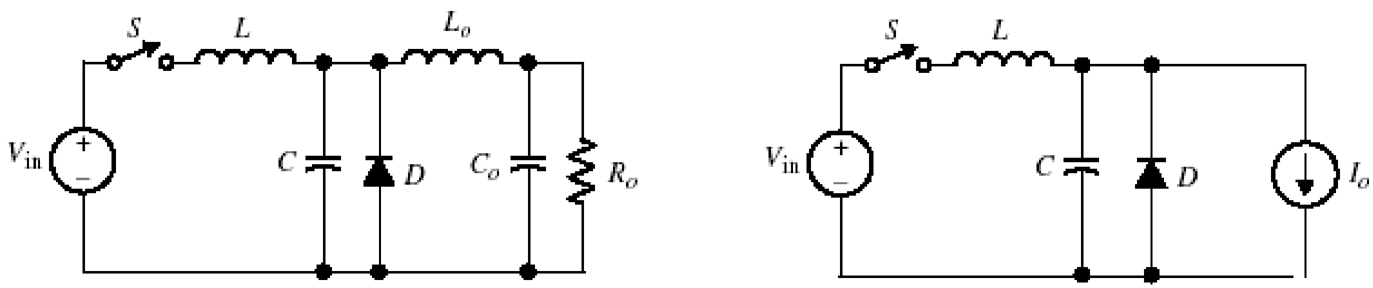
The Buck Resonant Converter

- Replacing the switch by the resonant-type switch, to obtain a quasi-resonant PWM buck converter
- It can be shown that there are four modes of operation under the steady-state condition



(a) (b)

Fig 6.8 (a) Conventional buck converter with L-type resonant switch. (b) Simplified equivalent circuit.

Buck Converter: Equivalent Modes

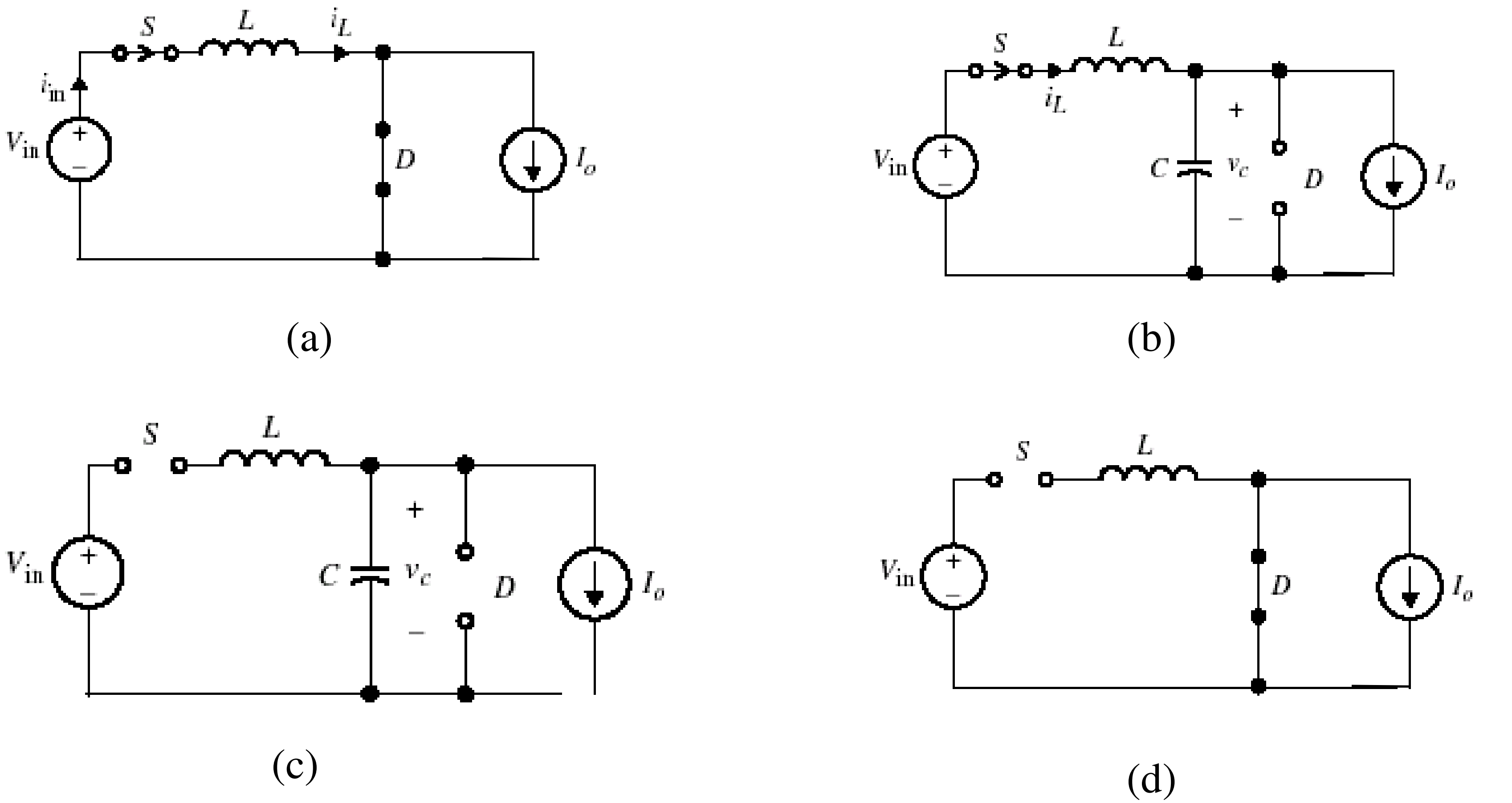


Fig 6.9 (a) Equivalent circuit for Mode I. (b) Equivalent circuit for mode II. (c) Equivalent circuit for mode III. (d) Equivalent circuit for mode IV.

Buck Converter: Steady-State Analysis

Mode I [$0 \le t < t_1$]

- Mode I starts at t = 0 when S is turned *ON*
- Assume for t > 0, both S and D are *ON*
- The capacitor voltage, v_c , is zero and the input voltage is equal to the inductor voltage

$$V_{in} = L\frac{di_L}{dt}$$

- The inductor current, i_L, is given by

$$i_L(t) = \frac{V_{in}}{L}t$$

- As long as the inductor current is less than I_o, the diode will continue conducting and the capacitor voltage remains at zero.

$$I_o = \frac{V_{in}}{L}t_1 \qquad (6.3)$$

- Hence, the time interval = t_1 is given by

$$\Delta t_1 = t_1 = \frac{LI_o}{V_{in}} \qquad (6.4)$$

- This is the inductor current charging state.

Steady-State Analysis (cont'd)

Mode II [$t_1 \le t < t_2$]

- Mode II starts at t_1, diode is open resonant stage between L and C
- The first-order differential equations that represent this mode are

$$C\frac{dv_c}{dt} = i_L - I_o \quad (6.5a)$$

$$L\frac{di_L}{dt} = V_{in} - v_c \quad (6.5b)$$

- Inductor current is given by

$$\frac{d^2 i_L}{dt^2} + \frac{1}{LC} i_L = \frac{I_0}{LC} \quad (6.6)$$

- The general solution for $i_L(t)$ is given by

$$i_L(t) = A_1 \sin \omega_o (t - t_1) + A_2 \cos \omega_o (t - t_1) + A_3 \quad (6.7)$$

- Resonant angular frequency and constants:

$$\omega_o = \sqrt{\frac{1}{LC}} \qquad A_1 = \frac{V_{in}}{L\omega_o}$$

- $A_2 = 0$, $A_3 = I_o$

Steady-State Analysis (cont'd)

Equations i_L and v_c are given by

$$i_L(t) = I_o + \frac{V_{in}}{Z_o}\sin\omega_o(t-t_1) \quad (6.8)$$

$$v_c(t) = V_{in}\left[1-\cos\omega_o(t-t_1)\right] \quad (6.9)$$

$Z_o = \sqrt{\frac{L}{C}}$ is known as the characteristic impedance

The time interval in this mode can be derived at $t = t_2$ by setting $i_L(t_2) = 0$,

$$i_l(t_2) = I_o + \frac{V_{in}}{Z_o}\sin\omega_o(t_2-t_1) \quad (6.10)$$

$$= 0$$

therefore,

$$\Delta t_2 = t_2 - t_1 = \frac{1}{\omega_o}\sin^{-1}\frac{-Z_o I_o}{V_{in}} = \frac{1}{\omega_o}\left(\pi + \sin^{-1}\frac{Z_o I_o}{V_{in}}\right) \quad (6.11)$$

Mode III starts at $t = t_2$, when the switch is turned *OFF*.

Steady-State Analysis (cont'd)

Mode III $[t_2 \le t < t_3]$:

At t_2, the inductor current becomes zero, and the capacitor linearly discharges from $v_c(t_2)$ to zero during t_2 to t_3.

The capacitor current equals to I_o as given by,

$$i_C = C\frac{dv_c}{dt} = -I_o \qquad (6.12)$$

The capacitor voltage $v_c(t)$ is obtained from Eq. (6.12) from t_2 to t_3 with $V_c(t_2)$ as the initial value,

$$v_c(t) = \frac{-I_o}{C}(t - t_2) + V_c(t_2) \qquad (6.13)$$

The initial value $V_c(t_2)$ is obtained from previous mode,

$$V_c(t_2) = V_{in}\left[1 - \cos\omega_o(t_2 - t_1)\right] \qquad (6.14)$$

Substituting Eq. (6.14) into Eq. (6.13),

$$v_c(t) = \frac{-I_o}{C}(t - t_2) + V_{in}\left[1 - \cos\omega(t_2 - t_1)\right] \qquad (6.15)$$

At $t = t_3$, the capacitor voltage becomes zero,

$$\Delta t_3 = t_3 - t_2 = \frac{C}{I_o}V_{in}\left[1 - \cos\omega_o(t_2 - t_1)\right] \qquad (6.16)$$

At this point, the diode turns *ON* and the circuit enters Mode IV.

Steady-State Analysis (cont'd)

Mode IV [$t_3 \le t < t_4$]:

At this mode the switch remains *OFF*, but the diode starts conducting at t = t_3. Mode IV continues as long as the switch is *OFF*, and the output current starts the free-wheeling stage through the diode.

Initial conditions

$$i_l(t) = 0$$

$$v_c(t) = 0$$

By turning *ON* the switch at t = T_s , the cycle repeats these four modes. The time Δt_4 is given by,

$$\Delta t_4 = T_s - \Delta t_1 - \Delta t_2 - \Delta t_3 \qquad (6.17)$$

Typical Steady-State Waveforms

Voltage Gain

The expression for the voltage gain, $M = \dfrac{V_o}{V_{in}}$

Figure 6.10 shows the steady state waveforms for v_c and i_L for the buck converter with L-type switch.

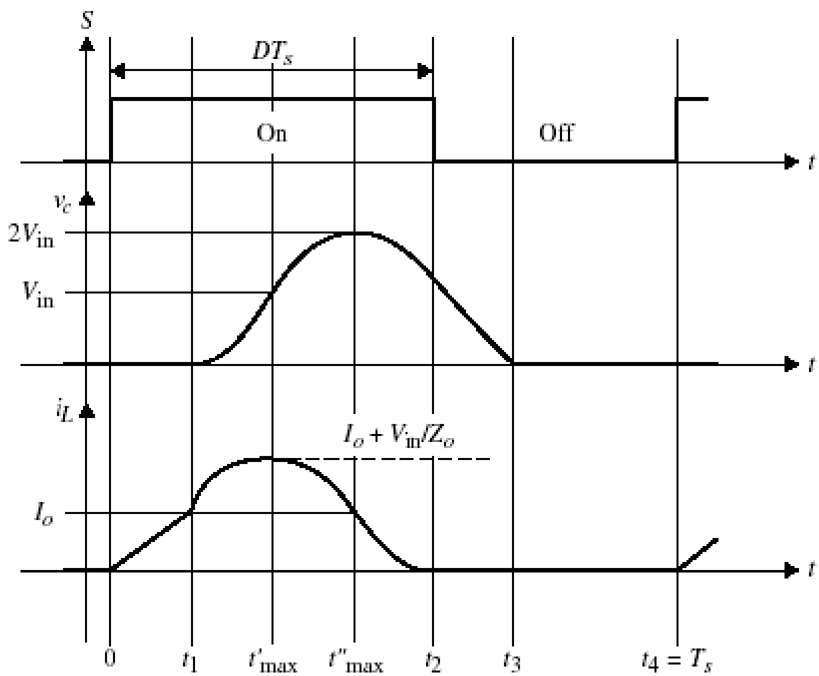


Fig 6.10 Steady-state current and voltage waveforms of buck L-type

Voltage Gain – 1st method

Substitute for $v_c(t)$ from intervals (t_2-t_1) and (t_3-t_2), to yield,

$$V_o = \frac{1}{T_s}\left[\int_{t_1}^{t_2} V_{in}(1-\cos\omega_o(t-t_1))dt + \int_{t_2}^{t_3}\left(\frac{-I_o}{C}(t-t_2)+V_c(t_2)\right)dt\right] \quad (6.18)$$

The voltage gain ratio is given by,

$$\frac{V_o}{V_{in}} = \frac{1}{T_s}\left[(t_2-t_1)-\frac{\sin\omega_o(t_2-t_1)}{\omega_o}-\frac{I_o}{V_{in}C}\frac{(t_3-t_2)^2}{2}+\frac{V_c(t_2)}{V_{in}}(t_3-t_2)\right] \quad \textbf{\textit{Book correction}}\ (6.19)$$

Substitute for (t_2-t_1), (t_3-t_2) and $V_C(t_2)$ from the above modes, a closed form expression for M in terms of the circuit parameters can be obtained.

Voltage Gain – 2nd method

The total input energy over one switching cycle,

$$E_{in} = \int_0^{T_s} i_{in} V_{in}\, dt \qquad (6.20)$$

Since i_{in} is equal to $i_L(t)$, Eq. (6.20) is rewritten as,

$$E_{in} = \int_0^{t_1} i_L(t) V_{in}\, dt + \int_{t_1}^{t_2} i_L(t) V_{in}\, dt \qquad (6.21)$$

Substituting for i_L(t) from Eqs. (6.2) and (6.8) into the above integrals, Eq. (6.21) becomes,

$$E_{in} = V_{in}\left\{\frac{V_{in}}{2L}t_1^2 + I_o(t_2 - t_1) - \frac{V_{in}}{Z_o\omega_o}\left[\cos\omega_o(t_2 - t_1) - 1\right]\right\} \qquad (6.22)$$ ***Book correction***

Substituting for $\cos\omega_o(t_2 - t_1) = 1 - \frac{I_o(t_3 - t_2)}{CV_{in}}$ from (6.16)

$$E_{in} = V_{in}\left\{\frac{t_1}{2} I_o + I_o(t_2 - t_1) + \frac{V_{in}}{Z_o\omega_o}\left[\frac{I_o(t_3 - t_2)}{CV_{in}}\right]\right\} \qquad (6.23)$$

with, $Z_o\omega_o = \frac{1}{C}$, Eq. (6.23) becomes,

$$E_{in} = V_{in} I_o\left[\frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2)\right] \qquad (6.24)$$

Voltage Gain (cont'd)

The output energy over one switching cycle is:

$$E_o = \int_0^{T_S} I_o V_o \, dt = I_o V_o T_s \quad (6.25)$$

Equating the input and output energy expressions

$$I_o V_o T_s = V_{in} I_o \left[\frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right] \quad (6.26)$$

From Eq. (6.26) the voltage gain is expressed by,

$$\frac{V_o}{V_{in}} = \frac{1}{T_s} \left[\frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right] \quad (6.27)$$

Substituting for t_1, (t_2-t_1) and (t_3-t_2) from previous equations, the voltage gain becomes

$$\frac{V_o}{V_{in}} = \frac{1}{T_s} \left\{ \frac{LI_o}{2V_{in}} + \frac{1}{\omega_o} \sin^{-1} \frac{-Z_o I_o}{V_{in}} + \frac{CV_{in}}{I_o} [1 - \cos \omega_o (t_2 - t_1)] \right\} \quad (6.28)$$

Normalization

To simplify and generalize the gain equation, the following normalized parameters are defined:

$M = \dfrac{V_o}{V_{in}}$	normalized output voltage	(6.29a)
$Q = \dfrac{R_o}{Z_o}$	normalized load	(6.29b)
$I_o = \dfrac{V_o}{R_o}$	average output current	(6.29c)
$f_{ns} = \dfrac{f_s}{f_o}$	normalized switching frequency	(6.29d)

By substituting Eq. (6.29) into Eq. (6.28), the final voltage gain is simplified into

$$M = \frac{f_{ns}}{2\pi}\left[\frac{M}{2Q} + \alpha + \frac{Q}{M}(1-\cos\alpha)\right] \quad (6.30)$$

where,

$$\alpha = \sin^{-1}\frac{-M}{Q} \quad (6.31)$$

Buck-Control Characteristic Curve

A plot of the control characteristic curve of M vs. f_{ns} under various normalized loads is given in Fig. 6.11

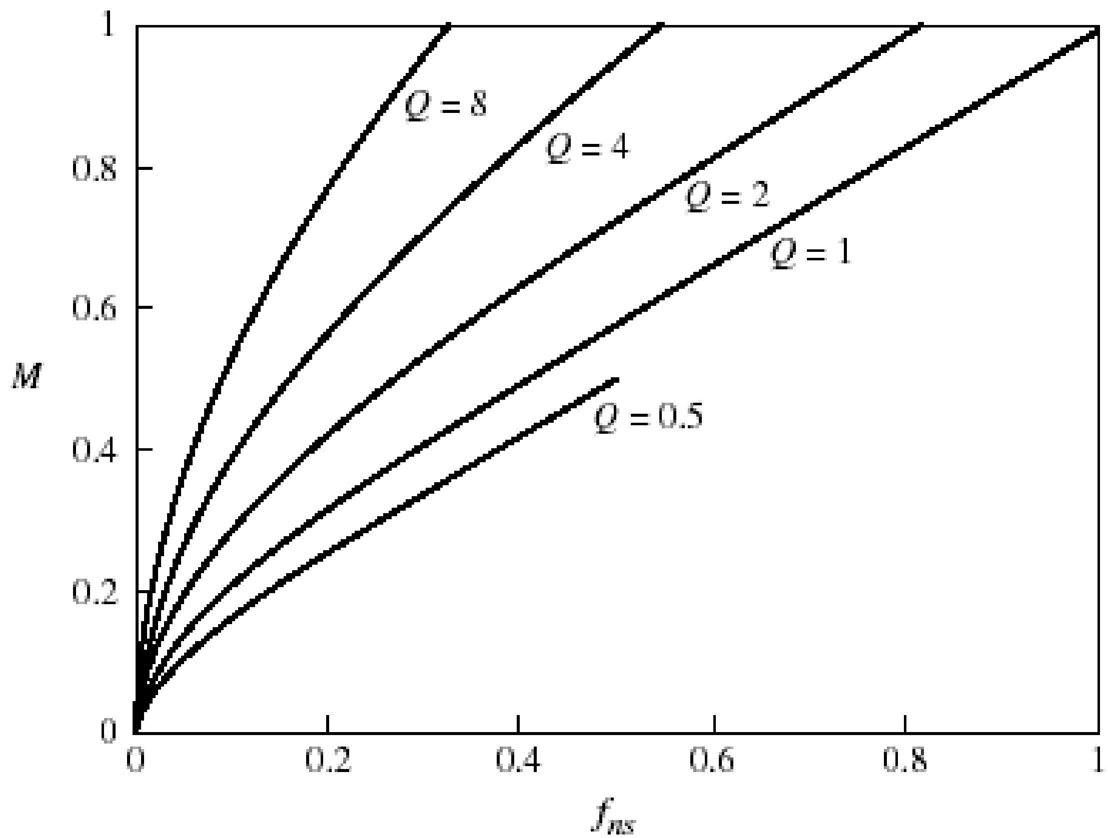


Fig 6.11 Control characteristic curve of ***M*** vs. f_{ns} for the ZCS buck converter.

ZCS Buck Converter

Example 6.1

Consider the following specifications for a ZCS buck converter of Fig. 6.8(a). Assume the parameters are: V_{in} = 25V, V_o = 12V, I_o = 1A, f_s = 250kHz
Design for the resonant tank parameters L and C and calculate the peak inductor current, and peak capacitor voltage. Determine the time interval for each mode.

Solution:

The voltage gain is $M = \frac{V_o}{V_{in}} = \frac{12}{25} = 0.48$. Select $f_{ns} = 0.4$. Determine Q from either the control characteristic curve of Fig. 6.11 or from the gain equation of Eq. (6.30). This results in Q approximately equaling 1. Since $R_o = \frac{V_o}{I_o}$, the characteristic impedance is given by,

$$Z_o = \frac{R_o}{Q} = 12\Omega \qquad = \sqrt{\frac{1}{LC}} = 12\Omega \qquad (6.32)$$

The second equation in terms of L and C is obtained from f_o. From the normalized switching frequency, f_o may be given by,

$$f_o = \frac{f_s}{f_{ns}}$$

$$f_o = \frac{f_s}{0.4} = 625kHz$$

In terms of the angular frequency, ω_o,

$$\omega_o = 2\pi f_o = \sqrt{\frac{1}{LC}} \qquad (6.33)$$

Solving Eqs. (6.32) and (6.33) for L and C, we obtain,

$$L = \frac{Z_o}{\omega_o} = \frac{12\Omega}{2\pi \times 625 \times 10^3\ rad/\sec} = 3.06 \times 10^{-6} \approx 3\mu H$$

$$C = \frac{1}{Z_o \omega_o} = \frac{1}{12 \times 2 \times \pi \times 625 \times 10^3} \approx 0.02\mu F$$

Example 6.1 (cont'd)

The peak inductor current, is given by,

$$I_{l,peak} = I_o + \frac{V_{in}}{Z_o}$$
$$\approx 3\,A$$

The peak capacitor voltage is:

$$\mathrm{v}_{\mathrm{c,peak}} = 2\,\mathrm{V}_{\mathrm{in}}$$
$$= 50\,\mathrm{V}$$

The time intervals are calculated from the following expressions:

$$t_1 = \frac{I_o L}{V_{in}} = \frac{(1A)\times(3\times10^{-6}H)}{25V} \approx 0.122\mu s$$

$$t_2 = t_1 + \frac{1}{\omega_o}\sin^{-1}\left(\frac{-Z_o I_o}{V_{in}}\right)$$
$$\approx 0.122 + \frac{1}{2\pi f_o}\sin^{-1}\left(\frac{-12\times1}{25}\right)$$
$$\approx 0.795\,\mu s$$

$$t_3 = t_2 + \frac{C V_{in}(1-\cos\omega_o(t_2-t_1))}{I_o}$$
$$= 0.795 + \frac{(0.02\times25\times10^{-6})(1-\cos\omega_o 0.67)}{1A} \approx 1.79\,\mu s$$

For t'_{max} we have

$$\omega_0(t_1'-t_1) = \frac{\pi}{2} \rightarrow t_1' = \frac{\pi/2}{\omega_0} + t_1$$
$$= 0.4\mu s + 0.122\mu s = 0.522\mu s$$
$$t_4 = 4\mu s = T_s$$

Exercise 6.2 for practice