Chapter 8

Phase-Controlled Converters

INTRODUCTION

We saw in the preceding chapter how diodes can be used to rectify an ac input voltage to produce an uncontrolled dc output. These circuits—whether half-wave or full-wave configurations under resistive, inductive, or capacitive loads—have one common feature: The level of the output voltage is a function only of the circuit parameters and the peak voltage and frequency of the applied voltage source. For this reason such circuits are known as *uncontrolled rectifier* circuits. In this chapter *controlled rectifier circuits*, using the silicon-controlled rectifier (SCR) instead of the diode, will be discussed. Unlike in diode rectifier circuits, in controlled circuits the power may flow from the load side (dc side) to the source side (ac side) under some control condition. This negative direction of power flow is known as *inversion* and the circuits are known as *controlled inverter circuits*.

In controlled rectifier circuits, diodes are replaced by SCRs to allow control of the conduction period or the phase of the conducting waveform. Since the voltage or current at the output is controlled by varying or delaying the phase of the conducting waveforms, such circuits are also known as *phase-controlled converters*. As shown in Chapter 2, unlike a diode, an SCR does not turn on when only the anode-cathode voltage becomes positive; rather, an additional signal must be applied to a third terminal (gate). The source of control stems from the fact that the gate signal can be applied at *any time* in the period during which the anode-cathode voltage is positive.

Controlled SCR rectifiers have a wide range of industrial and residential applications, especially applications in which power flows in both directions. For example, in the electrochemical industry, phase-controlled rectifiers are used to control the power in electroplating to the dc side, and in rotary machines they are used to control the speed of dc and ac motors in both directions. In residential applications, medium-power phase-controlled rectifiers are used in light dimmers and variable-speed appliances.

As in Chapter 7, half- and full-wave circuit configurations under resistive, inductive, and capacitive loads will be discussed. Also, because of their wide use in high-power applications, three-phase controlled circuits will be investigated. The SCR is used in such circuits because of its high current and voltage capabilities, its simple gating circuits, and its ability to control large anode currents with a small gate signal. Throughout this chapter, it will be assumed that the gate signal used to turn on the SCR is very short in duration and does not interfere with the circuit operation, that the turn-on and turn-off times are negligible, and that the anode-cathode voltage is zero in the conducting state (*on* state) and the anode current is zero in the nonconducting state (*off* state).

8.1 BASIC PHASE CONTROL CONCEPTS

Figure 8.1 shows block diagram representations for single- and three-phase SCR-controlled circuits.

The ac side consists of the line-frequency input voltage supply, which is either single-phase or three-phase (delta or wye connection), with an inductance, L_s. As in Chapter 7, the input power factor will be defined.

The SCR circuit block consists of one or more SCRs and possibly diodes, resulting in half- and full-wave and other configurations. The control signals are externally applied gating signals to turn on the SCRs in order to control the average output voltage. There are various integrated circuits commercially available to generate gating signals for such applications. Such ICs and other gating techniques are outside the scope of this textbook. The final block is labeled *load,* which represents the dc side of the converter.

Depending on the type of load used, the load current can be either continuous or discontinuous. Both cases will be analyzed in this chapter. Under highly inductive loads, i_o will be assumed constant. This will simplify the analysis significantly.

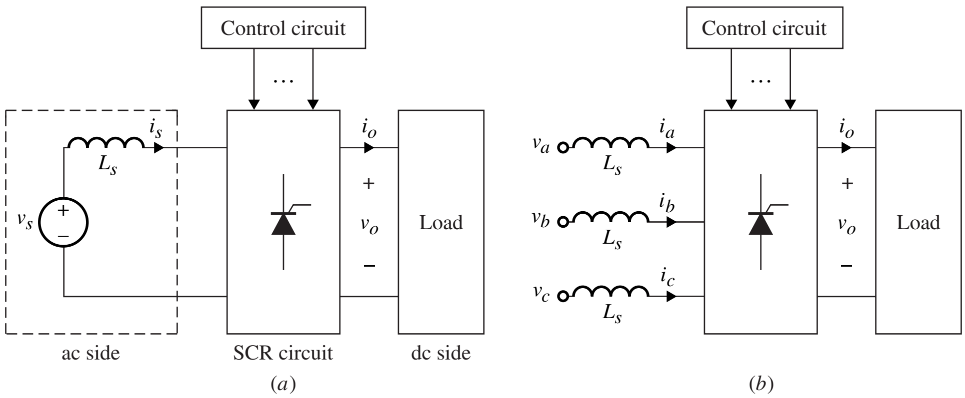


Figure 8.1 Block diagram representation for phase-controlled converters. (*a*) Single-phase. (*b*) Three-phase.

Depending on the SCR circuit configuration, the control signals, and the type of load, it is possible to produce a dc load current, I_o, and a dc load voltage, V_o, with two polarities, resulting in a four-quadrant mode of operation, as shown in Fig. 8.2.

In the two- and four-quadrant modes of operation, power can flow from the dc side to the ac side, resulting in what is known as a *phase-controlled inverter*. It will be shown later that for the inversion process to take place, the dc side must contain a type of energy source capable of delivering energy from the dc circuit to the ac supply side. The four-quadrant mode of operation is attainable if two circuits of the two-quadrant type are connected in a series or a back-to-back configuration.

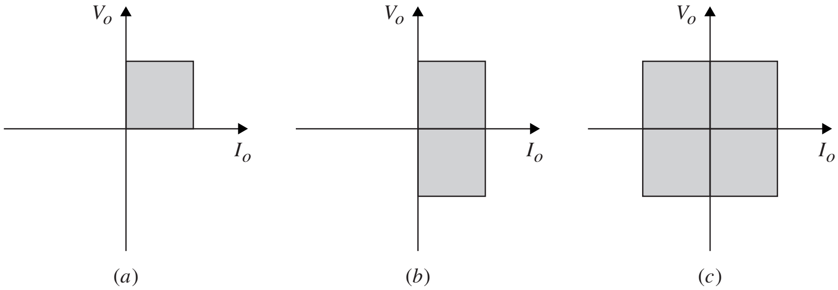


Figure 8.2 Quadrant diagram representation for three different modes of operation for Fig. 8.1. (*a*) One-quadrant mode. (*b*) Two-quadrant mode. (*c*) Four-quadrant mode.

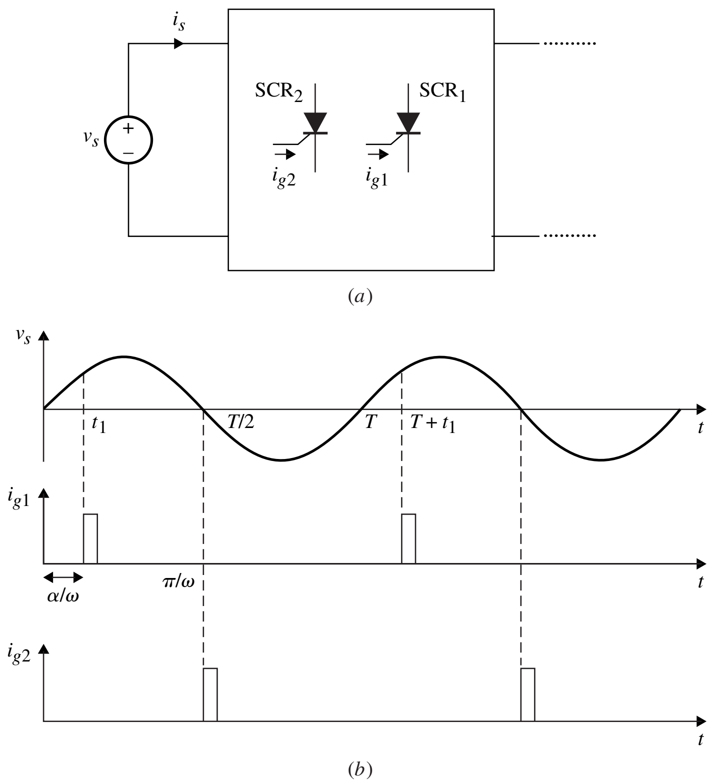


Figure 8.3 (*a*) SCR representation with gate trigger signals i_{g1} and i_{g2}. (*b*) Example with firing at t_1 and $T/2$ or angles at α and π.

Gating of the SCR is done by means of a gate trigger signal as shown in Fig. 8.3(*a*). The time t_1 is known as the firing time or the delay time. It is common to represent this instant in terms of α, which is known as the *firing angle* or the *delay angle*,

$$\alpha = \omega t_1$$

where ω is the frequency (rad/s) of the supplied voltage. For example, the delay angle for SCR_1 is α and for SCR_2 it is π, as shown in Fig. 8.3(*b*).

Since we assume an ideal SCR, the height and width of the gate trigger signal are not of importance. We should point out that time and angle notations will be used interchangeably.

8.2 HALF-WAVE CONTROLLED RECTIFIERS

In this section, we will discuss half-wave phase-controlled rectifiers under resistive and inductive loads. These circuits are mainly used in medium-power applications involving several kilowatts.

8.2.1 Resistive Load

When the diode is replaced by an SCR in the single-phase half-wave rectifier circuit, the resultant circuit is known as a half-wave phase-controlled rectifier, shown in Fig. 8.4.

When the source voltage is positive, the SCR will not conduct until the gate signal is applied at $t = t_1$, as shown in Fig. 8.5. At this time, the output voltage becomes equal to the input voltage. The output voltage, v_o, is sketched in Fig. 8.5.

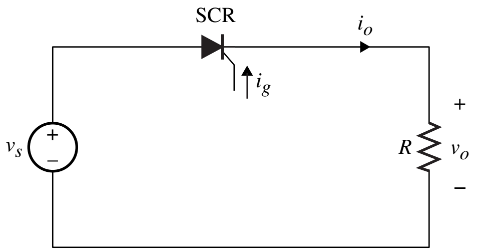


Figure 8.4 Half-wave phase-controlled rectifier.

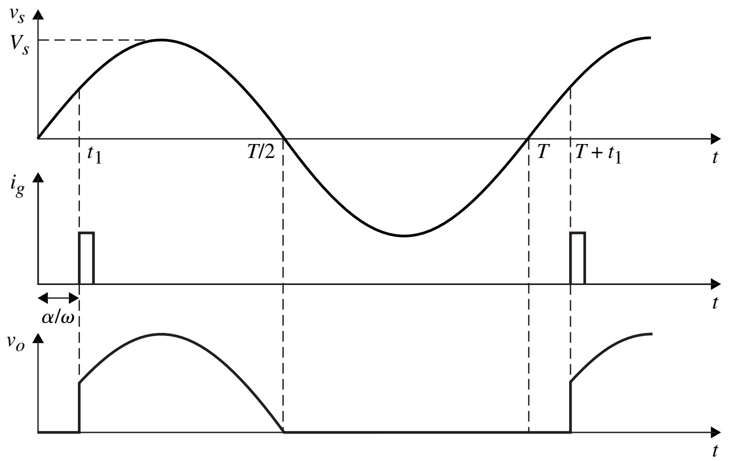


Figure 8.5 Waveforms for Fig. 8.4 when the SCR is triggered at $t = t_1$.

We must restate that once the SCR starts conducting, the gate signal can be removed. The delay angle (α) range is 0° to 180°. At $t = T/2$ the input voltage becomes negative, causing the SCR to turn off. The average output voltage is evaluated from the following equation:

$$V_o = \frac{1}{T}\int_0^T v_o\, dt = \frac{1}{T}\int_{t_1}^{T/2} v_s\, dt = \frac{V_s}{2\pi}(1 + \cos\alpha) \tag{8.1}$$

where the normalized voltage is given by

$$\begin{aligned} V_{no} &= \frac{V_o}{V_s/\pi} \\ &= \frac{1 + \cos\alpha}{2} \end{aligned} \tag{8.2}$$

$\alpha = \omega t_1$ is the firing angle, the varying of which allows the controllability of the output voltage. Figure 8.6 shows the normalized average output voltage V_{no} as a function of the delay angle, α. This control characteristic curve will allow us to determine the rectifier firing angle for a desired normalized output.

The rms value can be evaluated from Eq. (8.3) or Eq. (8.4):

$$V_{o,\,\text{rms}} = \sqrt{\frac{1}{T}\int_{t_1}^{T/2} (V_s \sin\omega t)^2 dt} \tag{8.3}$$

$$= \frac{V_s}{2}\sqrt{\left(1 - \frac{\alpha}{\pi}\right) + \frac{\sin 2\alpha}{2\pi}} \tag{8.4}$$

The input power factor (pf) and the total harmonic distortion (THD) for Fig. 8.4 are given by Eqs. (8.5) and (8.6), respectively:

$$\text{pf} = \frac{\sqrt{2}}{2}\sqrt{\left(1 - \frac{\alpha}{\pi}\right) + \frac{\sin 2\alpha}{\pi}} \tag{8.5}$$

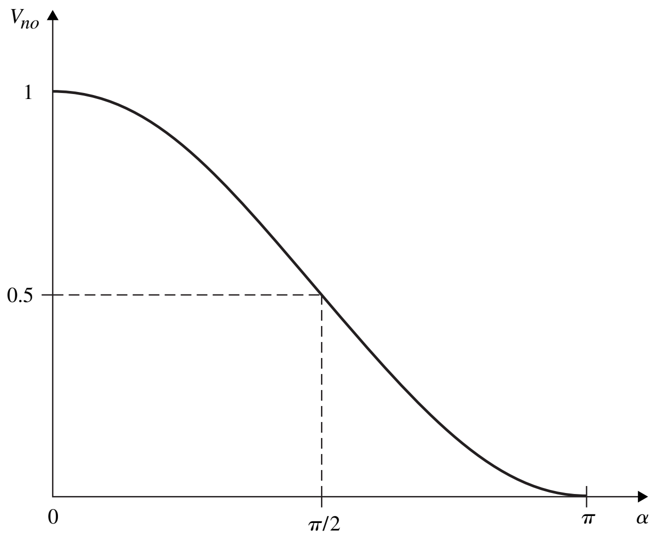


Figure 8.6 Normalized output voltage as a function of the delay angle.

$$\mathrm{THD} = \sqrt{\frac{2}{\left(1 - \frac{\alpha}{\pi}\right) + \left(\frac{\sin 2\alpha}{2\pi}\right)} - 1} \tag{8.6}$$

The reader is invited to verify these equations (see Problem 8.1). Since the average output voltage is positioned for $0 \le \alpha \le \pi$, and the average output current is V_o/R, this inverter operates in the first quadrant. Comparing this inverter with the uncontrolled half-wave diode rectifier circuit, we note that the average output voltage can be controlled by varying α. The phase-controlled current and voltage waveforms are related by the delay angle, α. This is why α is also known as the *angle of retard*. Both the THD and the input power factor are reduced in the phase-controlled rectifier due to the presence of the displacement angle α.

8.2.2 Inductive Load

Figure 8.7(*a*) shows a half-wave phase-controlled converter with an inductive-resistive load. The relevant waveforms are depicted in Fig. 8.7(*b*).

When the SCR is fired at $t = t_1$, the load current starts flowing until it reverses direction at $t = t_2$, at which the SCR is naturally turned off. It can be shown that the output current equation in the interval $t_1 \le t \le t_2$ is given by

$$i_o(t) = \frac{V_s}{|Z|}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{-(t-t_1)/\tau}\right] \tag{8.7}$$

where $\alpha = \omega t_1$, $\tau = L/R$, $\theta = \tan^{-1}(\omega L/R)$, and $|Z| = \sqrt{R^2 + (\omega L)^2}$.

The average value of $v_o(t)$ can be obtained by evaluating the following integral:

$$V_o = \frac{1}{T}\int_{t_1}^{t_2} V_s \sin \omega t \; dt \tag{8.8}$$

$$= \frac{V_s}{2\pi}(\cos\alpha - \cos\beta) \tag{8.9}$$

The numerical value of $\beta = \omega t_2$ can be obtained by evaluating Eq. (8.7) at $t = t_2$ with $i_o(t_2) = 0$.

Since the average value of the inductor voltage is zero in the steady state, the output average value is the same as the value across the load resistance. As a result, the average value of the load current, I_o, is given by

$$I_o = \frac{V_s}{2\pi R}(\cos\alpha - \cos\beta) \tag{8.10}$$

Similarly, we normalize the average output voltage by

$$V_{no} = \frac{V_o}{V_s/\pi} = \frac{\cos\alpha - \cos\beta}{2}$$

Figure 8.7(*c*) shows the plot of V_{no} versus α under different values of β. Notice that for $\beta = \pi$, the curve gives the resistive-load case shown in Fig. 8.6.

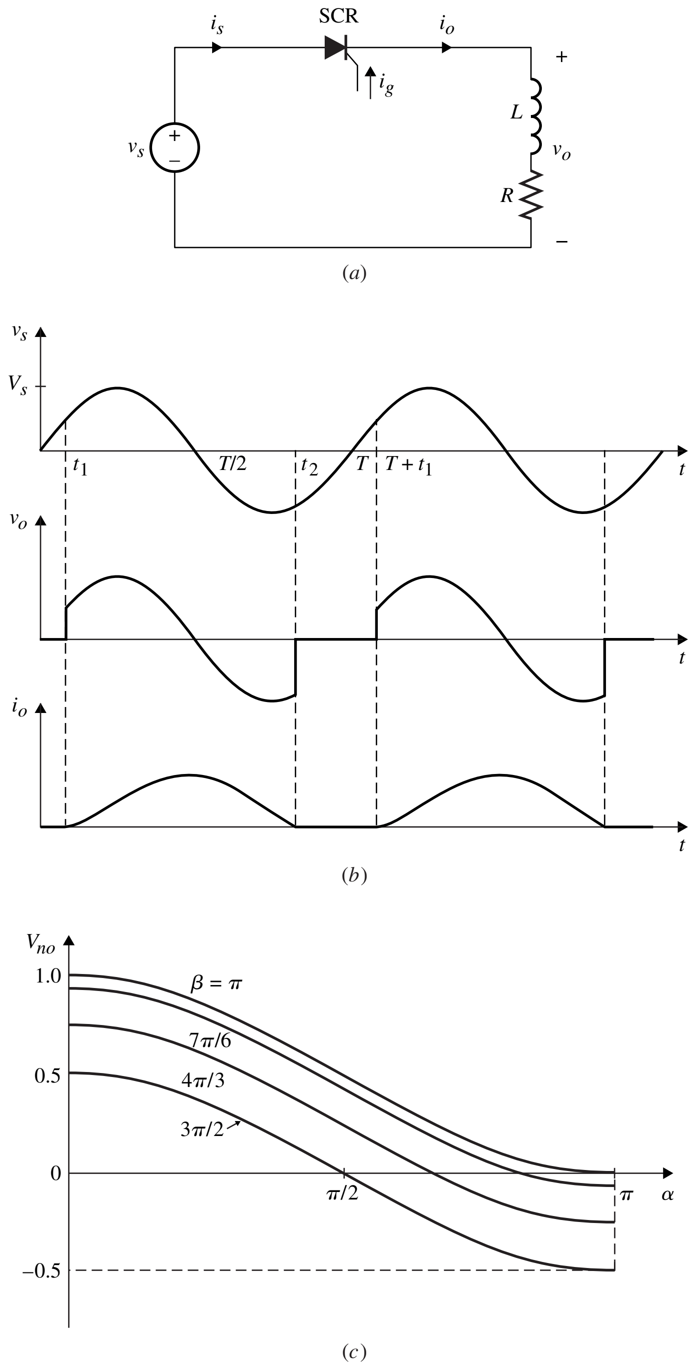


Figure 8.7 (a) Half-wave phase-controlled circuit with inductive load. (b) Relevant waveforms. (c) V_{no} versus α under different values of β.

EXERCISE 8.1

For the half-wave phase-controlled rectifier of Fig. 8.4, assume the *on*-state voltage drop of the SCR is V_{SCR}.

(a) Sketch the waveforms for v_o and i_o.

(b) Derive the expression for the average output voltage, V_o.

(c) Calculate V_o, pf, THD, and $V_{o,\text{rms}}$ using the following values: $V_s = 25$ V, $R = 10\ \Omega$, $V_{\text{SCR}} = 1.5$ V, and $\alpha = \pi/5$.

ANSWER 16 V, 0.76, 95%, 13.5 V

EXERCISE 8.2

For the half-wave phase-controlled rectifier of Fig. 8.7(*a*), use $v_s = 78\sin 377t$, $L = 10$ mH, $R = 5\ \Omega$, to design α for $V_o = 16$ V, obtain t_1 and t_2 in Fig. 8.7 (*b*).

ANSWER 37°, 1.2 ms, 17.9 ms

Consider the case where a free-wheeling diode is added to Fig. 8.7(*a*) as shown in Fig. 8.8(*a*). The presence of the diode results in a continuous load current, and the output voltage is always positive. The corresponding waveforms are shown in Fig. 8.8(*b*).

Notice that the principal operation of this circuit is similar to that of the half-wave diode rectifier with free-wheeling diode discussed in Chapter 3, with the exception that the intervals in which the two circuit equations are applied are shifted by α.

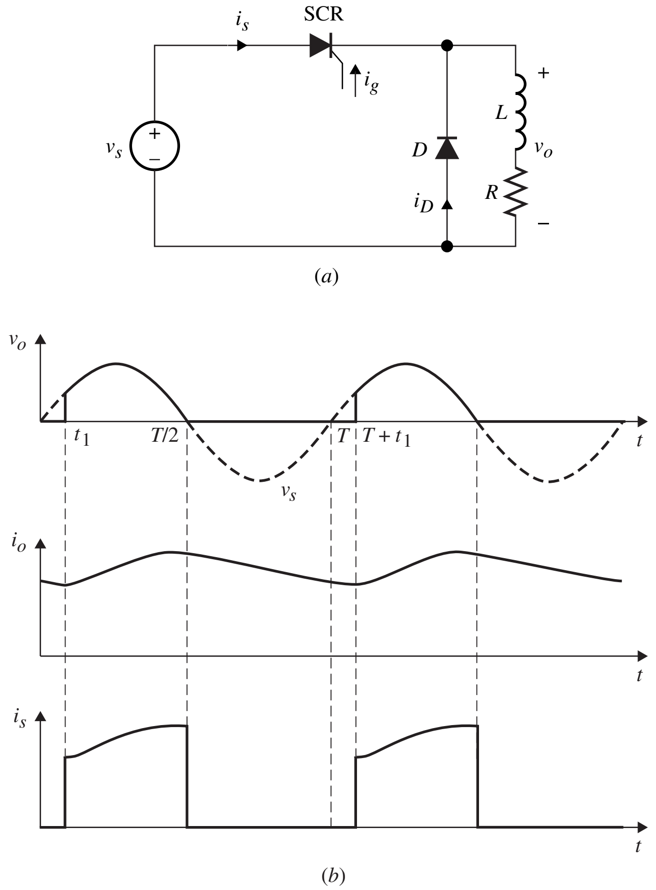


Figure 8.8 (*a*) Half-wave phase-controlled rectifier with free-wheeling diode. (*b*) Typical waveforms.

For the time interval in which the free-wheeling diode D is conducting, the circuit equation is given by

$$L\frac{di_o}{dt} + Ri_o = 0 \qquad \pi \le \omega t \le 2\pi + \alpha \tag{8.11}$$

and for the time interval in which the SCR is conducting and D is reverse biased,

$$L\frac{di_o}{dt} + Ri_o = V_s \sin\omega t \qquad \alpha \le \omega t \le \pi \tag{8.12}$$

By applying the proper boundary conditions at $\omega t = \alpha$ and $\omega t = \pi$, an expression for i_o can be derived (see Problem 8.3). It is interesting to note that since v_o does not go negative, the average output is no longer a function of the load inductance. It is given by

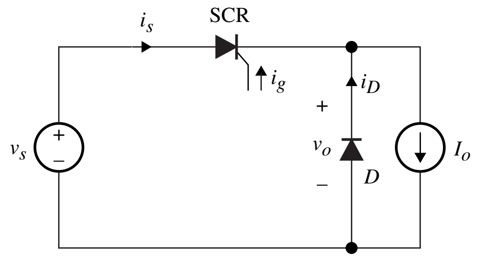
$$V_o = \frac{1}{T}\int_{t_1}^{T/2} V_s \sin\omega t \; dt = \frac{V_s}{2\pi}(1 + \cos\alpha) \tag{8.13}$$

Note that this circuit does not support a negative output voltage (no inversion).

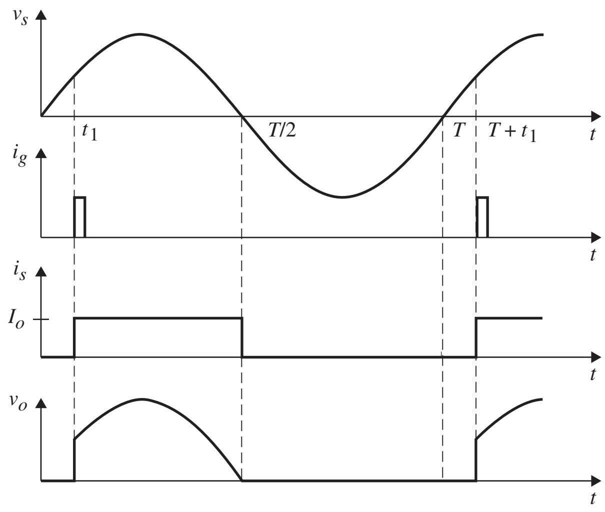
EXAMPLE 8.1

Draw the waveforms for i_s and v_o in Fig. 8.8(a). Assume $L/R >> T/2$.

SOLUTION The equivalent circuit of Fig. 8.8(a) is shown in Fig. 8.9(a), and the relevant waveforms are shown in Fig. 8.9(b). The control characteristic curve is the same as the one given in Fig. 8.6.



(a)



(b)

Figure 8.9 (a) Equivalent circuit for Fig. 8.8(a) with infinitely large L. (b) Relevant waveforms.

8.3 FULL-WAVE PHASE-CONTROLLED RECTIFIERS

8.3.1 Resistive Load

A phase-controlled full-wave bridge rectifier is shown in Fig. 8.10. This circuit is more practical than the phase-controlled half-wave rectifier shown in Fig. 8.4.

Like the uncontrolled full-wave bridge rectifier, the diagonal pairs SCR_1-SCR_2 and SCR_3-SCR_4 conduct during the positive and negative half-cycles, respectively. Assume the gating signal pairs are both delayed by α in their respective half-cycles as shown in Fig. 8.11.

The output average value is calculated from the following integral:

$$\begin{aligned} V_o &= \frac{2}{T}\int_{t1}^{T/2} V_s \sin \omega t \; dt \\ &= \frac{V_s}{\pi}(\cos\alpha + 1) \end{aligned} \tag{8.14}$$

From Eq. (8.14), it is clear that the control characteristic curve of the average output voltage versus α for this full-wave topology is doubled compared to the half-wave topology given in Fig. 8.6.

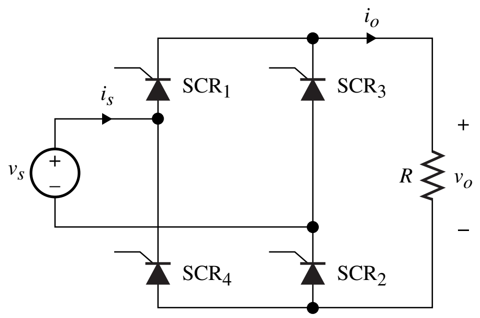


Figure 8.10 Full-bridge phase-controlled rectifier.

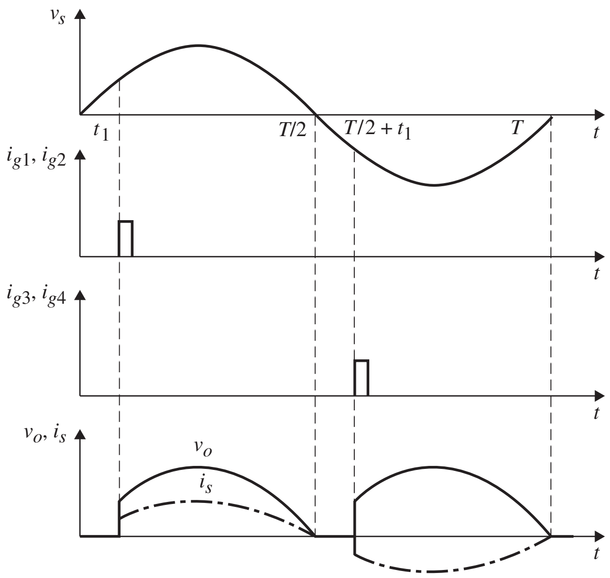


Figure 8.11 Waveforms for Fig. 8.10.

EXAMPLE 8.2

Determine the input power factor and the THD for the full-bridge topology of Fig. 8.10.

SOLUTION The rms value of v_s is $V_s/(\sqrt{2})$, and the rms value for i_s, calculated from the waveform of Fig. 8.11, is

$$I_{s,\,\mathrm{rms}} = \sqrt{\frac{1}{T/2}\int_{t_1}^{T/2}\left(\frac{V_s}{R}\sin\omega t\right)^2 dt} = \frac{V_s}{\sqrt{2}R}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}} \tag{8.15}$$

The average input power can be calculated from the following relation:

$$P_{\mathrm{in}} = \frac{1}{T/2}\frac{1}{R}\int_{t_1}^{T/2}(V_s\sin\omega t)^2 dt = \frac{V_s^2}{2R}\left[\left(1-\frac{\alpha}{\pi}\right)+\frac{\sin 2\alpha}{2\pi}\right] \tag{8.16}$$

From the definition of the power factor, and using Eqs. (8.15) and (8.16), we can obtain the following relation:

$$\mathrm{pf} = \frac{P_{\mathrm{ave}}}{I_{s,\mathrm{rms}}V_{s,\mathrm{rms}}} = \sqrt{\left(1-\frac{\alpha}{\pi}\right)+\frac{\sin 2\alpha}{2\pi}} \tag{8.17}$$

A plot of pf versus α is given in Fig. 8.12.

The THD can be found as

$$\mathrm{THD} = \sqrt{\frac{2\alpha+2\cos(2\alpha)-1}{2\pi-2\alpha-2\cos(2\alpha)+1}}$$

EXERCISE 8.3

Calculate the input power factor for $\alpha = 30°$ and $150°$ using Fig. 8.10.

ANSWER 0.75, 0.36

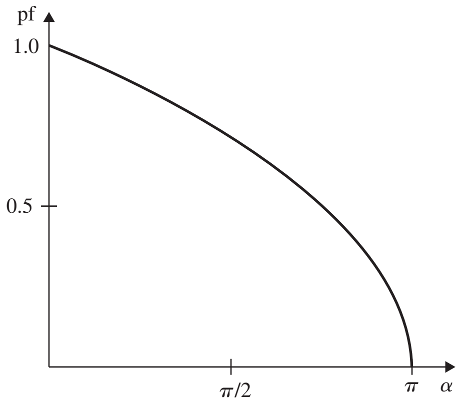


Figure 8.12 Power factor versus the delay angle for the circuit of Fig. 8.10.

EXERCISE 8.4

Consider the waveform for $i_s(t)$ given in Fig. 8.11.

(a) Derive the expression of the fundamental component, $i_{s1}(t)$, for the input current i_s.

(b) Calculate the THD for $\alpha = 30°$ and $\alpha = 150°$.

ANSWER 10.2%, 18.9%

8.3.2 Inductive Load

The full-wave phase-controlled rectifier under an inductive load is shown in Fig. 8.13. Let us first consider the waveforms for i_o and v_o and assume that the time constant L/R is not large compared to $T/2$. Again, the thyristor pairs SCR_1-SCR_2 and SCR_3-SCR_4 are triggered diagonally, each delayed by α in the respective half-cycles. It can be shown that, as far as the load current is concerned, there are two modes of operation: discontinuous and continuous conduction modes. If we assume that in steady-state operation, the inductor current value reaches zero before $t = T/2 + t_1$, then the rectifier is known to operate in the discontinuous conduction mode (dcm). For dcm to occur, the load current must reach zero before the opposite pair of SCRs are fired. However, if the inductor current (load) is not allowed to reach zero, i.e., the second diagonal pair of SCRs are triggered while the current is flowing, then since the current in the inductor does not change instantaneously, the current commutates instantaneously in the other thyristors. Under this case, the rectifier is known to operate in the continuous conduction mode (ccm). Figures 8.14 and 8.15 show the waveforms for v_o and i_o under dcm and ccm, respectively.

EXAMPLE 8.3

Derive the expression for $i_o(t)$ under both the continuous and discontinuous conduction modes, and determine the condition at which the boundary between ccm and dcm occurs.

SOLUTION Since the dcm is a special case of the ccm, let us first consider the continuous conduction mode case. As shown in the previous chapter, the total solution for $i_o(t)$ is given by

$$i_o(t) = Ae^{-(t-t_1)/\tau} + \frac{V_s}{|Z|}\sin(\omega t - \theta) \quad (8.18)$$

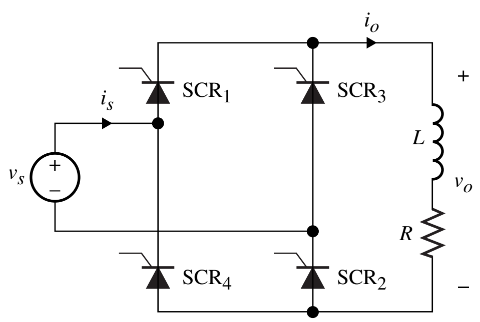


Figure 8.13 Full-wave phase-controlled rectifier with inductive load.

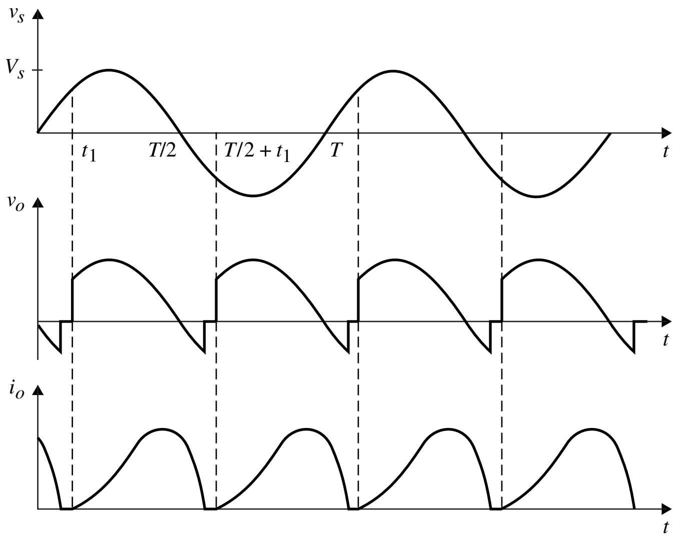


Figure 8.14 Waveforms for Fig. 8.13 under discontinuous conduction mode.

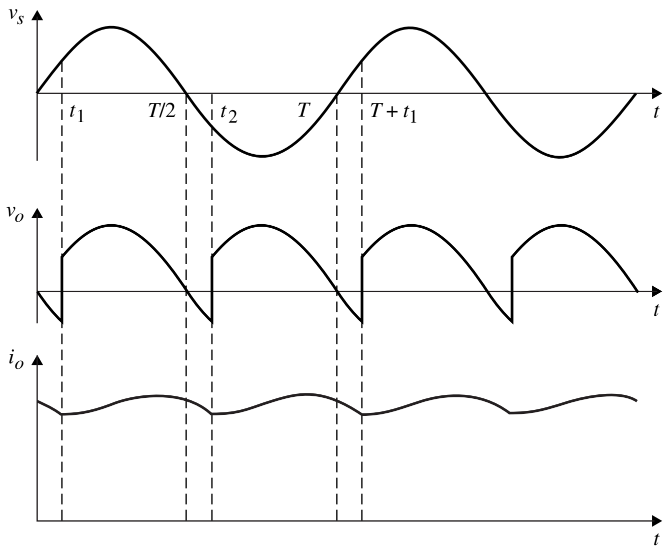


Figure 8.15 Waveforms for Fig. 8.13 under continuous conduction mode.

where A is a constant and must be determined from the initial condition, and the other parameters are given by

$$\tau = L/R$$

$$|Z| = \sqrt{(\omega L)^2 + R^2}$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

If we let the initial condition of $i_o(t)$ at $t = t_1$ be I_{o1}, from Eq. (8.18) we have

$$i_o(t_1) = I_{o1} = A + \frac{V_s}{Z}\sin(\alpha - \theta)$$

Solving for A, we obtain

$$A = I_{o1} - \frac{V_s}{|Z|}\sin(\alpha - \theta)$$

Therefore, the expression for the load current is given by

$$i_o(t) = \left[I_{o1} - \frac{V_s}{|Z|}\sin(\alpha - \theta)\right]e^{(t - t_1)/\tau} + \frac{V_s}{|Z|}\sin(\omega t - \theta) \tag{8.19}$$

In steady-state operation, $i_o(t_1)$ must equal $i_o(T/2 + t_1)$; hence, evaluating Eq. (8.19) at $t = T/2 + t_1$ and solving for I_{o1}, we obtain

$$I_{o1} = \frac{-\frac{V_s}{|Z|}(e^{-\pi/\omega\tau} - 1)\sin(\alpha - \theta)}{1 - e^{-\pi/\omega\tau}} \tag{8.20}$$

This initial condition determines the mode of operation as follows:

$$I_{o1}:\begin{cases} < 0 & \text{for} \quad \alpha > \theta \quad \text{dcm} \\ = 0 & \text{for} \quad \alpha = \theta \quad \text{boundary condition} \\ > 0 & \text{for} \quad \alpha < \theta \quad \text{ccm} \end{cases}$$

If we assume that the load time constant L/R is much larger than the half-period of the applied voltage, then we can model the load current by a constant current source, I_o. In this case, the waveform of the current i_s is a squarewave of magnitude $\pm I_o$, shifted by the triggering angle $\alpha = \omega t_1$ as shown in Fig. 8.16.

The average output voltages of the full-bridge phase-controlled circuits with the waveforms of Figs. 8.14, 8.15, and 8.16 are the same and given by

$$\begin{aligned} V_o &= \frac{1}{T/2}\int_{t_1}^{T/2 + t_1} V_s \sin\omega t \ dt \\ &= \frac{2V_s}{\pi}\cos\alpha \end{aligned} \tag{8.21}$$

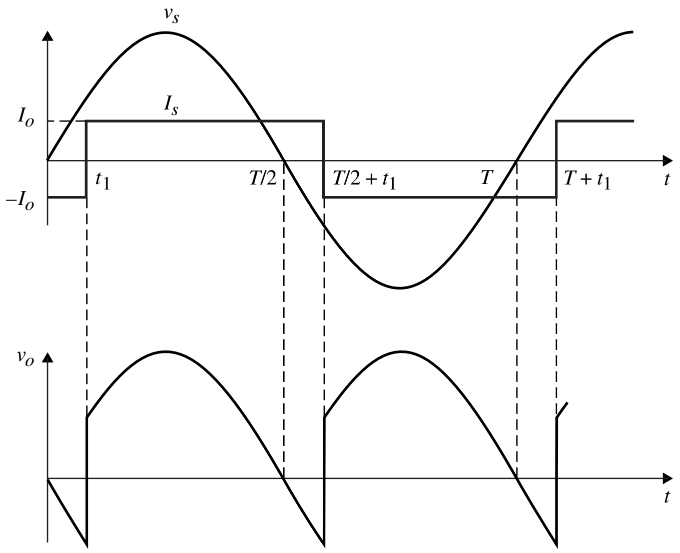


Figure 8.16 Waveforms of Fig. 8.13 under infinite load inductance.

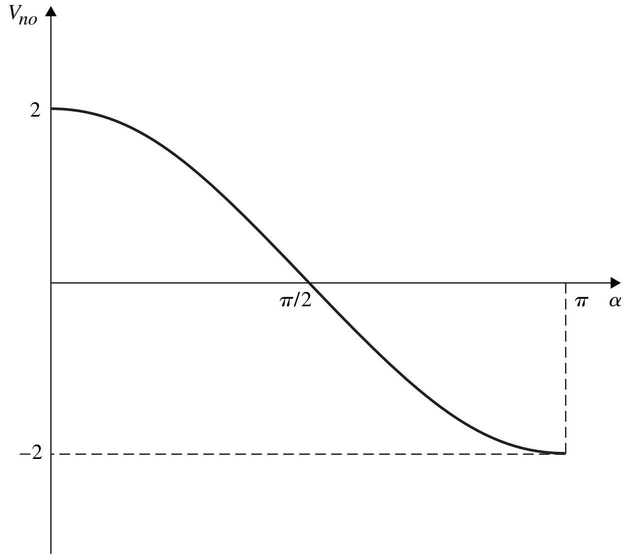


Figure 8.17 Control characteristic curve for Fig. 8.16.

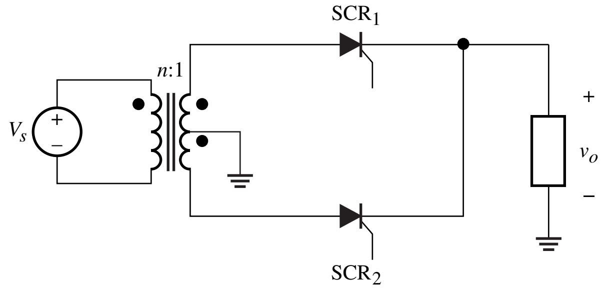


Figure 8.18 Full-wave controlled rectifier with center-tap transformer.

The control characteristic curve for the normalized average output voltage $V_{no} = \pi V_o / V_s$ is shown in Fig. 8.17. Notice that when $\pi/2 \leq \alpha \leq \pi$, this circuit produces a negative average output voltage. Since the average load current is always positive, the direction of the power flow in this range of α is from the dc output side to the ac input side. Consequently, as stated before, the circuit is known to operate in the inversion mode, whereas for $0 \leq \alpha \leq \pi/2$ the circuit operates in the rectification mode.

If the application requires the use of an isolated transformer, a center-tap full-wave controlled rectifier can be obtained using two SCRs as shown in Fig. 8.18. All waveforms are similar to the waveforms under resistive and inductive loads.

EXAMPLE 8.4

Derive the power factor for the phase-controlled full-wave bridge rectifier under a constant load current as shown in Fig. 8.19(*a*).

SOLUTION The waveform for $i_s(t)$ is redrawn in Fig. 8.19(*b*). From the Fourier series, we have

$$i_s(t) = I_{dc} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin(n\omega t)$$

From the symmetry of $i_s(t)$, the dc average value is zero and only its odd harmonics exist.

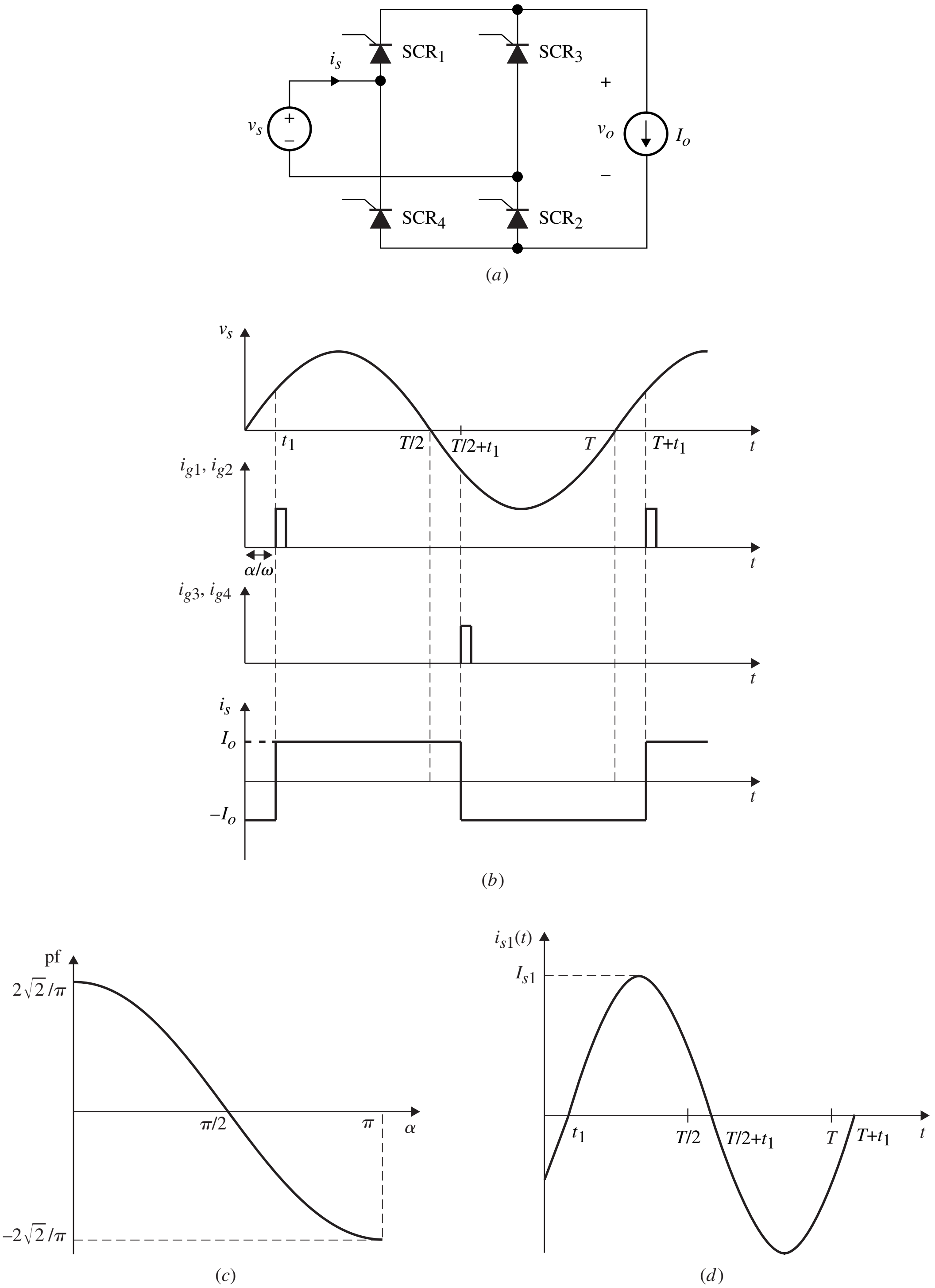


Figure 8.19 (*a*) Full-bridge phase-controlled circuit under constant load current. (*b*) Source voltage and current waveforms. (*c*) Power factor. (*d*) Fundamental component of $i_s(t)$.

The fundamental component is given by

$$i_{s1}(t) = I_{s1}\sin(\omega t + \theta)$$

where

$$I_{s1} = \sqrt{a_1^2 + b_1^2}$$

$$\theta = \tan^{-1}\frac{a}{b}$$

The coefficients a_1 and b_1 are evaluated from the following relations:

$$a_1 = \frac{2}{T}\left[\int_{t_1}^{T/2+t_1} I_o\cos(\omega t)\ dt + \int_{T/2+t_1}^{T+t_1} -I_o\cos(\omega t)\ dt\right]$$

$$= -\frac{4I_o}{\pi}\sin\alpha$$

$$b_1 = \frac{2}{T}\left[\int_{t_1}^{T/2+t_1} I_o\sin(\omega t)\ dt + \int_{T/2+t_1}^{T+t_1} -I_o\sin(\omega t)\ dt\right]$$

$$= -\frac{4I_o}{\pi}\cos\alpha$$

Hence, the peak value of the fundamental component is given by

$$I_{s1} = \frac{4I_o}{\pi}$$

and its rms value is

$$I_{s1,\text{rms}} = \frac{4I_o}{\sqrt{2}\pi}$$

The rms value of $i_s(t)$ is I_o.

From the preceding expression, we obtain the power factor:

$$\text{pf} = \frac{4}{\sqrt{2}\pi}\cos\theta$$

where the displacement angle is given by

$$\theta = -\tan^{-1}\left(\frac{\sin\alpha}{\cos\alpha}\right) = -\tan^{-1}(\tan\alpha) = -\alpha$$

The power factor is expressed in terms of the delay angle, α, as

$$\text{pf} = \frac{4}{\sqrt{2}\pi}\cos\alpha$$

which is sketched in Fig. 8.19(*c*). The fundamental component is shown in Fig. 8.19(*d*).

EXAMPLE 8.5

Consider the full-wave phase-controlled rectifier given in Fig. 8.20(*a*), with two SCRs and a flyback diode D_3.

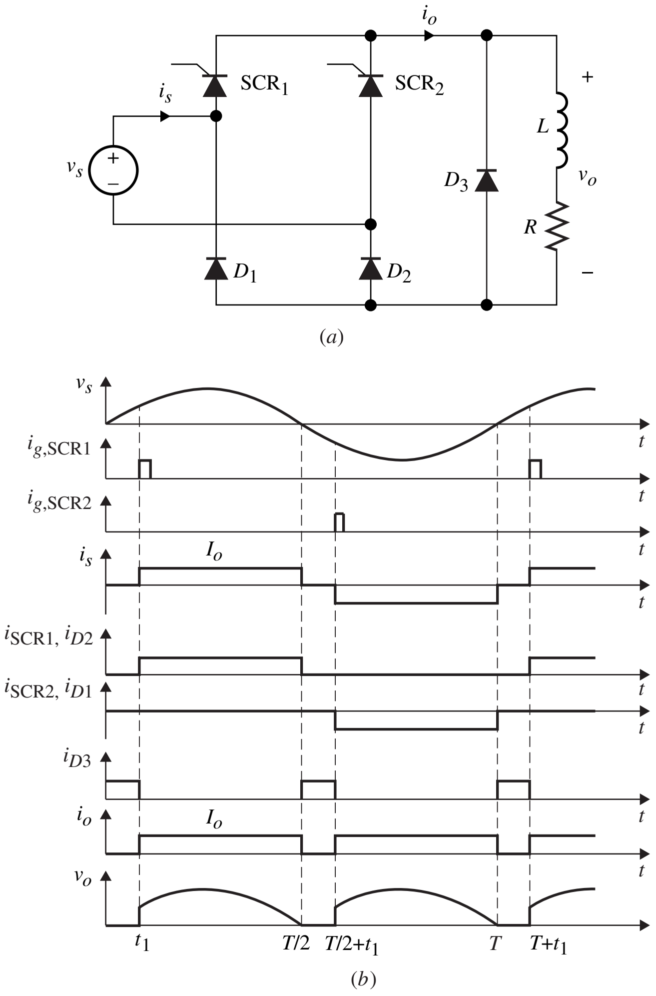


Figure 8.20 (*a*) Full-wave rectifier with two SCRs and a flyback diode. (*b*) Voltage and current waveforms.

(a) Assume $L/R >> T/2$. Draw the waveforms for i_s, i_{SCR1}, i_{SCR2}, i_{D1}, i_{D2}, i_{D3}, i_o, and v_o for $\alpha = 30°$.

(b) Show that the fundamental component of $i_s(t)$ is given by

$$i_{s1}(t) = I_{s1}\sin\left(\omega t - \frac{\alpha}{2}\right)$$

where

$$I_{s1} = \frac{2\sqrt{2}I_o}{\pi}\sqrt{1 + \cos 2\alpha}$$

I_o is the load current, assumed constant, and $\alpha = \omega t_1$.

(c) Derive the expression for the power factor and THD.

SOLUTION

(a) To sketch the waveforms, we must understand the basic circuit operation. Since we assume $L/R >> T/2$, i_o is considered constant as I_o. Let the firing times for SCR_1 and SCR_2 occur at t_1 and $T/2 + t_1$, respectively, as shown in Fig. 8.20(*b*). Before SCR_1 is gated at $t = t_1$, the load current is flowing in the flyback diode D_3, since SCR_2 is off because of the negative source voltage. At $t = t_1$, SCR_1 and D_2 turn on. At $t = T/2$, the negative source voltage turns D_3 on, and SCR_1

and D_2 cease to conduct. At $t = T/2 + t_1$, SCR_2 is gated, turning it on and causing D_3 to turn off and D_1 to turn on. This mode of operation continues until $t = T$, when SCR_2 and D_1 turn off, causing D_3 to turn on again. The cycle repeats at $t = T = t_1$, when SCR_1 is gated again.

(b) The fundamental component of $i_s(t)$ is given by

$$i_{s1}(t) = I_{s1}\sin(\omega t + \theta)$$

The parameters I_{s1} and θ are obtained from

$$I_{s1} = \sqrt{a_1^2 + b_1^2}$$

$$\theta = \tan^{-1}\frac{a_1}{b_1}$$

where

$$\begin{aligned} a_1 &= \frac{2}{T}\int_0^T i_s(t)\cos\omega t\ dt \\ &= \frac{2}{T}\left[\int_{t_1}^{T/2} I_o\cos\omega t\ dt - \int_{T/2+t_1}^{T} I_o\cos\omega t\ dt\right] \\ &= \frac{-2I_o}{\pi}\sin\alpha \end{aligned}$$

and

$$\begin{aligned} b_1 &= \frac{2}{T}\int_0^T i_s(t)\sin\omega t\ dt \\ &= \frac{2}{T}\left[\int_{t_1}^{T/2} I_o\sin\omega t\ dt - \int_{T/2+t_1}^{T} I_o\sin\omega t\ dt\right] \\ &= \frac{2I_o}{\pi}(1+\cos\alpha) \end{aligned}$$

Hence, I_{s1} is given by

$$I_{s1} = \frac{2\sqrt{2}I_o}{\pi}\sqrt{1+\sin 2\alpha}$$

(c) The rms expression is given by

$$\begin{aligned} I_{s,\text{rms}}^2 &= \frac{1}{2\pi}\left[\int_\alpha^\pi I_o^2\,d\omega t + \int_{\pi+\alpha}^{2\pi} I_o^2\,d\omega t\right] \\ &= \frac{1}{2\pi}[I_o^2(\pi-\alpha) + I_o^2(2\pi-(\pi+\alpha))] \\ &= \frac{I_o^2(-2\alpha+2\pi)}{2\pi} \\ &= I_o^2\left(1-\frac{\alpha}{\pi}\right) \end{aligned}$$

Hence, the power factor is expressed as

$$\text{pf} = \frac{2}{\pi}\frac{\sqrt{1+\cos\alpha}}{\sqrt{1-\frac{\alpha}{\pi}}}\cos\frac{\alpha}{2}$$

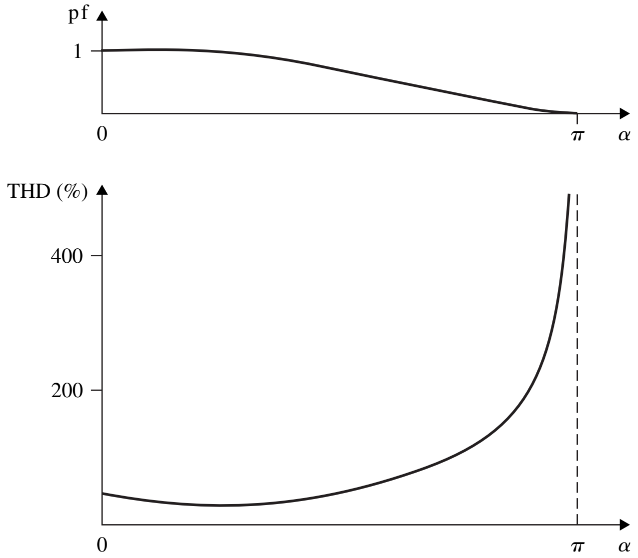


Figure 8.21 pf and THD as a function of α.

and the total harmonic distortion is given by

$$\text{THD} = \sqrt{\frac{\pi(\pi - \alpha)}{4(1 + \cos\alpha)} - 1}$$

The pf and THD plots are given in Fig. 8.21.

It is possible to design a full-wave phase-controlled converter that can provide bidirectional flow of the output voltage and the output current. The system of Fig. 8.22(*a*) can operate in the four quadrants as shown in Fig. 8.22(*b*).

The preceding general characteristics can be achieved by using dual full-wave phase-controlled converters connected in parallel, with their SCR connections opposite to each other. Two other configurations for full-wave bridge converters are known as series and parallel converters, shown in Fig. 8.23 and Fig. 8.24, respectively.

Figure 8.25 shows a phase-controlled full-wave circuit with a dc voltage source in the load side. Again, in the analysis we assume that the time constant L/R is very large compared to the period of the applied voltage. Hence, we can model the inductor load current as a constant I_o. The waveforms of the output voltage, v_o, and the source current, i_s, are the same as those of Fig. 8.19(*a*). In the steady state, the average output current is given by

$$I_o = \frac{V_o}{R} - \frac{V_{dc}}{R} = 2\frac{V_s}{\pi R}\cos\alpha - \frac{V_{dc}}{R} \tag{8.22}$$

EXERCISE 8.5

Consider the phase-controlled full-wave rectifier shown in Fig. 8.25 with $v_s = 110\sin 2\pi(60)t$, $L = 38$ mH, $R = 25\ \Omega$, and $V_{dc} = 12$ V. Assume the firing angle is 45° and the load current is constant. Find:

(a) The average output voltage

(b) The average output current

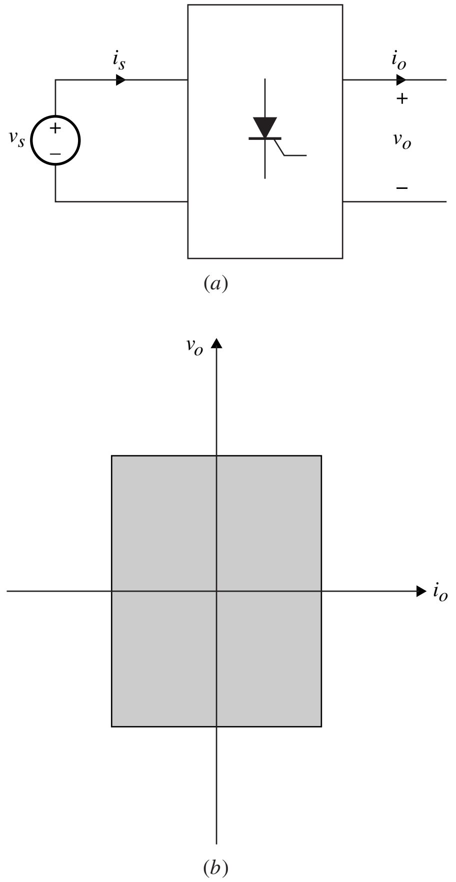


Figure 8.22 (*a*) Block diagram representation for a single-phase controlled rectifier. (*b*) Four-quadrant operation.

(c) The input and output average powers

(d) The power factor

ANSWER 49.5 V, 1.5 A, 74.26 W, 0.64

EXAMPLE 8.6

Determine the range of the delay angle α in Fig. 8.25 so that the input power factor is at least 0.75 and the minimum output power delivered to a 5 Ω load resistor is 1.2 kW. Assume $v_s = 240 \sin 377t$, $L = 125$ mH, and $V_{dc} = 20$ V.

SOLUTION The time constant is $L/R = 25$ ms, which is larger than $T/2$. Thus, the ripple current is considered very small. For an output power equal to 1.2 kW, the load current I_o equals 2.83 A from Eq. (8.22), and $\alpha = 50.37°$. For a power factor of 0.75, the conduction angle must be smaller than $33.6°$. As a result, the range of α is $0° \le \alpha \le 33.6°$. The power delivered to the load at $\alpha = 0°$ and $\alpha = 0°$ is 3.53 kW and 2.304 kW, respectively.

8.4 EFFECT OF AC-SIDE INDUCTANCE

In this section, we turn our attention to the analysis of the SCR rectifier and inverter circuits by including the ac-side inductance L_s. As with the diode rectifier circuits, here both the half-wave and the full-wave configurations will be considered.

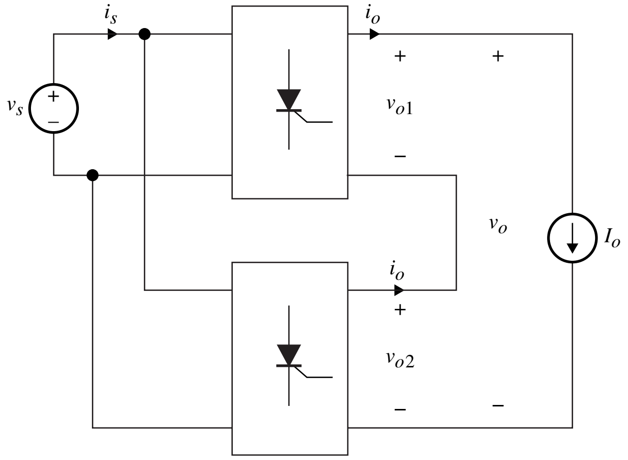


Figure 8.23 Series-connected full-wave SCR circuits.

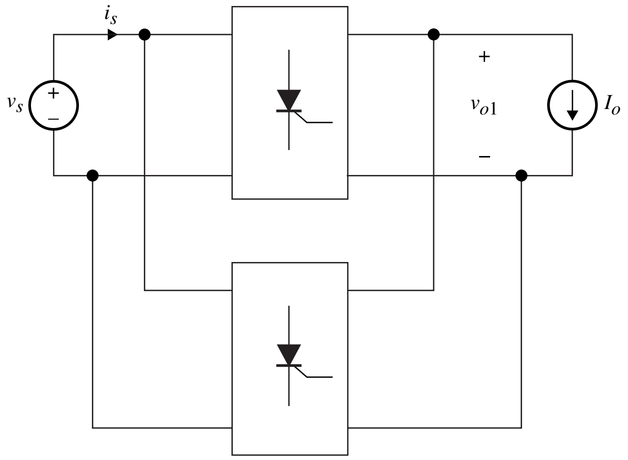


Figure 8.24 Parallel-connected full-wave SCR circuits.

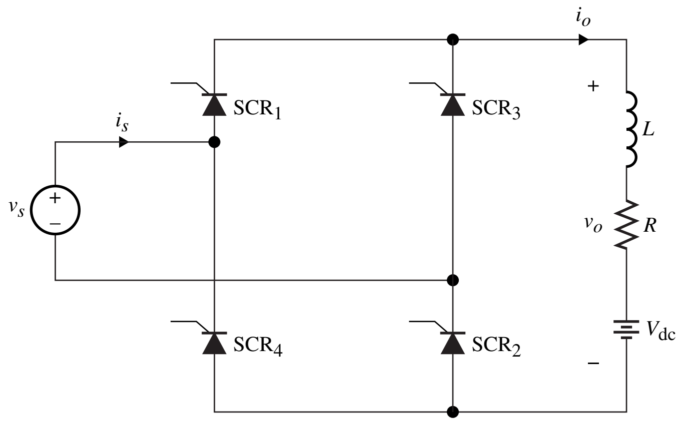


Figure 8.25 Phase-controlled full-wave circuit with load voltage source.

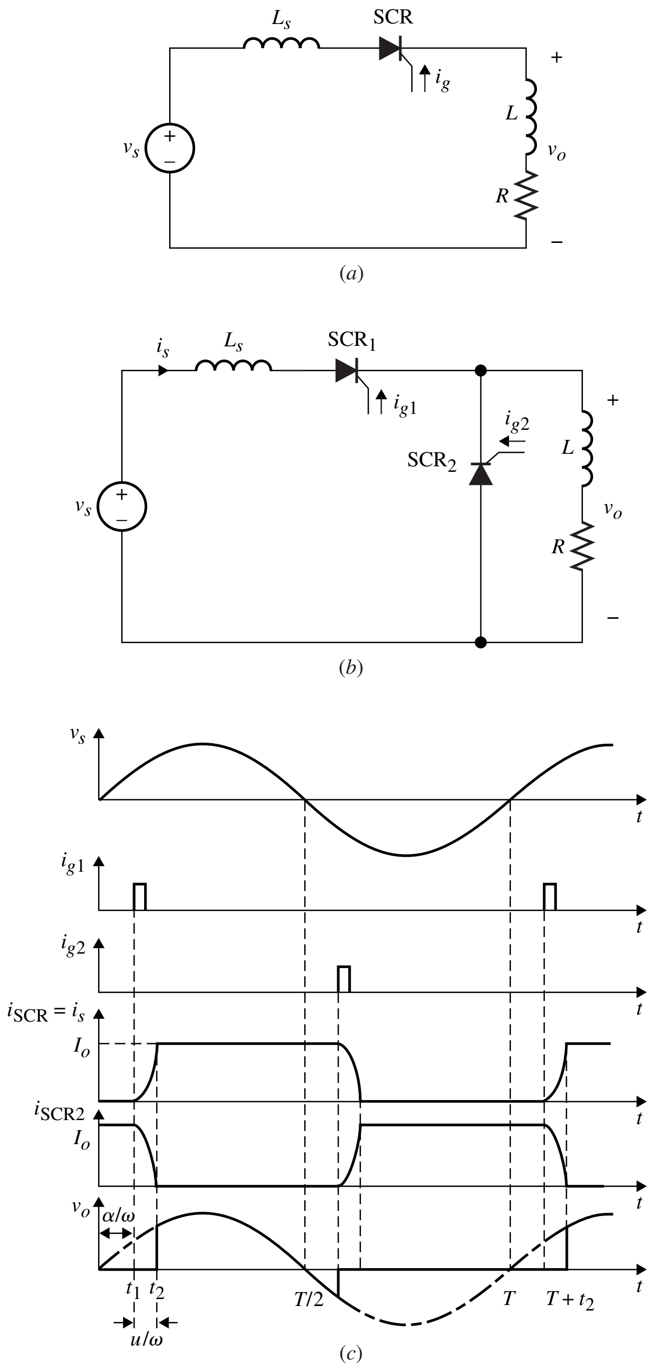


Figure 8.26 Half-wave SCR rectifier with ac-side reactance: (*a*) without flyback SCR, (*b*) with flyback SCR_2. (*c*) Voltage and current waveforms.

8.4.1 Half-Wave Circuits

Figure 8.26(*a*) shows the half-wave phase-controlled circuit including L_s with an inductive load. Without a flyback SCR across R-L, the circuit behaves like the half-wave rectifier under an inductive load, discussed in Section 8.2. Figure 8.26(*b*) shows the same circuit including a flyback SCR_2.

The analysis of this circuit is easily carried out by assuming $(L/R) >> T$. Figure 8.26(c) shows the waveforms for i_s, i_{SCR1}, and v_o under a constant load current.

Notice that at $t = T/2$, SCR_2 cannot be turned on because its gate signal is not available until time $t = t_1 + T/2$. The operation of this circuit is similar to that of the diode circuit with an ac-side inductance discussed in the previous chapter. At $t = t_1 + T/2$, when SCR_2 is fired, the current in SCR_1 cannot change to zero because of L_s; hence, both SCR_1 and SCR_2 remain on for a period of $(t_2 - t_1) = \mu/\omega$ shown in Fig. 8.26(c).

$$V_o = \frac{1}{T}\int_{t_2}^{T/2 + t_1} V_s \sin\omega t \; dt \tag{8.23}$$

During this interval, the current in SCR_1 decreases to zero and the current in SCR_2 increases to I_o at the same rate. The average output voltage is given by evaluating Eq. (8.23) and using $\alpha = \omega t_1$ and $u = \omega(t_2 - t_1)$, leading to the following equation for V_o:

$$V_o = \frac{V_s}{2\pi}(\cos\alpha + \cos(\alpha + u)) \tag{8.24}$$

The load current in terms of the commutation angle u and the firing angle α is obtained by integrating Eq. (8.25) from $t = t_1$ to t.

$$L_s\frac{di_s}{dt} = v_s(t) \tag{8.25}$$

Equation (8.25) is obtained from Fig. 8.26(a) when both SCR_1 and SCR_2 are conducting. From Eq. (8.25), we obtain $i_s(t)$ using the initial condition $i_s(t_1) = 0$, to yield

$$\begin{aligned} i_s(t) &= \frac{-V_s}{L_s\omega}(\cos\omega t - \cos\omega t_1) \\ &= \frac{V_s}{L_s\omega}(\cos\alpha - \cos\omega t) \end{aligned} \tag{8.26}$$

Evaluating Eq. (8.26) at $t = t_2$, where $i_s(t_2) = I_o$, we obtain

$$I_o = \frac{V_s}{\omega L_s}[\cos\alpha - \cos(u + \alpha)] \tag{8.27}$$

Substitute the above equation into Eq. (8.24), to give

$$V_o = \frac{V_s}{\pi}\left(\cos\alpha - \frac{\omega L_s I_o}{2V_s}\right) \tag{8.28}$$

In terms of the normalized output voltage and the normalized output current, Eq. (8.28) may be written as follows:

$$V_{no} = \frac{1}{\pi}\left(\cos\alpha - \frac{I_{no}}{2}\right) \tag{8.29}$$

The characteristic curve for I_{no} as a function of V_{no} under different firing angles is shown in Fig. 8.27.

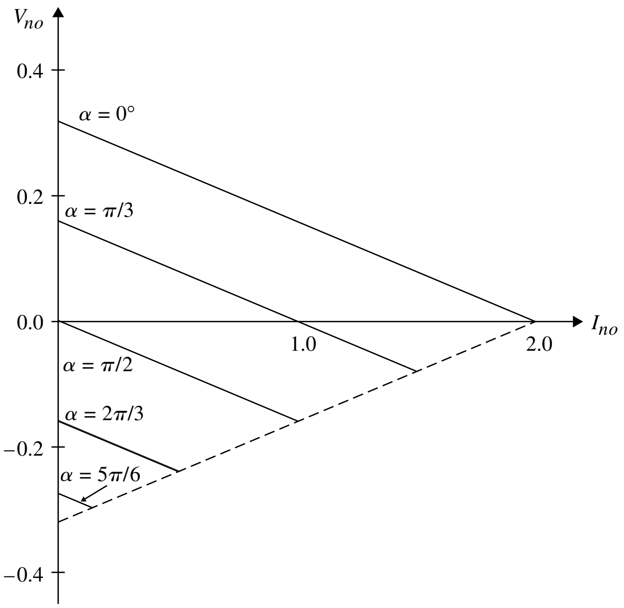


Figure 8.27 Characteristic curves for V_{no} versus I_{no} under different values of α.

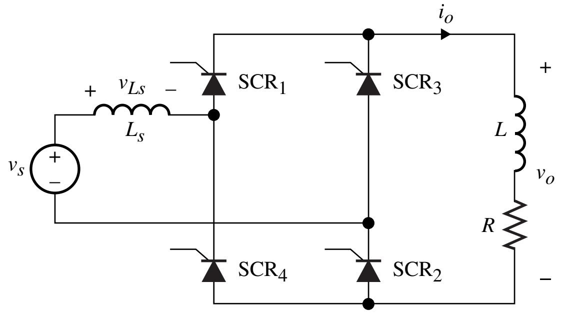


Figure 8.28 Controlled full-bridge rectifier with ac-side inductance.

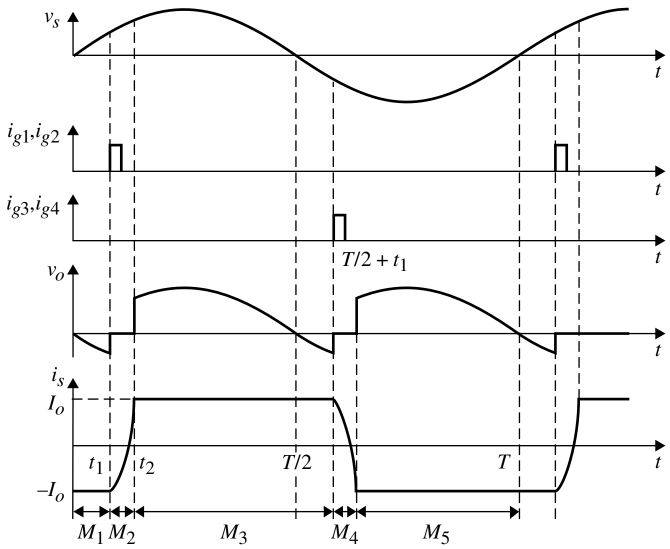


Figure 8.29 Output voltage and source current waveforms.

8.4.2 Full-Wave Bridge Circuits

Figure 8.28 shows the controlled full-wave bridge rectifier with an ac-side inductance. Assuming constant I_o, the waveforms for v_s, i_s, and v_o are shown in Fig. 8.29.

The basic circuit operation is similar to that of the full-wave diode rectifier with an ac-side inductance. We first assume that SCR_1 and SCR_2 are off during the negative cycle of $v_s(t)$ for $t \le t_1$. The voltage across L_s is zero; hence, the entire source voltage appears across the output as shown in Fig. 8.30(*a*), which is represented as mode 1 (M_1).

In this mode we have, $v_{Ls} = 0$, $i_s = -I_o$, and $v_o = -v_s$. This mode continues while SCR_1 and SCR_2 remain off, until $t = t_1$, when they are triggered on. At this time, SCR_3 and SCR_4 remain on to maintain continuity of the inductor current. The resultant circuit is shown in Fig. 8.30(*b*) and designated as mode 2.

The inductor current, i_s, is obtained from the following relation:

$$v_{Ls} = L_s \frac{di_s}{dt} \qquad (8.30)$$
$$= v_s(t)$$

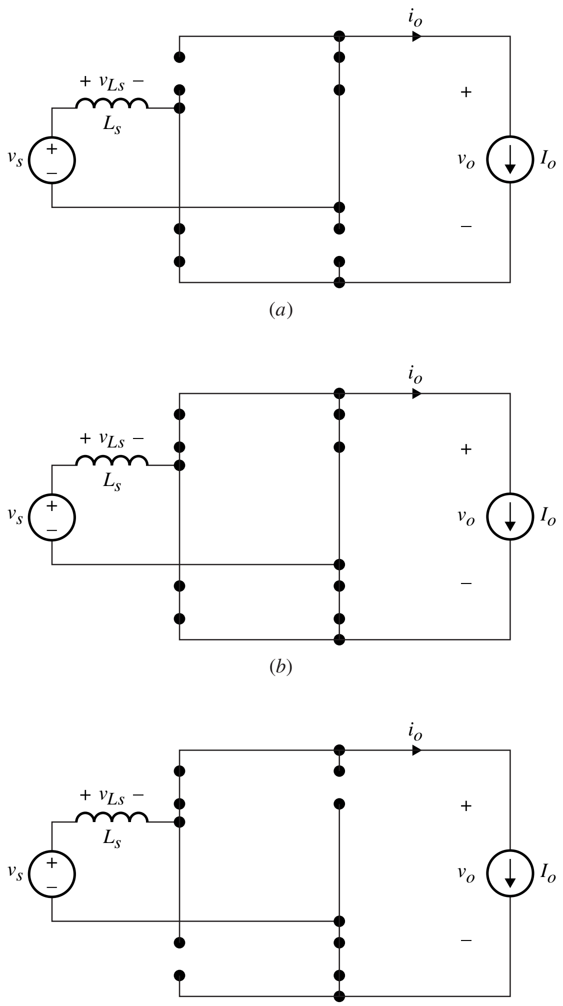


Figure 8.30 (*a*) Mode 1: $0 \le t \le t_1$.
(*b*) Mode 2: $t_1 \le t \le t_2$.
(*c*) Mode 3: $t_2 \le t \le T/2 + t_1$.

Using the initial condition $i_s(t_1) = -I_o$ in Eq. (8.30), we obtain

$$\begin{aligned} i_s(t) &= \int_{t_1}^{t} \frac{V_s}{L_s} \sin \omega t \; dt - I_o \\ &= \frac{V_s}{L_s \omega}(\cos \alpha - \cos \omega t) - I_o \end{aligned} \tag{8.31}$$

At $t = t_2$, the currents in SCR_3 and SCR_4 reach zero and the circuit enters mode 3, as shown in Fig. 8.30(*c*). In this mode we have $i_s = +I_o$, $v_{Ls} = 0$, and $v_o = v_s$.

Mode 4 starts at $t = T/2 + t_1$, when SCR_3 and SCR_4 are turned on, resulting in the same equivalent circuit of mode 2 with $i_s(t)$ given by

$$i_s(t) = \frac{-V_s}{L_s \omega}(\cos \alpha + \cos \omega t) + I_o \tag{8.32}$$

At $t = T/2 + t_2$, the current $i_s(t)$ reaches $-I_o$, resulting in mode 5, where SCR_3 and SCR_4 are on, and SCR_1 and SCR_2 are off, which is similar to mode 1. The cycle repeats at $t = T + t_1$, when SCR_1 and SCR_2 are triggered. To obtain an expression for the commutation angle, u, we evaluate Eq. (8.32) at $t = T/2 + t_2$ with $i_s(T/2 + t_2) = -I_o$; hence, we have

$$-I_o = I_o - \frac{V_s}{L_s \omega}[\cos \omega(T/2 + t_2) + \cos \omega t_1]$$

Using $u = \omega(t_2 - t_1)$, the commutation angle may be given by

$$u = \cos^{-1}\left(\cos \alpha - \frac{2 I_o \omega L_s}{V_s}\right) - \alpha \tag{8.33}$$

In terms of the normalized load current, Eq. (8.33) gives

$$u = \cos^{-1}(\cos \alpha - 2 I_{no}) - \alpha \tag{8.34}$$

The average output voltage is

$$\begin{aligned} V_o &= \frac{2}{T} \int_{t_2}^{T/2 + t_1} v_s(t) \; dt \\ &= \frac{2 V_s}{2\pi}(\cos \omega t_1 + \cos \omega t_2) \\ &= \frac{V_s}{\pi}(\cos \alpha + \cos(\alpha + u)) \end{aligned} \tag{8.35}$$

Substituting for $\cos(\alpha + u) = \cos \alpha - 2 L_s I_o / V_s$, we get

$$V_o = \frac{2 V_s}{\pi}\left(\cos \alpha - \frac{\omega L_s I_o}{V_s}\right) \tag{8.36}$$

In terms of normalized values, Eq. (8.36) becomes

$$V_{no} = \frac{2}{\pi}(\cos \alpha - I_{no}) \tag{8.37}$$

Figure 8.31 shows the regulation curve for I_{no}, v_s, and V_{no} under different firing angles.

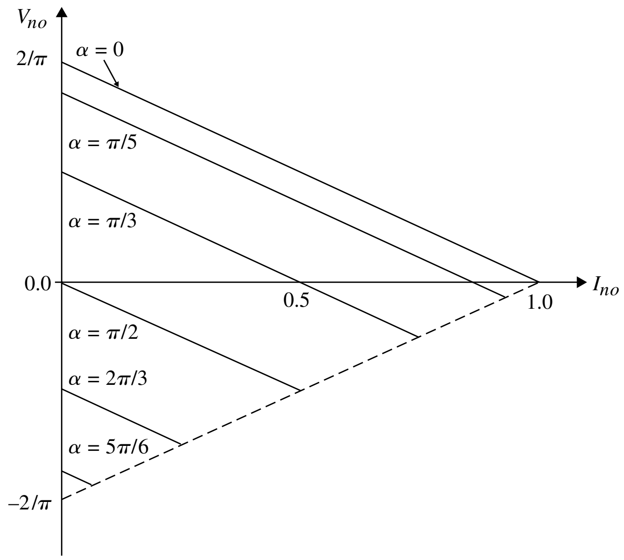


Figure 8.31 Regulation curve for the phase-controlled full-wave bridge converter with ac-side inductance.

EXERCISE 8.6

Consider the circuit given in Fig. 8.28. Assume $v_s = 110 \sin 2\pi(60)t$, $\alpha = 60°$, $L_s = 20$ mH, and $I_o = 10$ A. Determine the average output voltage V_o and the approximate rms input current. (*Hint*: Assume current commutation is linear.)

ANSWER –13 V, 8.15 A

EXERCISE 8.7

Consider the full-wave circuit of Fig. 8.28 with $L_s = 5$ mH, $V_s = 110$ V, $\alpha = 60°$, and $\omega = 377$ rad/s. Use Fig. 8.31 to determine:

(a) The average output voltage for $I_o = 10$ A

(b) The range of α if the local current changes from 10 A to 20 A while V_o remains the same as in part (*a*)

(c) The range of α if we assume the input voltage V_s changes by ±20% of its nominal value of 110 V while the output voltage remains constant as given in part (*a*)

ANSWER 23 V, $47.8° < \alpha < 60°$, $51.3° < \alpha < 65.4°$

8.5 THREE-PHASE CONTROLLED CONVERTERS

8.5.1 Half-Wave Converters

Figure 8.32(*a*) shows a phase-controlled half-wave rectifier circuit under a resistive load. Its current and voltage waveforms are shown in Fig. 8.32(*b*). Figure 8.32(*c*) shows the waveforms under a highly inductive load.

If we assume all SCRs are triggered simultaneously at $\alpha = \omega t_1$, then it is apparent from the circuit that only one positive source current can flow at any given time. This type of arrangement resembles the operation of an OR gate in digital systems. In practice, each SCR is triggered at α within its cycle; i.e., SCR_1, SCR_2, and SCR_3 are triggered at α, $2\pi/3 + \alpha$, and $4\pi/3 + \alpha$, respectively, as shown in Fig. 8.32(*b*). Figure 8.32(*b*) and

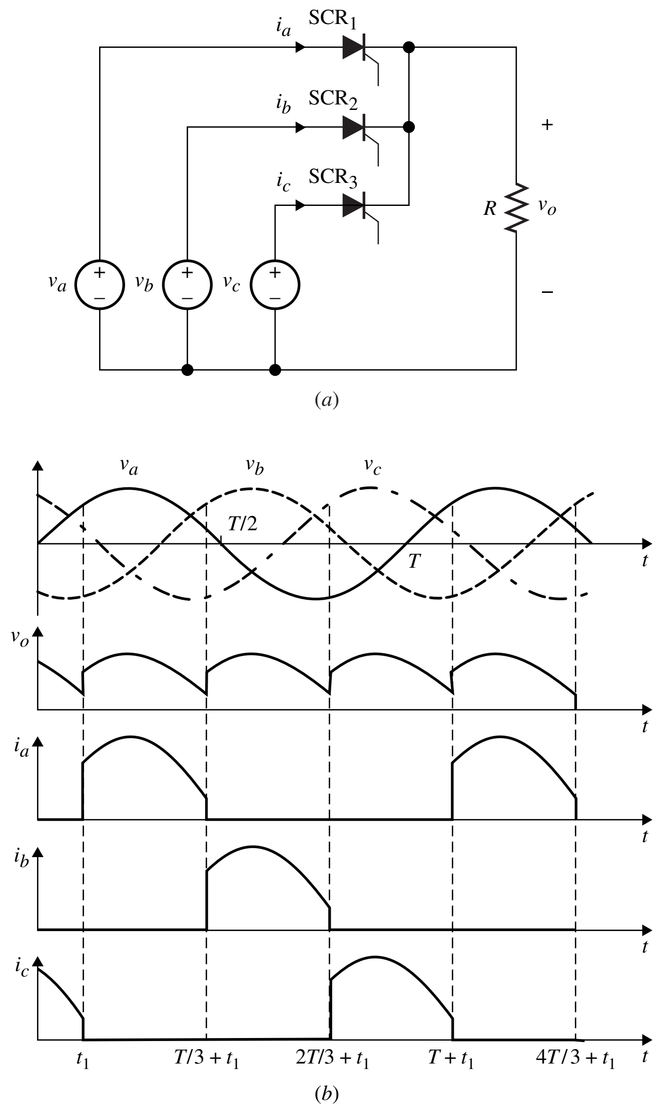


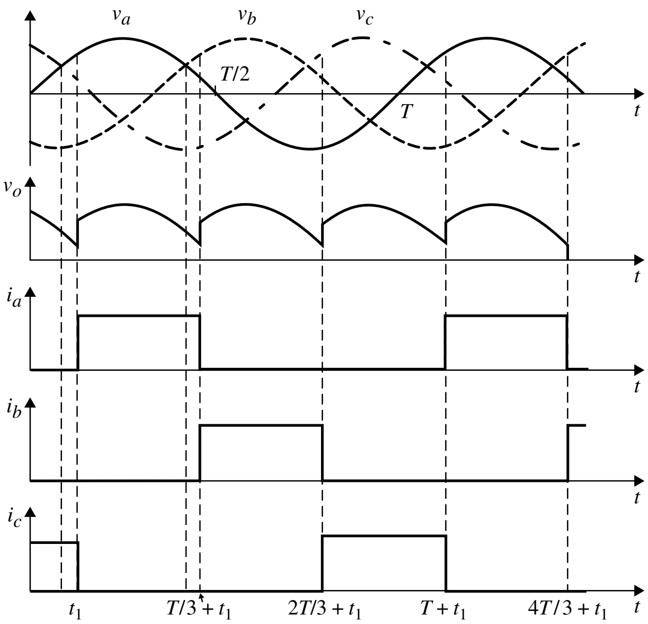
Figure 8.32 (*a*) Three-phase controlled half-wave rectifier. (*b*) Output voltage and line current waveforms under resistive load.

(*c*) shows relevant current and voltage waveforms under a purely resistive load, and a highly inductive load, respectively. Here we have assumed the three-phase voltages are position sequence with their line to natural voltages given as $v_a = V_s \sin \omega t$, $v_b = V_s \sin(\omega t - 120°)$, and $v_c = V_s \sin(\omega t - 240°)$.

The average output voltage is given by

$$\begin{aligned} V_o &= \frac{3}{T}\int_{t_1}^{T/3 + t_1} V_s \sin \omega t \; dt \\ &= \frac{-3V_s}{T\omega}[\cos \omega(T/3 + t_1) - \cos \omega t_1] \\ &= \frac{3V_s}{2\pi}[\cos \alpha - \cos \omega(T/3 + t_1)] \end{aligned} \tag{8.38}$$

where $\alpha = \omega t_1$.



(*c*)

Figure 8.32 (*continued*) (*c*) Waveforms under highly inductive load.

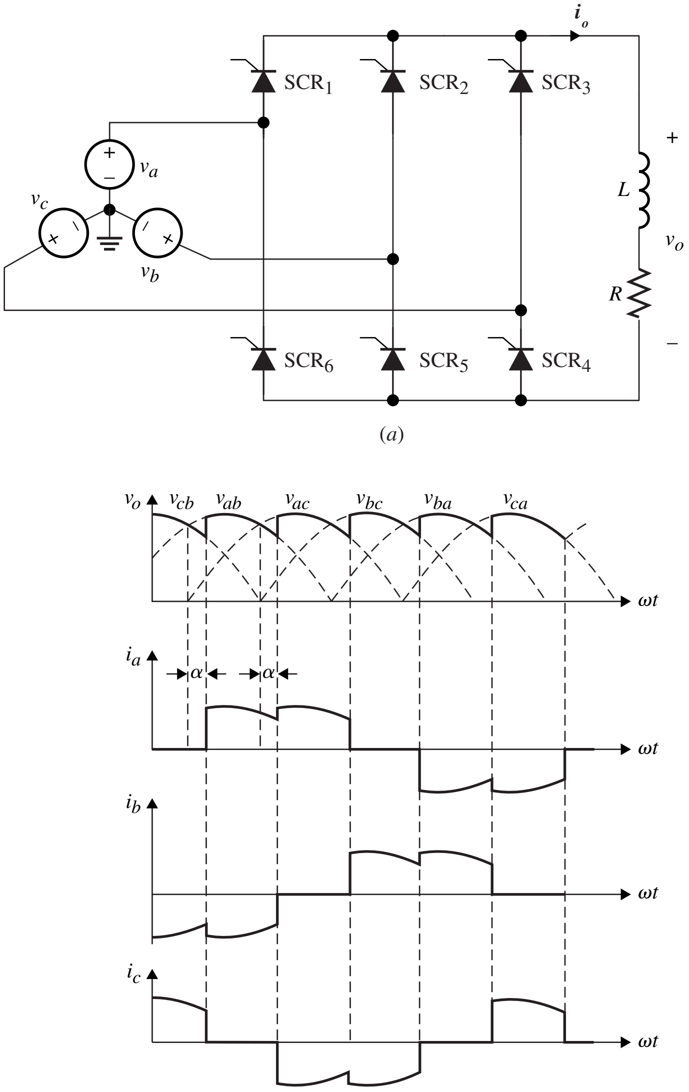
8.5.2 Full-Wave Converters

Figure 8.33(*a*) shows the phase-controlled full-wave three-phase converter under an inductive-resistive load with a Y-connected three-phase voltage source.

The waveforms for $L = 0$ and $L = \infty$ are shown in Fig. 8.33(*b*) and (*c*), respectively, using $\alpha = 15°$. To illustrate how the full-wave bridge circuit works, we will give a detailed mode-by-mode operation for $\alpha = 15°$. If we select a positive set of voltages as in the half-wave case, then we must obtain a line-to-line voltage sequence for the converter operation.

The line-to-line voltage set is v_{ab}, v_{bc}, and v_{ca}, given in Chapter 7 and whose waveforms are redrawn in Fig. 8.34(*a*), showing mode 1 to mode 6 for $\alpha = 0°$ (SRCs act like diodes) with the corresponding conducting SCRs. Figure 8.34(*b*) shows the firing sequence for $\alpha = 0°, 30°, 60°, 90°, 120°, 150°$, and $180°$. For $\alpha > 90°$, the voltage starts to become negative and the converter operates in the *inversion mode*. For simplicity, these waveforms are plotted against angle ωt. As shown in Fig. 8.34(*b*), the most positive rectified voltage in mode 1 (M_1) is $|v_{bc}|$. Hence, the conducting SCRs are SCR_3 and SCR_5, and the equivalent circuit is shown in Fig. 8.35(*a*). In this mode, $v_o = -v_{bc}$, $i_a = 0$, $i_b = -i_c = -I_o$ and the SCR voltages are: $v_{SCR1} = -v_{ca}$, $v_{SCR2} = v_{bc}$, $v_{SCR4} = v_{bc}$, $v_{SCR6} = -v_{ab}$. It is clear from these voltages that all voltages across the other SCRs are negative.

At $\omega t = 30° + \alpha$, v_{ab} becomes the most positive and SCR_1 and SCR_5 are triggered to start mode 2. At 60° later, as shown in Fig. 8.35, it can be shown that all the SCR voltages are negative in this mode also. Note from Fig. 8.35 that during each voltage phase, two gating signals are applied during the positive and negative cycles. For example, during the positive cycle of $v_{bc}(t)$, SCR_2 and SCR_6 are triggered with $\alpha = 45°$ at $\omega t = \pi/2 + \pi/4$, and SCR_3 and SCR_5 are triggered in the negative cycle at $\omega t = 3\pi/2 + \pi/4$.



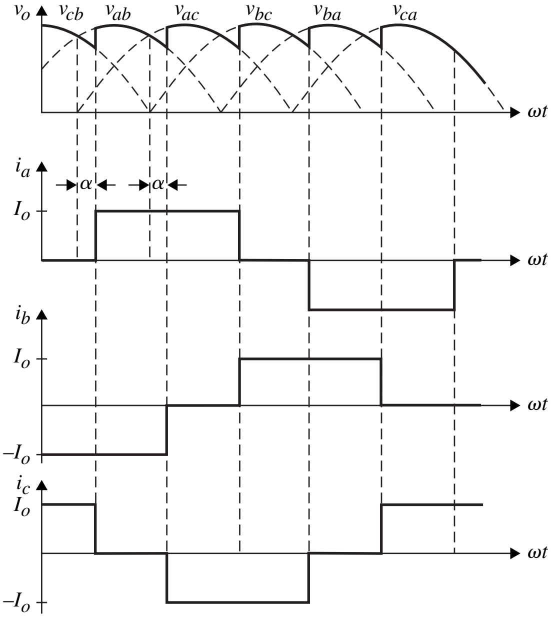
(b)

Figure 8.33 (*a*) Full-bridge three-phase controlled rectifier. (*b*) Output voltage and current waveforms under purely resistive load.

The output voltages for $\alpha = 0°$, $\alpha = 30°$, $\alpha = 60°$, and $\alpha = 90°$ are shown in Fig. 8.36(*a*), and the output voltages for $\alpha = 120°$, $\alpha = 150°$, and $\alpha = 180°$ are shown in Fig. 8.36(*b*). For clarity, Fig. 8.36 shows the absolute voltage values for v_{ab}, v_{bc}, and v_{ca}.

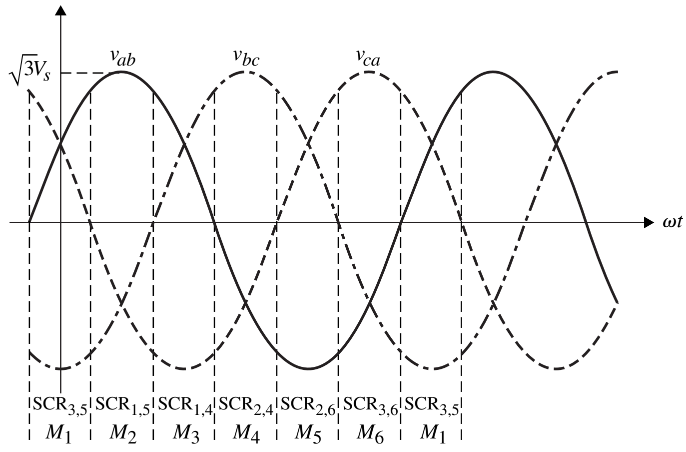
The average output voltage can be evaluated over any of the given six pulse intervals. Let us select the second pulse between t_1 and t_2 indicated in the phase voltages of Fig. 8.36(*a*). The average output voltage is given by

$$V_o = \frac{6}{T}\int_{t_1}^{t_2} v_{ab}\, dt$$



(*c*)

Figure 8.33 (*continued*) (*c*) Output voltage and current waveforms under highly inductive load.

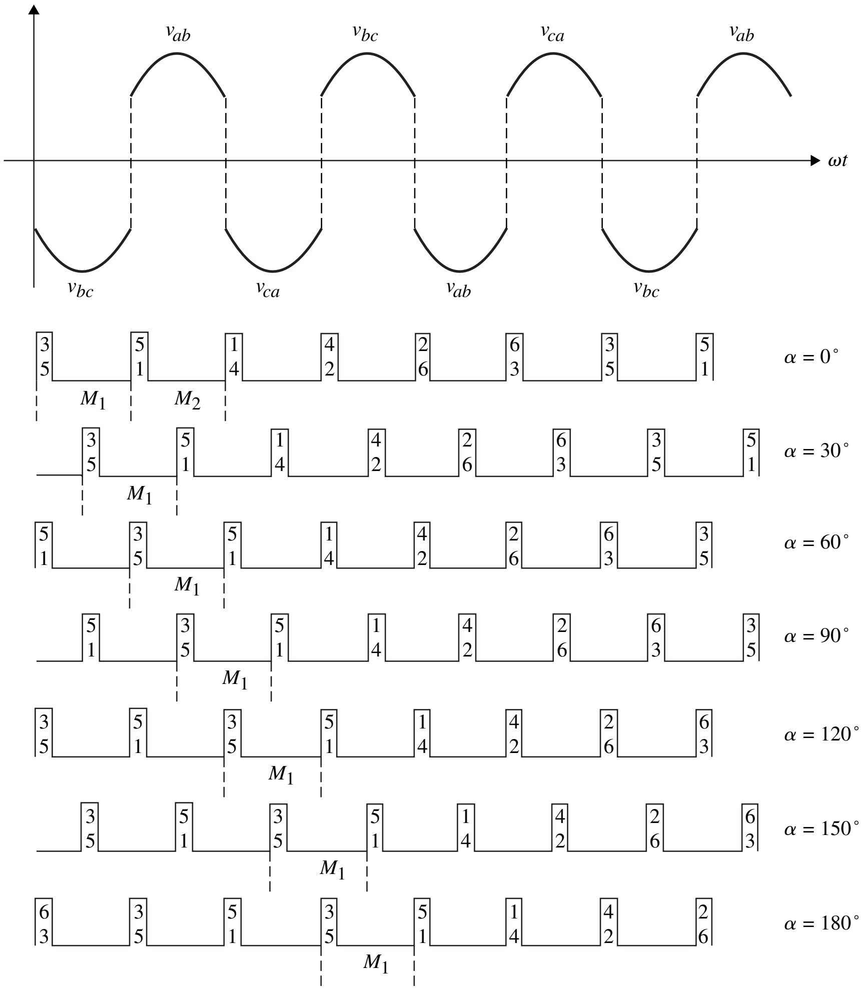


(*a*)

Figure 8.34 Three-phase line-to-line voltage. (*a*) Corresponding conduction modes.

Substituting for $v_{ab} = \sqrt{3}V_s \sin(\omega t + 30°)$ and $\omega t_1 = 30° + \alpha$ and $\omega t_2 = 90° + \alpha$, we have

$$
\begin{aligned}
V_o &= \frac{3}{\pi}\int_{\pi/6+\alpha}^{\pi/2+\alpha} \sqrt{3}V_s \sin\left(\omega t + \frac{\pi}{6}\right) d\omega t \\
&= \frac{-3\sqrt{3}}{\pi}V_s\left[\cos\left(\alpha + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{\pi}{6}\right)\right] \qquad (8.39) \\
&= \frac{3\sqrt{3}V_s}{\pi}\cos\alpha
\end{aligned}
$$



(*b*)

Figure 8.34 (*continued*) Three-phase line-to-line voltage. (*b*) Corresponding firing angles.

The natural conduction is at $\omega t = \pi/6$ if all SCRs are triggered continuously. In this case modes 1 to 6 are the same as in the previous chapter. In this section we are investigating the effect of delaying the trigger by α with respect to the angle of natural conduction. In Fig. 8.36, the natural conduction angle in M_1 is $-\pi/6$ when SCR_3 and SCR_5 are acting like diodes. Now let us consider M_1 by delaying the triggering of SCR_3 and SCR_5 by α. Prior to $\omega t = \alpha$, v_{ca} is the most positive; hence, SCR_3 and SCR_6 are conducting.

For illustration purposes, consider the case for $\alpha = 60°$ We notice that for $\alpha > 60°$, the output voltage starts becoming negative. For $\alpha = 90°$, the positive and negative areas are equal, resulting in a zero average output voltage.

EXERCISE 8.8

Show that the average output power for Fig. 8.35 with $L = \infty$ is given by Eq. (8.40).

$$P = \frac{3\sqrt{3}}{2\pi} V_s I_o \cos\alpha \tag{8.40}$$

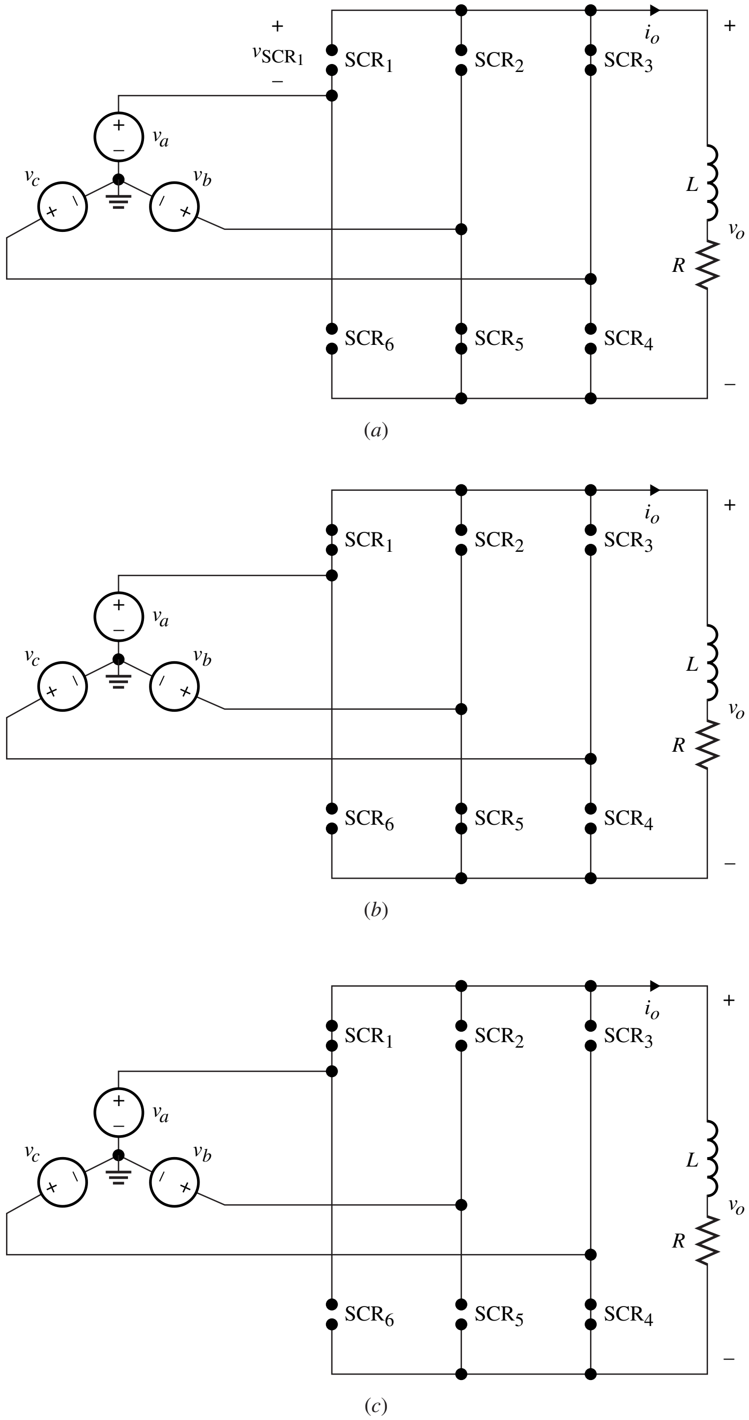
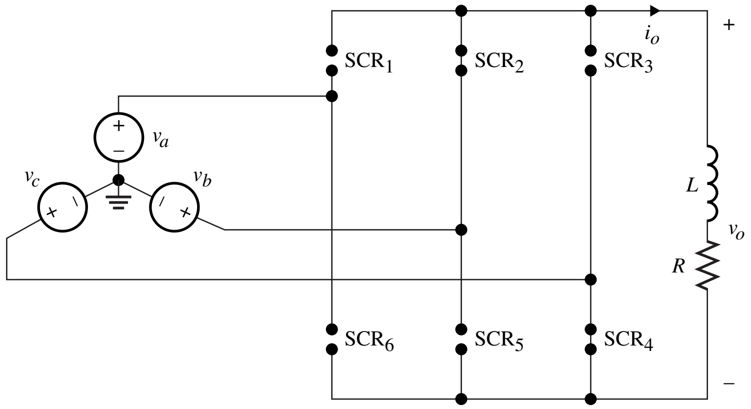
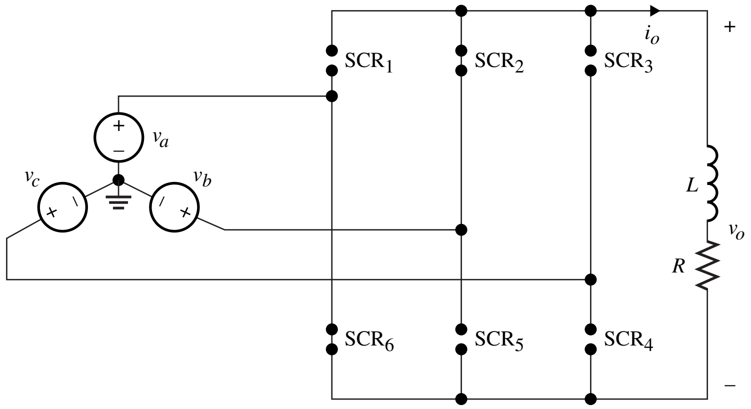


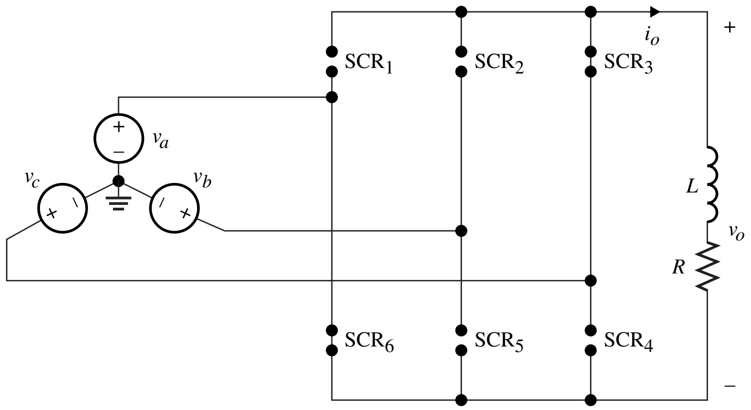
Figure 8.35 Equivalent circuits for the six modes of operation. (*a*) Mode 1. (*b*) Mode 2. (*c*) Mode 3.



(*d*)



(*e*)



(*f*)

Figure 8.35 (*continued*) Equivalent circuits for the six modes of operation. (*d*) Mode 4. (*e*) Mode 5. (*f*) Mode 6.

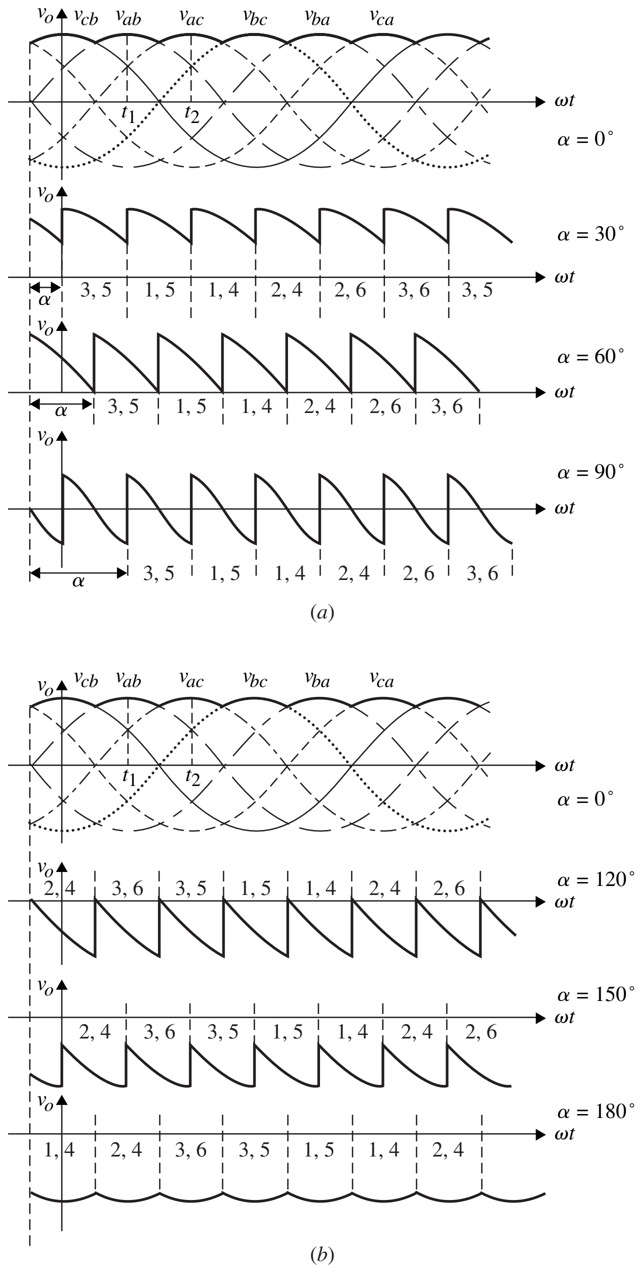


Figure 8.36 Output voltages for firing angles. (*a*) $\alpha = 0°$, $\alpha = 30°$, $\alpha = 60°$, and $\alpha = 90°$. (*b*) $\alpha = 120°$, $\alpha = 150°$, and $\alpha = 180°$

PROBLEMS

Single-Phase Half-Wave Rectifiers

8.1 Derive the power factor and THD expressions for the half-wave phase-controlled rectifier given in Eqs. (8.5) and (8.6).

8.2 Calculate the average power delivered to a resistive load in a half-wave phase-controlled rectifier with $v_s = 200 \sin 377t$, $\alpha = 30°$, and $R_L = 25\ \Omega$.

8.3 Derive the load current equation, i_o, for the inductive-load phase-controlled rectifier with a freewheeling diode in Fig. 8.8(*a*).

8.4 Determine the delay angle α such that the total power to be delivered to a $20\ \Omega$ load resistance with $\omega L = 20\ \Omega$ in Fig. 8.8(*a*) equals 40 W. Assume $v_s = 100 \sin 377t$.

Single-Phase Full-Wave Rectifiers

8.5 An alternative way to make a full-wave bridge rectifier is to use a center-tap transformer as shown in Fig. P8.5.

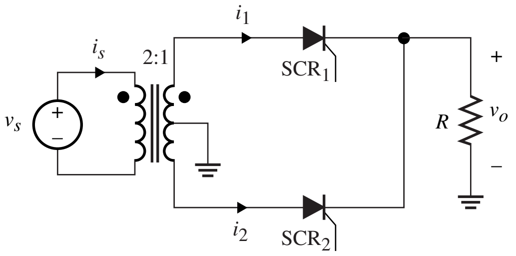


Figure P8.5

(a) Sketch the waveforms for i_s, i_1, i_2, and v_o and determine the power factor.

(b) Determine α for a power factor equal to 0.95. Assume $v_s = 100 \sin 377t$ and $\alpha = 40°$.

8.6 (a) Determine the input power factor for the inductive-load phase-controlled full-wave rectifier shown in Fig. P8.6. Assume $v_s = 100 \sin 377t$, $\alpha = 60°$, and the load current is constant.

(b) Determine α for a power factor equal to 0.75.

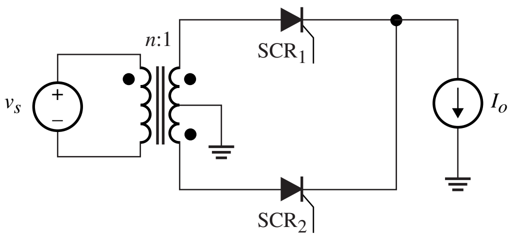


Figure P8.6

8.7 Figure P8.7 shows a full-wave phase-controlled rectifier with an inductive load and a free-wheeling diode.

(a) Sketch the waveform for v_o and give the expression for the average output voltage.

(b) Show that the load current is given by the following two equations:

$$i_o(t) = I_{o1}e^{-(t-t_1)/\tau} + \frac{V_S}{|Z|}\sin[\omega(t-t_1)]$$

$$t_1 \le t \le \frac{T}{2}$$

$$i_o(t) = I_{o2}e^{-(t-T/2)/\tau} \qquad \frac{T}{2} \le t \le T - t_1$$

where

$$|Z| = \sqrt{(\omega L)^2 + R^2},\ \alpha = \omega t_1,\ \theta = \tan^{-1}\frac{\omega L}{R},$$

$$\tau = L/R$$

$$I_{o1} = e^{-(\pi + \alpha/\tau\omega)} + e^{\alpha/\tau\omega}\sin(\alpha + \theta)$$

$$I_{o2} = e^{-(\pi/\tau\omega)} + e^{\alpha/\tau\omega}\sin(\alpha + \theta)$$

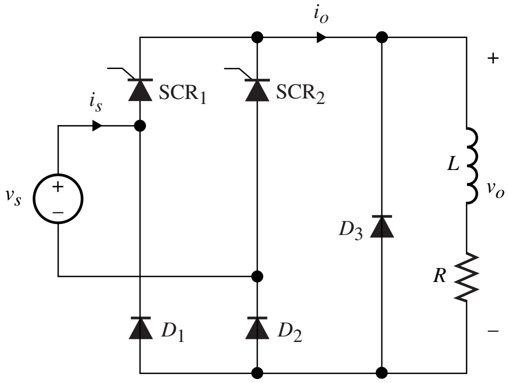


Figure P8.7

8.8 Derive the THD for the full-bridge phase-controlled rectifier of Fig. 8.13.

8.9 Show that the average power delivered to the load in Fig. 8.25 is zero when

$$\alpha = \cos^{-1}\frac{V_{dc}\pi}{2V_s}$$

8.10 Sketch the waveforms for i_s, i_o, and v_o, and determine the average output voltage for the circuit of Fig. P8.10, under the conditions (*a*) $C = 0$ and (*b*) $C = \infty$, respectively. Assume the devices are triggered in a similar way to those of Fig. 8.11.

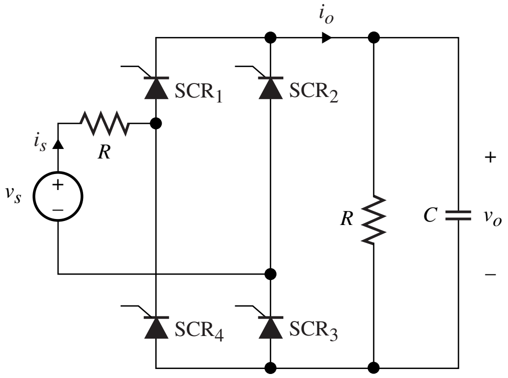


Figure P8.10

8.11 Consider a power electronic circuit that generates the waveforms i_s and v_s shown in Fig. P8.11. Determine the THD.

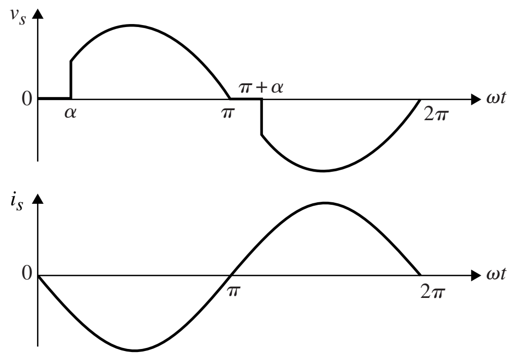


Figure P8.11

8.12 Sketch the waveforms for i_s and v_o for Fig. P8.12. Assume $L/R >> T$, $v_s = V_s \sin \omega t$, and $V_{dc} < V_s$.

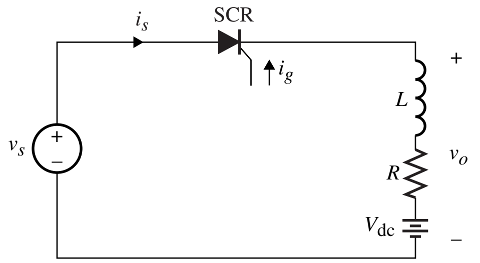


Figure P8.12

8.13 Repeat Exercise 8.5 for $\alpha = 150°$.

8.14 Sketch the waveforms for v_o, i_{SCR1}, and i_{SCR2} using the circuit shown in Fig. P8.15 and obtain the expression for the fundamental component of $i_s(t)$. Assume v_s is a squarewave of amplitude $\pm V_s$.

8.15 Calculate the input pf for $I_o = 5$ A and $\alpha = 60°$, and the current THD for Problem 8.14.

8.16 Consider the full-wave phase-controlled circuit shown in Fig. P8.16. Calculate: (*a*) the average output voltage V_o, (*b*) the average output current I_o, (*c*) the rms values of $i_s(t)$, and (*d*) the input power factor. Assume $L/R >> T/2$ and $v_s = 110 \sin 377t$ and $\alpha = 45°$.

8.17 Repeat Problem 8.16 for $\alpha = 150°$.

8.18 Figure P8.18 shows a half-wave phase-controlled converter with a flyback SCR_2. Assume $v_s = 110 \sin 377t$.

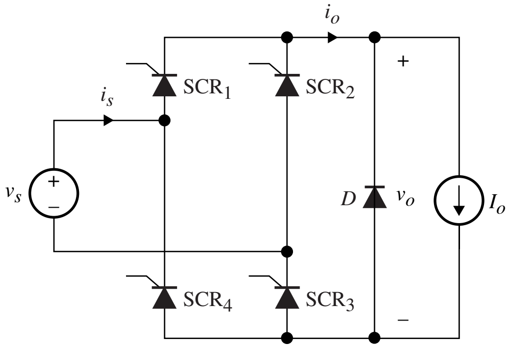


Figure P8.15

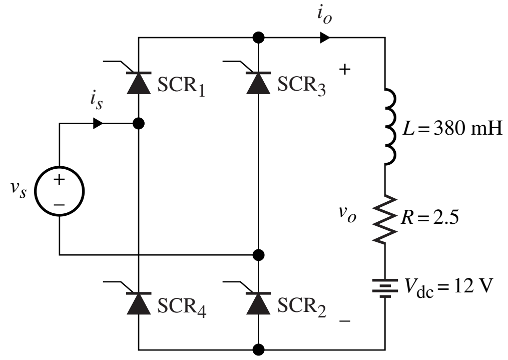


Figure P8.16

(a) Calculate V_o, u, and the power delivered to the load for $\alpha = 60°$.

(b) If the load resistance is reduced by 50%, find the new value of α for maintaining the same load current.

(c) Repeat part (*b*) by assuming L_s has increased by 20%.

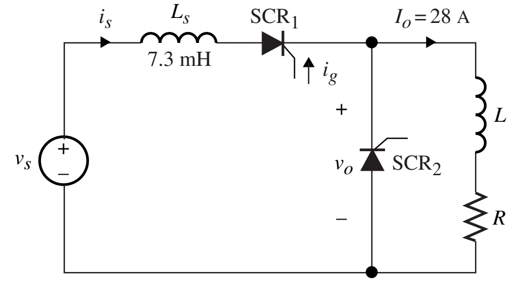


Figure P8.18

8.19 Calculate the rms value for $i_s(t)$ given in Problem 8.18 by assuming that the commutation period is linear.

8.20 Calculate the average load current in the circuit shown in Fig. P8.20. Assume $\alpha = 60°$ and $v_s = 110\sin377t$.

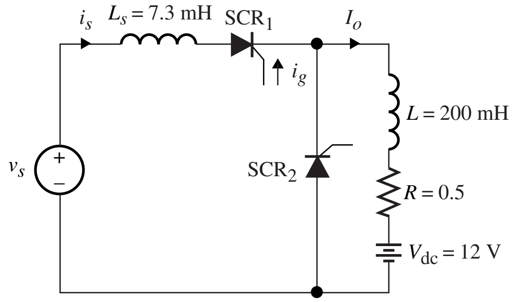


Figure P8.20

8.21 Draw the waveforms for i_s, i_{D1}, i_{s1}, and v_o in Fig. P8.21 under the following load conditions: (*a*) $L = 0$, (*b*) $L = \infty$. Label your axes and show which device(s) conduct for each time interval. Assume $v_s = V_s \sin \omega t$ and the firing angle is 30°.

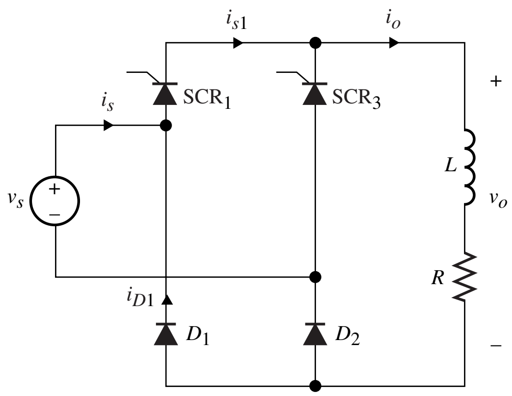


Figure P8.21

8.22 Consider the SCR circuit in Fig. P8.22 with $\alpha = 30°$. Assume v_{s1} and v_{s2} are 180° out of phase.

(a) Sketch the waveforms for i_{s1}, i_{s2}, and v_o for two periods.

(b) Derive the expressions for i_{s1}, i_{s2}, and v_o.

(c) Find the commutation interval during which both SCRs are conducting.

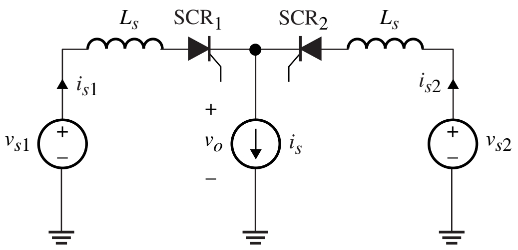


Figure P8.22

D8.23 Consider the half-controlled diode/SCR full-bridge circuit shown in Fig. P8.23. Assume the input is given by $v_s(t) = V_s \sin \omega t$. Assume the firing angle is $\alpha = 30°$.

(a) Sketch v_o, i_{D1}, i_{D2}, i_{SCR1}, i_{SCR2}, and i_s.

(b) Derive the expression for v_o.

(c) Design for α so that the average output voltage is 48 V. Assume $v_s(t) = 100 \sin 377t$ V.

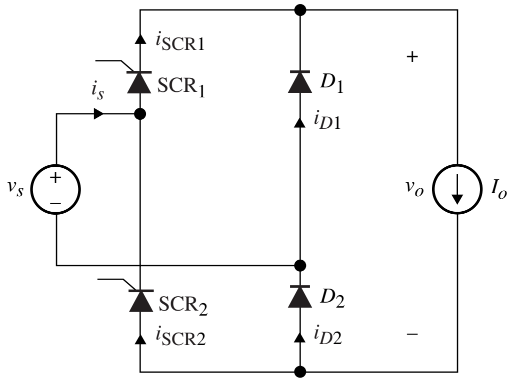


Figure P8.23

D8.24 Consider the controlled half-wave rectifier of Fig. 8.26(*a*) with $L_s = 7.3$ mH, $\alpha = 60°$, which supplies an average load current of 20 A. Assume $v_s = 110 \sin(2\pi 60t)$.

(a) Use Fig. 8.27 to determine the average output voltage.

(b) Determine the average power delivered to the load.

(c) Find the commutation angle.

(d) Design for the inductor L in the load side; $V_o = 8.75$ V and $R = 0.44\ \Omega$.

(e) If the load resistance is decreased by 50%, what is the new α needed to maintain the same average output voltage?

(f) If the source voltage is increased by 20%, what is the new α needed to maintain the same average load?

Three-Phase Converters

8.25 The three-phase half-wave controlled rectifier with a commutating diode across the load is shown in Fig. P8.25. Assume that the voltage drops across the thyristor are negligible and the supply voltage is 100 V rms. For a firing delay angle of 45°, sketch the waveform of v_o and determine its average value.

8.26 **(a)** Sketch the waveforms for v_o, i_{SCR3}, and i_o for the three-phase controlled rectifier shown in Fig. P8.26 for $\alpha = 30°$ and $\alpha = 60°$.

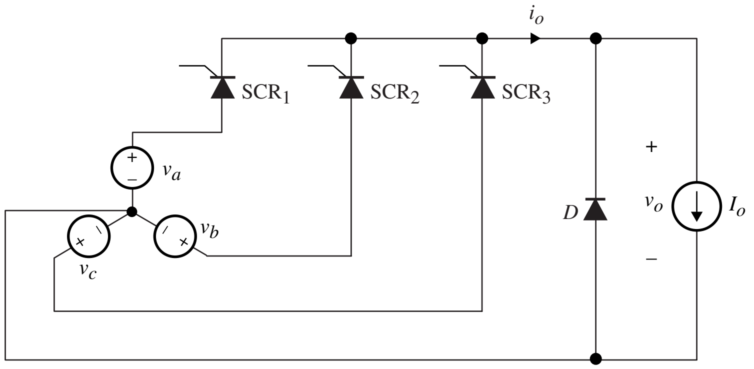


Figure P8.25

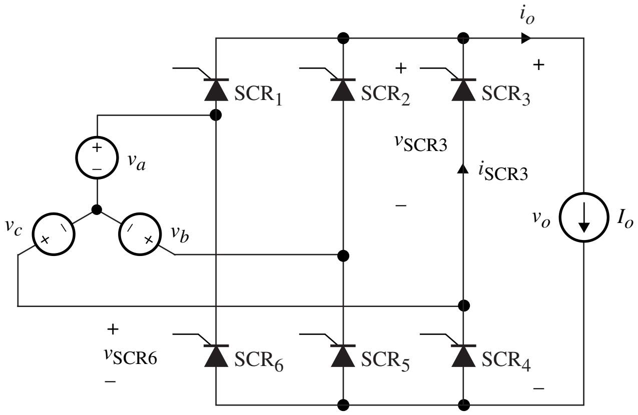


Figure P8.26

(b) Derive the expression for the average output voltage.

(c) Sketch the waveform of the voltage across SCR_6 and SCR_3. Assume $v_a = V_s \sin \omega t$, $v_b = V_s \sin(\omega t - 2\pi/3)$, and $v_c = V_s \sin(\omega t - 4\pi/3)$.

8.27 Determine the average output voltage for Problem 8.26 using $V_s = 110$ V.