot I^* = 10,000 \angle 41.41^\circ$$



MUST CONNECT CAPACITOR IN PARALLEL  
TO CANCEL INDUCTIVE EFFECT OF LOAD  
(TO SUPPLY REACTIVE PWR)

$$Q_{\text{LOAD}} = 6614 \text{ VARS (REACTIVE PWR REQUIRED)}$$

$$Q_C \text{ MUST} = -6614 \text{ VARS (REACTIVE PWR SUPPLIED)}$$

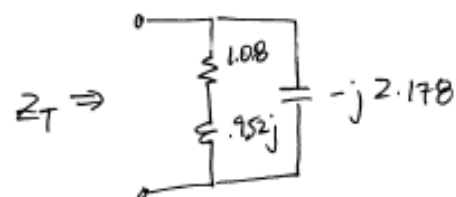
$$6614 = \frac{V^2}{X_C} = V^2 \omega C = 120^2 \cdot 2\pi f \cdot C$$

$$C = 1.218 \text{ mF}$$

CHECK

$$Z_C = \frac{-j}{\omega C} = -j \cdot \frac{1}{2\pi f \cdot 1.218 \text{ mF}}$$

$$= -j 2.178$$



$$Z_T = \frac{(1.08 + j.952) \cdot (-j 2.178)}{(1.08 + j.952 - j 2.178)}$$

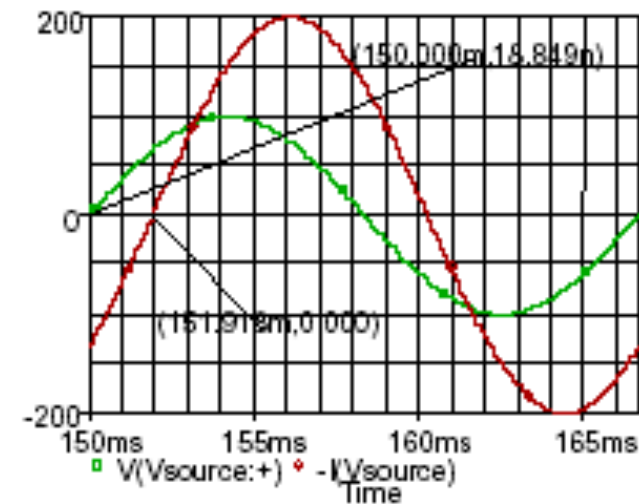
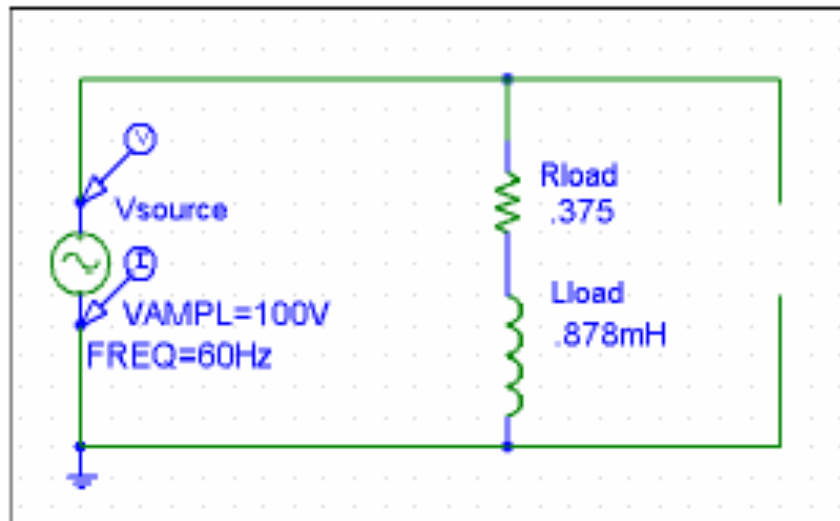
$$= 1.919 \angle 0^\circ$$

$$I = \frac{120 \angle 0^\circ}{1.919 \angle 0^\circ} = 62.53 \text{ A}_{\text{RMS}}$$

# Sinusoidal Systems

## Power Factor Correction

### Exercise 3.10



$$\omega := 2 \cdot \pi \cdot 60$$

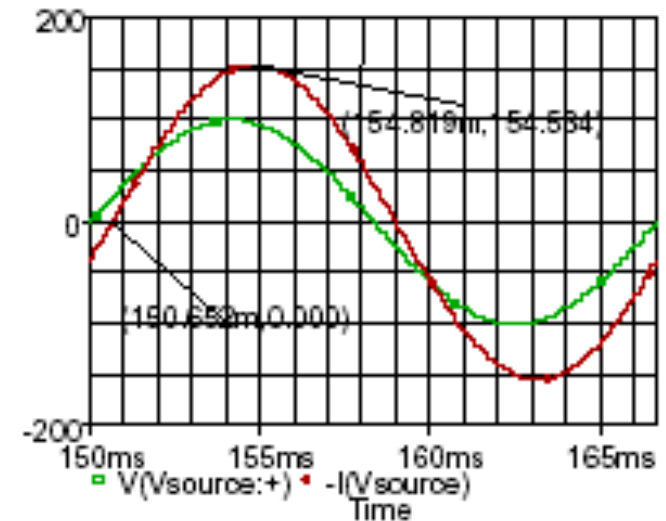
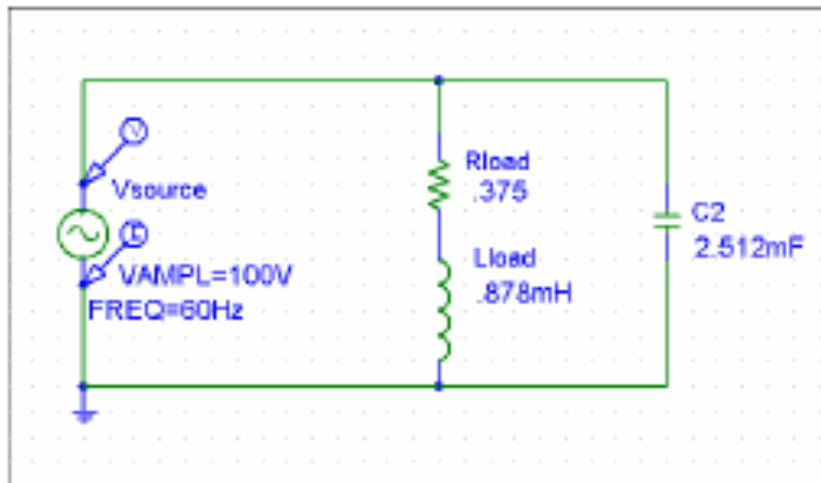
$$1.918 \cdot 10^{-3} \cdot \omega \cdot \frac{180}{\pi} = 41.429$$

$$\frac{100}{\sqrt{2}} \cdot \frac{200}{\sqrt{2}} = 1 \times 10^4$$

# Sinusoidal Systems

## Power Factor Correction

I - Correct PF to 0.97



$$\omega := 2 \cdot \pi \cdot 60$$

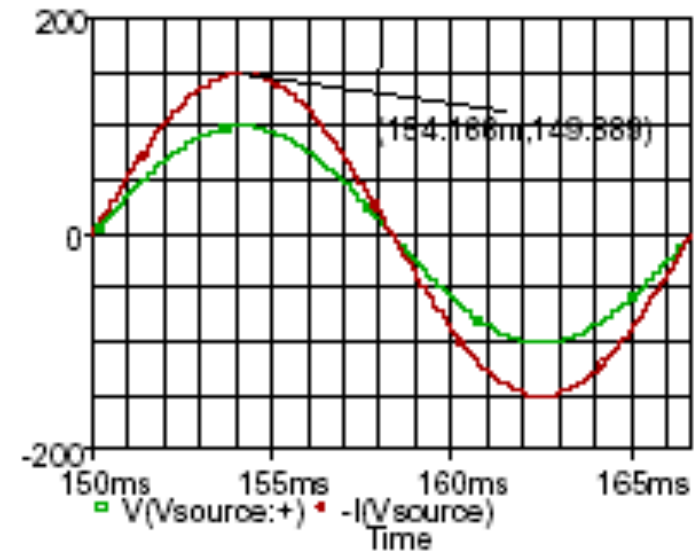
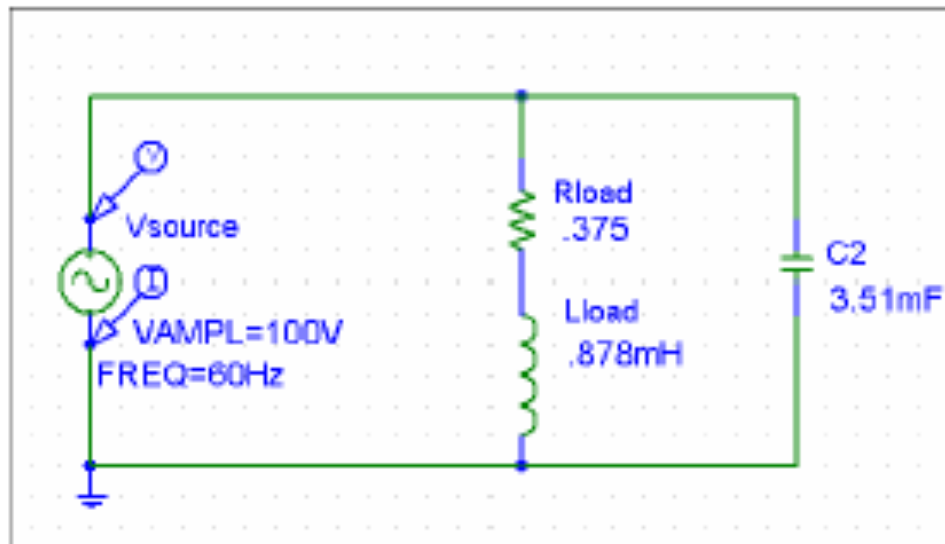
$$.652 \cdot 10^{-3} \cdot \omega \cdot \frac{180}{\pi} = 14.083$$

$$\frac{100}{\sqrt{2}} \cdot \frac{154.534}{\sqrt{2}} = 7.727 \times 10^3$$

# Sinusoidal Systems

## Power Factor Correction

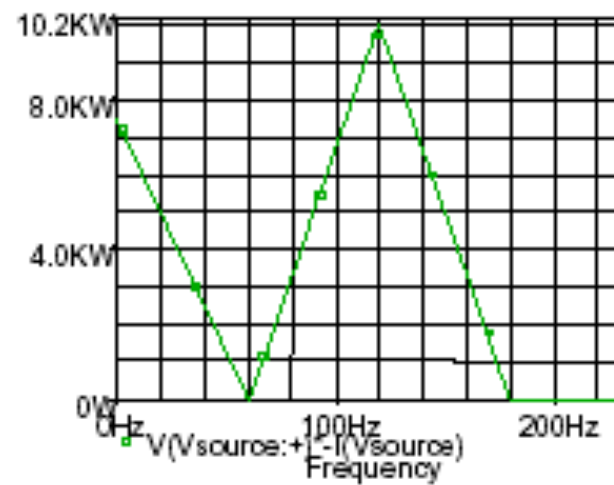
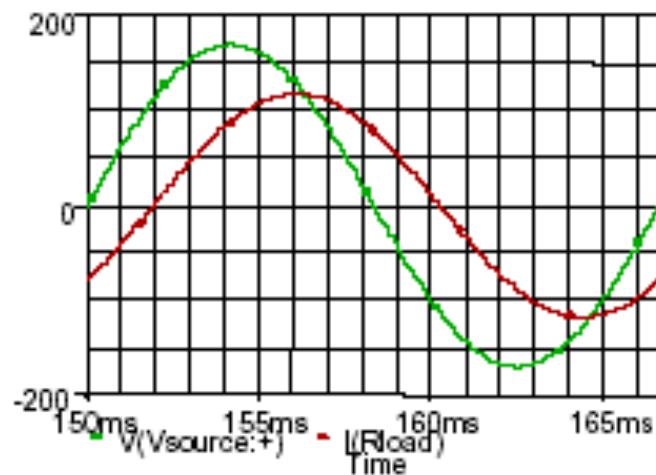
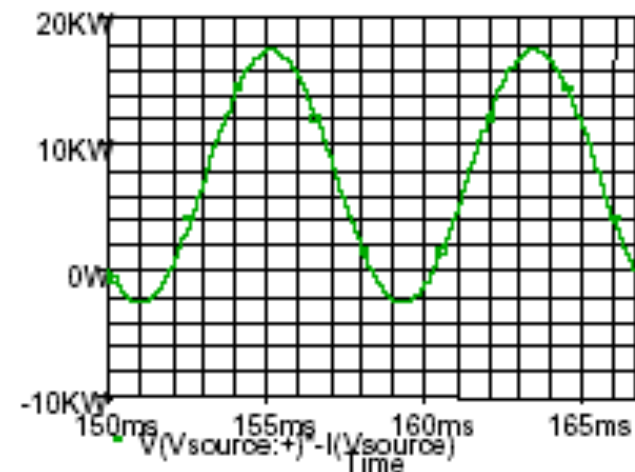
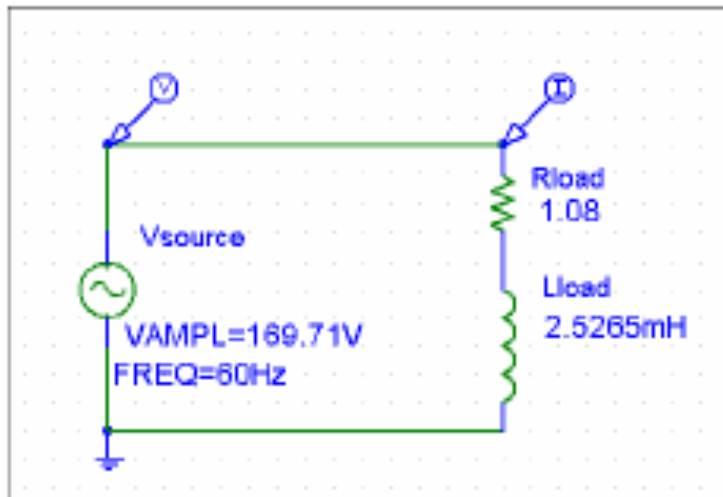
II - Correct PF to 1.0



$$\frac{100}{\sqrt{2}} \cdot \frac{150}{\sqrt{2}} = 7.5 \times 10^3 \text{ W}$$

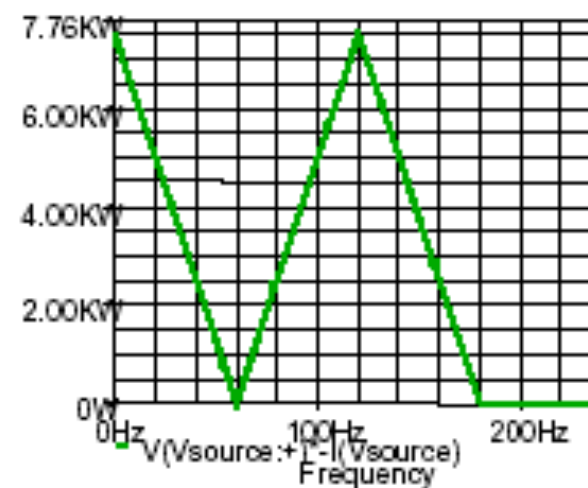
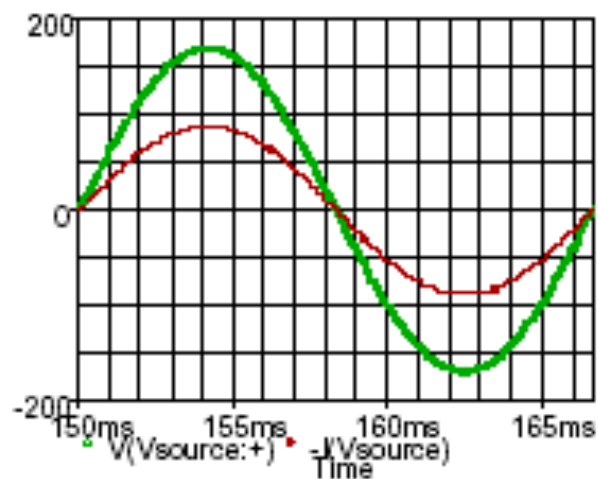
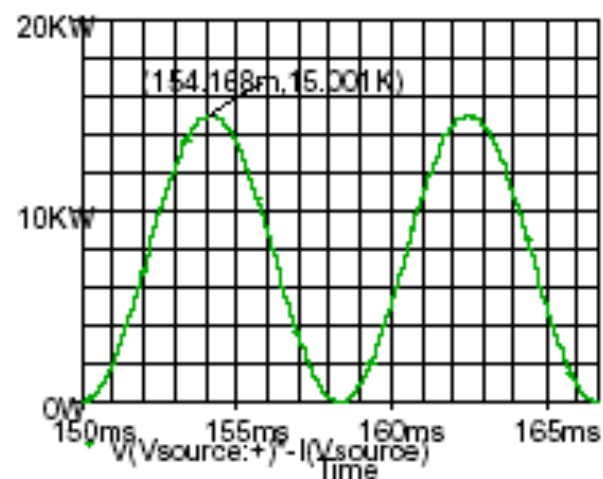
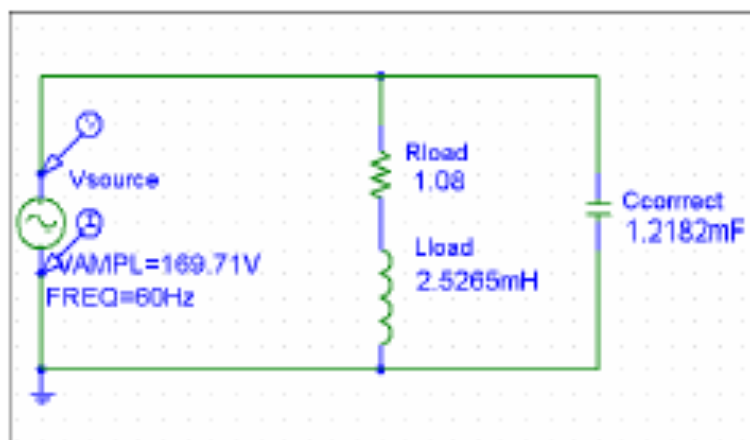
# Sinusoidal Systems

## Power Factor Correction



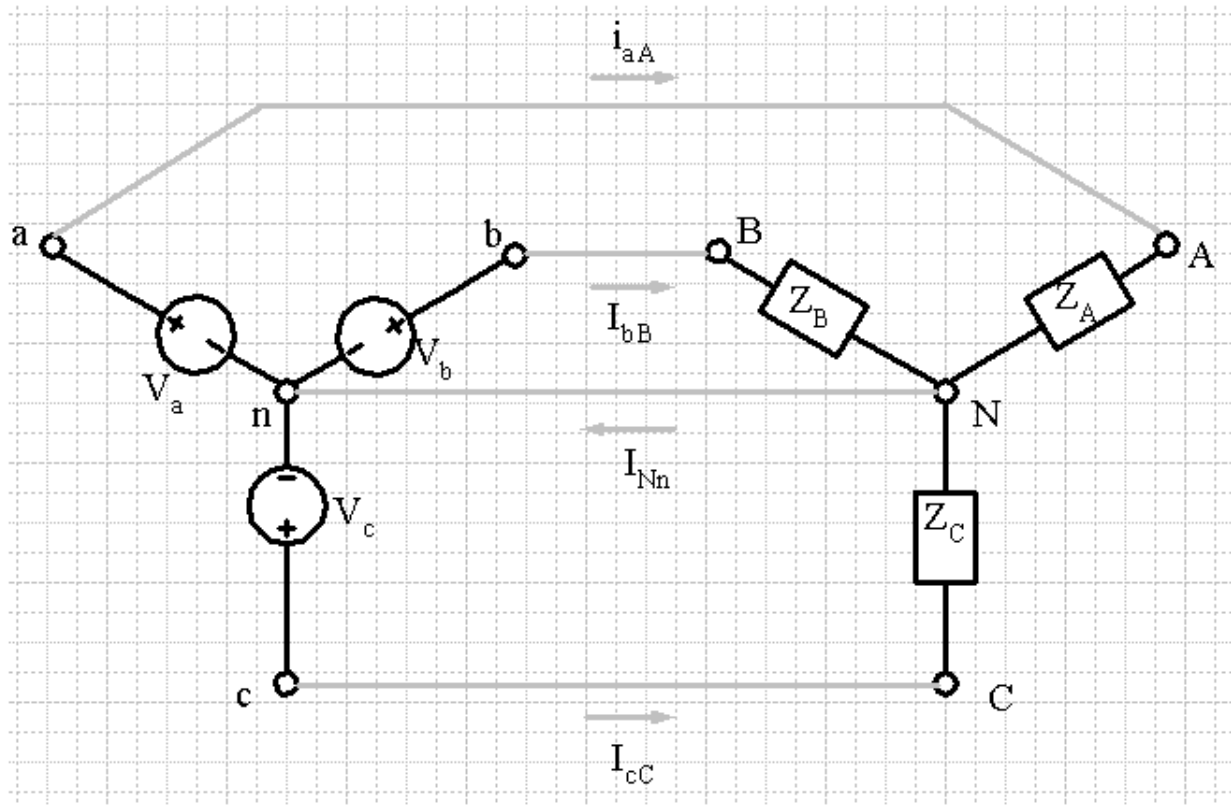
# Sinusoidal Systems

## Power Factor Correction



# Sinusoidal Systems

## Neutral Currents in 3 Phase 4 Wire Systems



- A balanced three phase load draws equal currents, each separated by 120 degrees as the source voltages are separated by 120 degrees

# Sinusoidal Systems

## Neutral Currents in 3 Phase 4 Wire Systems

EEL 5245

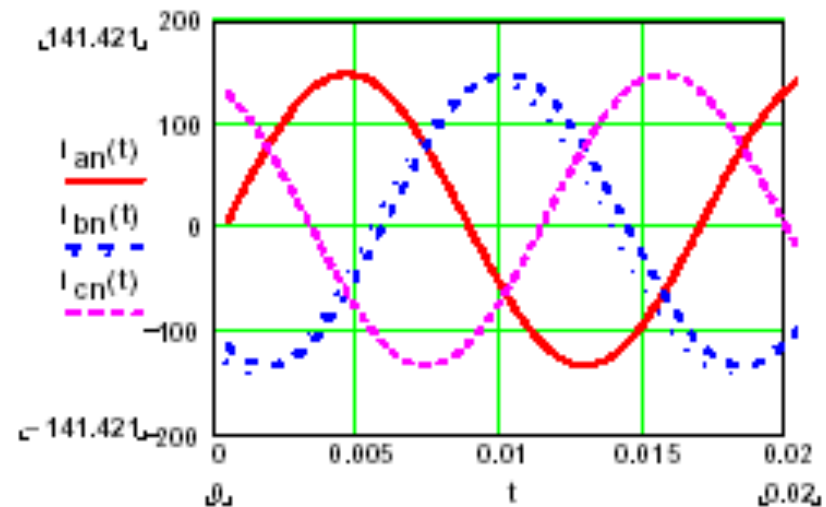
$$I_{\text{rms}} := 100$$

$$i_{\text{an}}(t) := I_{\text{rms}} \cdot \sqrt{2} \cdot \sin(377 \cdot t - 0)$$

$$i_{\text{bn}}(t) := I_{\text{rms}} \cdot \sqrt{2} \cdot \sin\left(377 \cdot t - \frac{2 \cdot \pi}{3}\right)$$

$$i_{\text{cn}}(t) := I_{\text{rms}} \cdot \sqrt{2} \cdot \sin\left(377 \cdot t - \frac{4 \cdot \pi}{3}\right)$$

$$i_{\text{n}}(t) := (i_{\text{an}}(t) + i_{\text{bn}}(t) + i_{\text{cn}}(t)) = 0$$



- The neutral current (by KCL) is the sum of the three phase currents for wye connected load
- If the three loads are “balanced”, neutral current is zero