## The ZCS Boost Converter

The boost-quasi-resonant converter with an M-type switch as shown in Fig. 6.13(a), with its equivalent circuit shown in Fig. 6.13(b).

(a)

(b)

Fig 6.13 (a) ZCS boost converter with M-type switch. (b) Simplified equivalent circuit.

## ZCS Boost: Equivalent Circuit Modes



Fig 6.14 (a) Equivalent circuit for mode I. (b) Equivalent circuit for mode II. (c) Equivalent circuit for mode III. (d) Equivalent circuit for mode IV.

## ZCS Boost Converter: Steady-State Analysis

## Mode I [ $0 \leq \boldsymbol{t}<\mathrm{t}_{1}$ ]:

Assume switch and the diode are both ON
The output voltage is given by

$$
V_{o}=L \frac{d i_{L}}{d t}
$$

The initial inductor current and capacitor voltage,

$$
i_{l}(0)=0 \quad v_{c}(0)=V_{o}
$$

Integrating Eq. (6.34), the inductor current becomes,

$$
i_{L}(t)=\frac{V_{o}}{L} t+i_{L}(0)=\frac{V_{o}}{L} t .
$$

When the resonant inductor current reaches the input current, $I_{i n}$, the diode turns OFF,

$$
\frac{V_{o}}{L} t_{1}=I_{i n}
$$

with $\mathrm{t}_{1}$ given by,

$$
t_{1}=\frac{I_{i n} L}{V_{o}}
$$

At $\mathrm{t}=\mathrm{t}_{1}$, the diode turns $O F F$ since $i_{L=I_{i n}}$, and the converter enters Mode II.

## Steady-State Analysis (cont'd)

## Mode II [ $\mathbf{t}_{1} \leq \mathbf{t}<\mathrm{t}_{\mathbf{2}}$ ]:

The switch remains closed, but the diode is $O F F$ at Mode II as shown in Fig. 6.14(b). This is a resonant mode during which the capacitor voltage starts decreasing resonantly from its initial value of Vo. When $i_{L}=I_{\text {in }}$, the capacitor reaches its negative peak. At $\mathrm{t}=\mathrm{t}_{2} . i_{L}$ equals zero, and the switch turns $O F F$, hence, switching at zero-current.

The initial conditions,

$$
v_{c}\left(t_{1}\right)=V_{o} \quad i_{L}\left(t_{1}\right)=I_{o}
$$

From Fig. 6.14(b), the first derivatives for $i_{\mathrm{L}}$ and $v_{\mathrm{c}}$ are,

$$
\begin{gathered}
L \frac{d i_{L}}{d t}=v_{c} \\
C \frac{d v_{c}}{d t}=I_{i n}-i_{L}
\end{gathered}
$$

Using the same solution technique used in the buck converter to solve the above differential equations, the expression for $i_{L}(\mathrm{t})$

$$
\begin{align*}
& i_{L}(t)=I_{i n}+\frac{V_{o}}{Z_{o}} \sin \omega_{o}\left(t-t_{1}\right)  \tag{6.37}\\
& v_{c}(t)=V_{o} \cos \omega_{o}\left(t-t_{1}\right) \tag{6.38}
\end{align*}
$$

where $\omega_{o}=\frac{1}{\sqrt{L C}}$
At , $t=t_{2} \quad i_{L}\left(t_{2}\right)=0$ and the time interval can be obtained from evaluating Eq. (6.37) at $t=t_{2}$
to yield,

$$
\begin{equation*}
\left(t_{2}-t_{1}\right)=\frac{1}{\omega_{o}} \sin ^{-1}\left(-\frac{I_{i n} Z_{o}}{V_{o}}\right) \quad=\frac{1}{\omega_{o}}\left[\pi+\sin ^{-1}\left(\frac{I_{i n} Z_{o}}{V_{o}}\right)\right] \tag{6.39}
\end{equation*}
$$

## Steady-State Analysis (cont'd)

## Mode III [ $\mathbf{t}_{2} \leq \mathbf{t}<\mathbf{t}_{3}$ ]:

Mode III starts at $t$, and the switch and the diode are both open as shown in Fig. 6.14(c). Since $v_{c}$ is constant, the capacitor starts charging up by the input current source. The capacitor voltage,

$$
\begin{align*}
v_{c} & =\frac{1}{C} \int_{t_{2}}^{t} I_{i n} d t \\
& =\frac{I_{i n}}{C}\left(t-t_{2}\right)+v_{c}\left(t_{2}\right) \tag{6.40}
\end{align*}
$$

The diode begins conducting at $t=t_{3}$ when the capacitor voltage is equal to the output voltage, i.e. $v_{\mathrm{c}}\left(\mathrm{t}_{3}\right)=\mathrm{V}_{\mathrm{o}}$.

$$
V_{o}=\frac{I_{\text {in }}}{C}\left(t_{3}-t_{2}\right)+v_{c}\left(t_{2}\right)
$$

Time interval in this period

$$
\begin{equation*}
t_{3}-t_{2}=\frac{I_{i n}}{C}\left[V_{o}-v_{c}\left(t_{2}\right)\right] \tag{6.41}
\end{equation*}
$$

## Mode IV $\left[\mathbf{t}_{\mathbf{3}} \leq \mathbf{t}<\mathbf{t}_{\mathbf{4}}\right.$ :

At $t_{3}$, the capacitor voltage is clamped to the output voltage, and the diode starts conducting again. The cycle of the mode will repeat again at the time of $T_{s}$ when S is turned $O N$ again

## ZCS Boost-Typical Steady-State Waveforms

Typical steady state waveforms are shown in Fig. 6.15.


Fig 6.15 Steady-state waveforms of the boost converter with M-type switch.

## ZCS Boost Converter

## Voltage Gain

Conservation of energy per switching cycle to express the voltage gain, $M=V_{o} / V_{\text {in }}$
The input energy is,

$$
\begin{equation*}
E_{i n}=V_{i n} I_{i n} T_{s} \tag{6.42}
\end{equation*}
$$

The output energy,

$$
\begin{equation*}
E_{o}=\int_{0}^{T_{s}} i_{o} V_{o} d t \tag{6.43}
\end{equation*}
$$

The output current equals $i_{o}=\mathrm{I}_{\mathrm{in}}-i_{L}$ and $i_{o}=\mathrm{I}_{\mathrm{in}}$ for intervals $0 \leq \mathrm{t} \leq \mathrm{t}_{1}$ and $\mathrm{t}_{3} \leq \mathrm{t}<\mathrm{T}_{\mathrm{s}}$,

$$
\begin{equation*}
E_{o}=\int_{0}^{t_{1}}\left(I_{i n}-i_{L}\right) V_{o} d t+\int_{t_{3}}^{T_{s}} I_{i n} V_{o} d t \tag{6.44}
\end{equation*}
$$

The input current is obtained from the conservation of output power as:

$$
I_{i n}=\frac{V_{o}^{2}}{V_{i n} R}
$$

## ZCS Boost-Voltage Gain

Substituting for the input current and by evaluating Eq. (6.44), the output energy becomes

$$
\begin{align*}
E_{o} & =V_{o} \int_{0}^{t_{1}}\left(I_{i n}-\frac{V_{o}}{L} t\right) d t+I_{i n} V_{o}\left(T_{s}-t_{3}\right) \\
& =V_{o}\left(I_{i n} t_{1}-\frac{1}{2} \frac{V_{o}}{L} t_{1}^{2}\right)+I_{i n} V_{o}\left(T_{s}-t_{3}\right) \tag{6.45}
\end{align*}
$$

with $t_{1}=\frac{I_{i n} L}{V_{o}}$ and $\left(T_{s}-t_{3}\right)=T_{s}-\left[t_{1}+\left(t_{2}-t_{1}\right)+\left(t_{3}-t_{2}\right)\right]$, and use the equations for $\left(t_{3}-t_{2}\right)$ and $\left(t_{2}-t_{1}\right)^{\circ}$ from Eqs. (6.39) and (6.41), Eq. (6.45) becomes,

$$
\begin{equation*}
E_{o}=-\frac{1}{2} I_{i n}^{2} L+V_{o} I_{i n}\left[T_{s}-\frac{\alpha}{\omega_{o}}-\frac{C}{I_{i n}} V_{o}(1-\cos \alpha)\right] \tag{6.46}
\end{equation*}
$$

The voltage gain expression is given by,

$$
\begin{equation*}
\frac{M-1}{M}=\frac{f_{n s}}{2 \pi}\left[\frac{M}{2 Q}+\alpha+\frac{Q}{M}(1-\cos \alpha)\right] \tag{6.47}
\end{equation*}
$$

where, $\alpha, \mathrm{M}, \mathrm{I}_{\mathrm{o}}$ and $f_{n s}$ are given as before.

## ZCS Boost-Control Characteristic Curve

Fig 6.16 shows the characteristic curve for M vs. $f_{n s}$ as a function of the normalized load.


Fig 6.16 Characteristic curve for $\boldsymbol{M}$ vs. $f_{\text {ns }}$ for the boost ZCS converter.

## ZCS Boost Converter

## Example 6.2

Design a boost ZCS converter for the following parameters: $\mathrm{V}_{\mathrm{in}}=20 \mathrm{~V}, \mathrm{~V}_{\mathrm{o}}=40 \mathrm{~V} . \mathrm{P}_{\mathrm{o}}$ $=20 \mathrm{~W}, \mathrm{f}_{\mathrm{s}}=250 \mathrm{kHz}$.

## Solution:

Solution:
The voltage gain is $M=\frac{V_{o}}{V_{i n}}=\frac{40}{20}=2$ Let us select $f_{n s}=0.38$. From the characteristic curve of Fig. 6.16, Q can be approximated to 6.0
The characteristic impedance is given by,

$$
\begin{equation*}
Z_{o}=\frac{R_{o}}{Q}=\frac{80 \Omega}{6}=13.33 \Omega \tag{6.48}
\end{equation*}
$$

Resonant frequency is,

$$
\begin{align*}
& f_{o}=\frac{f_{s}}{f_{n s}} \\
& f_{o}=\frac{250 \mathrm{kHz}}{0.38}=657.89 \mathrm{kHz} \tag{6.49}
\end{align*}
$$

Solve Eqs. (6.48) and (6.49) for L and C

$$
\begin{aligned}
& L=\frac{Z_{o}}{2 \pi f_{o}}=\frac{13.33 \Omega}{2 \pi \times 657.89 \times 10^{3}}=3.22 \times 10^{-6} \mathrm{H} \\
& C=\frac{1}{Z_{o} \omega_{o}}=\frac{1}{(13.33)\left(2 \pi \times 657.89 \times 10^{3}\right)}=18.14 \mathrm{nF}
\end{aligned}
$$

## ZCS Boost Converter

## Example 6.3

Design a boost converter with ZCS, with the following design parameters: $\mathrm{V}_{\text {in }}=25 \mathrm{~V}$, $\mathrm{P}_{0}=30 \mathrm{~W}$ at $\mathrm{I}_{0}=0.5 \mathrm{~A}$, and $f_{\mathrm{s}}=100 \mathrm{kHz}$. Assume the output voltage ripple $\Delta \mathrm{V}_{0}$ is $0.2 \%$

## Solution:

The load resistance, $R=\frac{P_{0}}{I_{0}^{2}}=\frac{30}{(0.5)^{2}}=120 \Omega$
$M=\frac{V_{0}}{V_{s}}=\frac{60}{25}=2.4$,
From the characteristic curve of Fig. 6.15, approximate Q to 6 when we assume $f_{n}=0.58$.

$$
f_{o}=\frac{100}{0.58}=172.4 k H z .
$$

From Q and $\mathrm{R}_{\mathrm{o}}$, the characteristic impedance is obtained from,

$$
\begin{aligned}
& Q=\frac{R_{0}}{Z_{0}}=\frac{120}{Z_{0}}=6, \text { and } Z_{0}=20 \quad \sqrt{\frac{L}{C}}=20 \\
& \omega_{o}=2 \pi\left(172 \times 10^{3}\right)=1080.7 \times 10^{3} \mathrm{rad} / \mathrm{sec} \\
& \sqrt{\frac{1}{L C}}=1080.7 \times 10^{3}
\end{aligned}
$$

Solving for C and L

$$
\begin{aligned}
& C=46.27 \eta F \\
& L=18.51 \mu H
\end{aligned}
$$

## Example 6.3 (cont'd)

The time intervals are given by

$$
\begin{aligned}
t_{1} & =\frac{L_{r} I_{i}}{V_{0}}=\frac{18.51 \times 10^{-3} \times 1.2}{60}=0.370 \mu s \\
t_{2} & -t_{1}=\frac{1}{\omega_{o}} \sin ^{-1}\left[\frac{-Z_{0} I_{i}}{V_{0}}\right] \\
& =\frac{1}{1080.7 \times 10^{3}} \sin ^{-1}\left[-\frac{20 \times 1.2}{60}\right]=3.29 \mu s \\
t_{3}- & t_{2}=\frac{1}{\omega_{n}} \frac{V_{0}}{Z_{0} I_{i}}(1-\cos \alpha) \\
& =\frac{1}{1080.7 \times 10^{3}} \frac{60}{20 \times 1.2}(1-\cos 3.553) \\
& =0.193 \mu s \\
t_{4}- & t_{3}=T-t_{1}-\left(t_{12}\right)-\left(t_{23}\right) \\
& =10-0.370-3.29-0.193=6.147 \mu s
\end{aligned}
$$

## Other ZCS Boost Converter

Figure 6.17(a) shows the quasi-resonant boost converter by using the L-type resonant switch, and the simplified circuit and its steady state waveforms are shown in Fig.6. 17(b) and (c), respectively.
(a)


(c)

Fig 6.17 (a) ZCS boost converter with L-type switch. (b) Simplified equivalent circuit. (c) Steady-state waveforms.

