Chapter 7

Uncontrolled Diode Rectifier Circuits

INTRODUCTION

For nearly a century, rectifier circuits have been the most common power electronic circuits used to convert ac to dc. The word *rectification* is used not because these circuits produce dc, but rather because the current flows in one direction; only the *average* output signal (voltage or current) has a dc component. Moreover, since these circuits allow power to flow only from the source to the load, they are often termed *unidirectional converters*. As will be shown shortly, when rectifier circuits are used solely, their outputs consist of dc along with high-ripple ac components. To significantly reduce or eliminate the output ripple, additional filtering circuitry is added at the output. In the majority of applications, diode rectifier circuits are placed at the front end of the power electronic 60 Hz systems, interfaced with the sine-wave voltage produced by the electric utility. In dc-to-dc application, at the rectified side or the dc side, a large filter capacitor is added to reduce the rectified voltage ripple. This dc voltage maintained across the output capacitor is known as raw dc or *uncontrolled dc*.

The principal circuit operations of the various configurations of rectifier circuits are similar, whether half- or full-wave, single-phase or three-phase. Such circuits are said to be *uncontrolled* since the rectified output voltage and current are a function only of the applied excitation, with no mechanism for varying the output level. In such circuits, regardless of the configuration, diodes are almost always used to achieve rectification. This is because diodes are inexpensive, are readily available with low and high power capabilities, and have terminal characteristics that are simple and well understood. However, since diode circuits are nonlinear, i.e., their circuit topological modes vary with time, their analysis can be challenging, as illustrated in Chapter 3. To simplify the analysis it will be assumed throughout this chapter that the diodes are ideal as discussed in Chapter 2. In this chapter, we will discuss the single- and three-phase uncontrolled rectifier circuits of both the half- and full-bridge configurations as shown in the general block diagram representation in Fig. 7.1.

The analyses of rectifier circuits will include resistive, inductive, and capacitive loads, as well as loads with dc sources. We should point out that *rectification* is a term generally used for converting ac to dc with power flowing unidirectionally to the load, whereas *inversion* is a term used when dc power is converted to ac power. In diode and SCR circuits (SCR will be discussed in Chapter 8), inversion refers to the process in which power flow is from the dc side to the ac side. Another important point is that the circuits in this chapter are also known as *naturally commutating* or *line-commutating converters* since the diodes are naturally turned off and on by being connected directly to a single- or three-phase ac line supply.

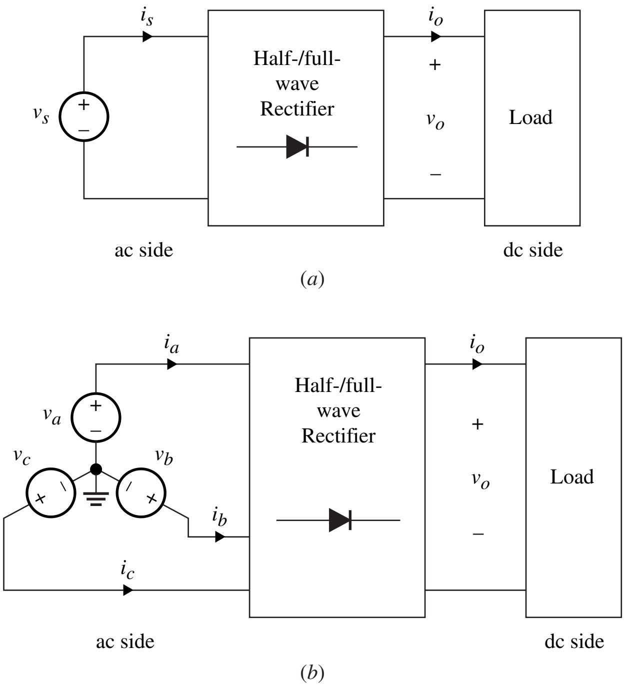


Figure 7.1 Block diagram representation for (*a*) single-phase and (*b*) three-phase rectifier circuits.

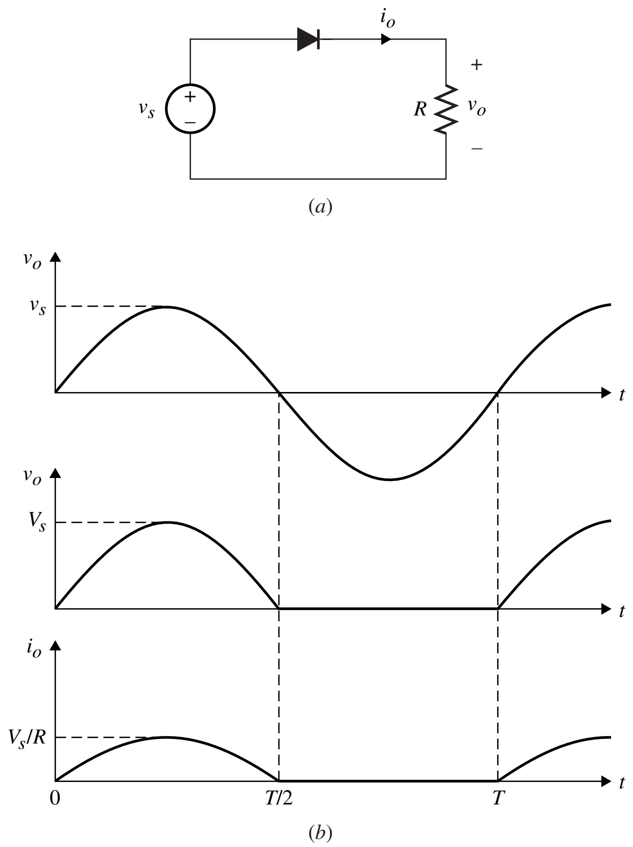


Figure 7.2 (*a*) Half-wave rectifier circuit. (*b*) Voltage and current waveforms.

7.1 SINGLE-PHASE RECTIFIER CIRCUITS

7.1.1 Resistive Load

Figure 7.2(*a*) shows the simplest single-phase, half-wave rectifier circuit under a resistive load. The diode conducts only when the source voltage v_s is positive, as shown in Fig. 7.2(*b*). Throughout this chapter and the next, the source voltage v_s is given by $v_s(t) = V_s \sin \omega t$, where V_s is the peak voltage of the source and ω is its frequency. At $t = T/2$, when v_s becomes negative, the diode ceases to conduct since it does not allow a negative current to flow through it.

The average value of the output voltage, V_o, which represents the dc component of the output signal, v_o, is calculated by evaluating the following integral over one period, T:

$$\begin{aligned} V_o &= \frac{1}{T}\int_0^T v_o(t)\,dt \\ &= \frac{1}{T}\int_0^{T/2} V_s \sin \omega t \; dt \\ &= \frac{V_s}{\pi} \end{aligned} \tag{7.1}$$

The average load current is given by

$$\begin{aligned} I_o &= \frac{V_o}{R} \\ &= \frac{V_s}{\pi R} \end{aligned} \tag{7.2}$$

We observe that the output voltage waveform contains an ac component (ripple) at the same frequency as the ac source; hence, the output dc component is only 32% of the peak applied voltage ($V_o = 0.32 V_s$). Because of their low dc output and high ripple voltages, half-wave rectifier circuits are not practical and are limited to low-voltage applications.

EXAMPLE 7.1

Consider the half-wave rectifier circuit of Fig. 7.2(a) with a resistive load of 25 Ω and a 60 Hz ac source of 110 V rms.

(a) Calculate the average values of v_o and i_o.

(b) Calculate the rms values of v_o and i_o.

(c) Calculate the average power delivered to the load.

(d) Repeat part (c) by assuming that the source has a resistance of 60 Ω.

SOLUTION

(a) The average values of v_o and i_o are given by

$$\begin{aligned} V_o &= \frac{V_s}{\pi} \\ &= \frac{\sqrt{2}(110\ \text{V})}{\pi} \\ &= 49.52\ \text{V} \end{aligned}$$

and

$$\begin{aligned} I_o &= \frac{V_o}{R} \\ &= 1.98\ \text{A} \end{aligned}$$

(b) The rms value of the output voltage is expressed as

$$\begin{aligned} V_{o,\text{rms}} &= \sqrt{\frac{1}{T}\int_0^T v_o^2\ dt} \\ &= \sqrt{\frac{1}{T}\int_0^{T/2} V_s^2 \sin^2 \omega t\ dt} \end{aligned} \tag{7.3}$$

Substituting for $\sin^2 \omega t = (1 - \cos 2\omega t)/2$, Eq. (7.3) gives $V_{o,\text{rms}} = 77.78$ V.

The rms current is given by

$$\begin{aligned} I_{o,\text{rms}} &= \frac{V_{o,\text{rms}}}{R} \\ &= 3.11\ \text{A} \end{aligned}$$

(c) The average power delivered to the load over one cycle is obtained from the following relation:

$$\begin{aligned} P_o &= \frac{1}{T}\int_0^T i_o v_o \, dt \\ &= \frac{1}{TR}\int_0^{T/2} v_o^2 \, dt \\ &= \frac{V_{o,\text{rms}}^2}{R} \\ &= 242 \text{ W} \end{aligned}$$

(d) With a 60 Ω source resistance, R_s, the average power is given by

$$\begin{aligned} P_o &= \frac{R V_{o,\text{rms}}^2}{(R+R_s)^2} \\ &= 20.93 \text{ W} \end{aligned}$$

EXERCISE 7.1

Calculate the efficiency of the circuit of Example 7.1 for parts (*c*) and (*d*).

ANSWER 100%, 54.5%.

A more practical circuit is the full-wave rectifier shown in Fig. 7.3(*a*). Its typical waveforms are shown in Fig. 7.3(*b*).

The circuit operates as follows: When $v_s > 0$, D_1 and D_4 turn on, and D_2 and D_3 turn off. The voltage across D_2 and D_3 is $-v_s$, which keeps them off. At $t = T/2$, the negative source voltage stops D_1 and D_4 from conducting and causes D_2 and D_3 to begin conducting, until $t = T$, when the cycle repeats. The average output voltage and average output current are twice that of the half-wave rectifier under a resistive load.

EXERCISE 7.2

Calculate the rms and average diode current for $v_s = 120\sqrt{2}\sin(2\pi 60t)$ V and $R = 25$ Ω in Fig. 7.2(*a*).

ANSWER 3.4 A rms, 2.16 A

EXERCISE 7.3

Repeat Example 7.1 for the full-wave rectifier circuit given in Fig. 7.3(*a*).

ANSWER 99 V, 3.96 A, 110 V rms, 4.4 A rms, 484 W, 41.87 W

7.1.2 Inductive Load

An increase in the conduction period of the load current of Fig. 7.2 can be achieved by adding an inductor in series with the load resistance as shown in Fig. 7.4(*a*), resulting in a half-wave rectifier circuit under an inductive load. This means the

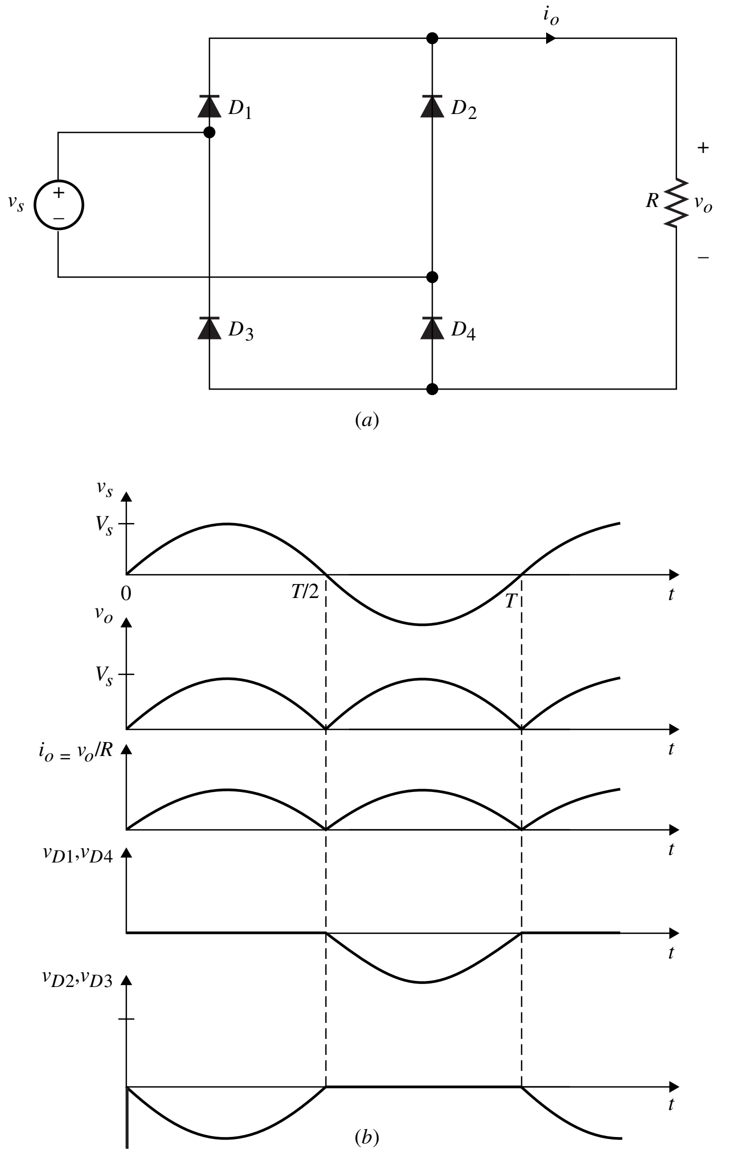


Figure 7.3 (*a*) Full-wave rectifier with resistive load. (*b*) Current and voltage waveforms.

load current flows not only during $v_s > 0$, but also for a portion of $v_s < 0$. The diode is kept in the *on* state by the inductor's voltage, which offsets the negative voltage of $v_s(t)$. The load current is present between $T/2$ and T, but never for the entire period, regardless of the inductor size. This can be easily explained by assuming that the diode conducts for the entire period. Consequently, the output voltage v_o must equal v_s, since the diode voltage is zero. This can occur only when the load current is alternating. This is clearly a contradiction, and there must be a time in which the diode stops conducting. Figure 7.4(*b*) shows the steady-state waveforms for the circuit of Fig. 7.4(*a*). The equivalent circuit mode for $t < t_2$ and $t > t_2$ are shown in Fig. 7.4(*c*) and (*d*), respectively, where t_2 is the time when D stops conducting.

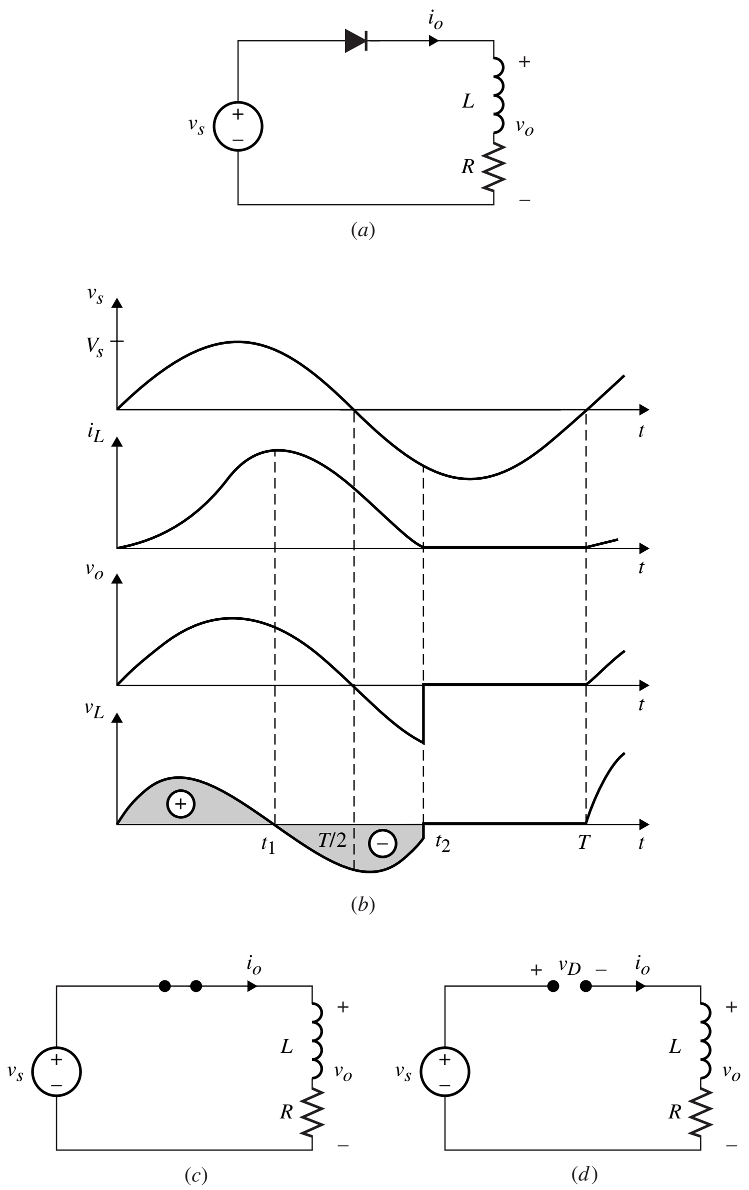


Figure 7.4 (*a*) Half-wave rectifier circuit with inductive load. (*b*) Current and voltage waveforms. (*c*) Equivalent circuit for $0 \le t \le t_2$. (*d*) Equivalent circuit for $t_2 \le t \le T$.

In steady state, the average value of the inductor voltage is zero. This is indicated by the equal positive and negative areas of v_L in Fig. 7.4(*b*). At $t = t_1$, the load current reaches the peak, at which $v_L = 0$. The average value of v_o is calculated from Fig. 7.4(*b*) as follows:

$$\begin{aligned} V_o &= \frac{1}{T}\int_0^{t_2} V_s \sin \omega t \; dt \\ &= \frac{V_s}{2\pi}(1 - \cos \omega t_2) \end{aligned} \tag{7.4}$$

The times t_1 and t_2 can be determined from the output current equation, to be derived next. A mathematical expression for the output current, i_o, can be obtained by considering the equivalent circuit for $0 \le t \le t_2$ shown in Fig. 7.4(c). The first-order differential equation that mathematically represents this circuit is given by

$$\frac{di_o}{dt} + \frac{R}{L} i_o = \frac{1}{L} V_s \sin \omega t \tag{7.5}$$

Solving Eq. (7.5) for i_o requires obtaining both the transient, $i_{o,\mathrm{tr}}(t)$, and steady-state, $i_{o,\mathrm{ss}}(t)$, solutions, as given by

$$i_o(t) = i_{o,\mathrm{tr}}(t) + i_{o,\mathrm{ss}}(t)$$

The transient current response, also known as the *natural response*, is obtained by setting the forced excitation, v_s, to zero, resulting in the following current equation:

$$i_{o,\mathrm{tr}}(t) = I_{o,i} e^{-t/\tau} \tag{7.6}$$

where

$I_{o,i}$ = initial value of $i_{o,\mathrm{tr}}(t)$ at $t = 0$

τ = circuit time constant, equal to L/R

The steady-state or *forced* response is obtained by using the equivalent phasor circuit shown in Fig. 7.5. The phasor current is given by

$$I_{o,\mathrm{ss}} = \frac{V_s \angle 0°}{|Z| \angle \theta} = \frac{V_s}{|Z|} \angle{-\theta} \tag{7.7}$$

where

$$|Z| = \sqrt{(\omega L)^2 + R^2}$$

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

In the time domain, we obtain the steady-state output current response as

$$i_{o,\mathrm{ss}}(t) = \frac{V_s}{|Z|} \sin(\omega t - \theta) \tag{7.8}$$

The total solution for i_o is obtained by adding Eqs. (7.6) and (7.8), to yield

$$i_o(t) = I_{o,i} e^{-t/\tau} + \frac{V_s}{|Z|} \sin(\omega t - \theta) \tag{7.9}$$

The initial value, $I_{o,i}$, is obtained from evaluating Eq. (7.9) at $t = 0^+$ with $i_o(0^+) = 0$, to give

$$I_{o,i} = \frac{V_s \omega L}{|Z|^2} \tag{7.10}$$

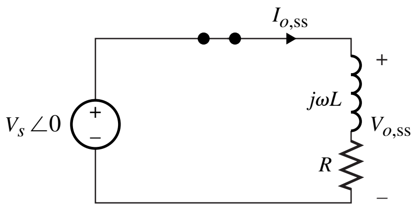


Figure 7.5 Equivalent phasor circuit for Fig. 7.4(c).

The time at which the diode stops conducting, $t = t_2$, must be determined in order to evaluate the average load voltage given in Eq. (7.4). Evaluating Eq. (7.9) at $t = t_2$, with $i_o(t_2) = 0$, we obtain the following expression in terms of t_2:

$$t_2 = -\tau \ln\left[-\sqrt{1+\left(\frac{R}{\omega L}\right)^2}\sin(\omega t_2 - \theta)\right] \tag{7.11}$$

Since t_2 is in both sides of Eq. (7.11), a numerical analysis method is required to solve for it. By normalizing the average output voltage as $V_{no} = V_o/V_s$, we can obtain a plot of this normalized value versus a normalized load resistance (R_n), which is defined as

$$R_n = \frac{\omega L}{R} \tag{7.12}$$

In terms of R_n, the average output voltage is expressed by

$$V_{no} = \frac{1-\cos\omega t_2}{2\pi} \tag{7.13a}$$

where

$$\omega t_2 = -R_n \ln\left[-\sqrt{1+\frac{1}{R_n^2}}\sin(\omega t_2 - \theta)\right] \tag{7.13b}$$

$$\theta = \tan^{-1} R_n \tag{7.13c}$$

Note that, unlike the resistive half-wave rectifier circuit, this circuit experiences load regulation since the average output voltage varies with the change of the load.

One important design criterion in power converters is what is known as *load regulation.* This parameter gives a measure of how the output voltage deviates from its desired value when the load changes. If we assume the output voltage takes on a minimum value at low load, V_{ol}, and a maximum value at full load, V_{of}, then the load regulation ($\mathrm{LR_{load}}$) is defined by

$$\mathrm{LR_{load}} = \left|\frac{V_{of} - V_{ol}}{V_{of}}\right| \times 100\% \tag{7.14}$$

Another important parameter is *line regulation*, $\mathrm{LR_{line}}$, which gives a measure of the output variation when the line input voltage changes. The line regulation is defined as

$$\mathrm{LR_{line}} = \frac{V_{o,\max} - V_{o,\min}}{V_{o,\min}} \times 100\% \tag{7.15}$$

where $V_{o,\max}$ and $V_{o,\min}$ are the average output voltages at maximum and minimum line voltages, respectively.

EXAMPLE 7.2

Consider the inductive-load half-wave rectifier circuit of Fig. 7.4(*a*) with $v_s = 120 \sin 377t$, $L = 60$ mH, and $R = 12\ \Omega$. Determine: (*a*) V_o, (*b*) I_o, (*c*) $\mathrm{LR_{line}}$ if V_s is changed by ±20%, and (*d*) $\mathrm{LR_{load}}$ if R decreases by 50%.

SOLUTION

(a) The normalized average output voltage for $\omega L/R \approx 1.9$ is obtained from Eqs. (7.11) and (7.13*a*).

$$V_{no} = 0.213$$

Then we obtain the average output voltage as

$$V_o = V_s V_{no,\mathrm{ave}}$$
$$= 25.56 \text{ V}$$

(b) The average output current is

$$I_o = \frac{V_o}{R}$$
$$= 2.13 \text{ A}$$

(c) The minimum and maximum peak input voltages are given as

$$V_{s,\min} = 96 \text{ V}$$
$$V_{s,\max} = 144 \text{ V}$$

The corresponding average outputs are

$$V_{o,\min} = 20.44 \text{ V}$$
$$V_{o,\max} = 30.67 \text{ V}$$

The line regulation is obtained from Eq. (7.15) as $\mathrm{LR}_{\mathrm{line}} = 50\%$.

(d) With the load resistance decreasing by 50%, the new value is $R = 6\ \Omega$, resulting in $\omega L/R = 3.77$ and $V_{no} = 0.156$. This gives a new value of the average output voltage equal to 18.72 V. The resultant load regulation is $\mathrm{LR}_{\mathrm{load}} = 26.73\%$. This is considered very poor load regulation.

A more useful half-wave rectifier circuit is one in which a diode is added across the output as shown in Fig. 7.6(a). Unlike Fig. 7.4(a), the load current in Fig. 7.6(a) is permitted to be continuous and the output voltage cannot go negative.

A careful investigation of Fig. 7.6(a) shows that D_1 and D_2 cannot conduct simultaneously. For example, if we assume both diodes are on, then the average output voltage would be zero and V_s simultaneously. This is not possible. In the steady-state condition, this circuit has two topological modes of operation.

Mode 1: D_1 Is On and D_2 Is Off Mode 1 occurs when v_s is positive, causing D_1 to become forward biased and D_2 reverse biased. The equivalent circuit model is shown in Fig. 7.6(b). The time interval for this mode is $0 \le t < T/2$. The equation that governs this mode is

$$\frac{di_o}{dt} + \frac{R}{L} i_o = \frac{1}{L} v_s(t) \tag{7.16}$$

Equation (7.16) is similar to Eq. (7.5) and has the following complete solution.

$$i_{o1}(t) = \frac{V_s}{|Z|} \sin(\omega t - \theta) + I_1 e^{-t/\tau} \tag{7.17}$$

where

$$|Z| = \sqrt{(\omega L)^2 + R^2}$$
$$\tau = \frac{L}{R}$$
$$\theta = \tan^{-1} \frac{\omega L}{R}$$

and I_1 is constant and to be determined from the initial conditions. The subscript of the load current i_{o1} denotes the current i_o of mode 1.

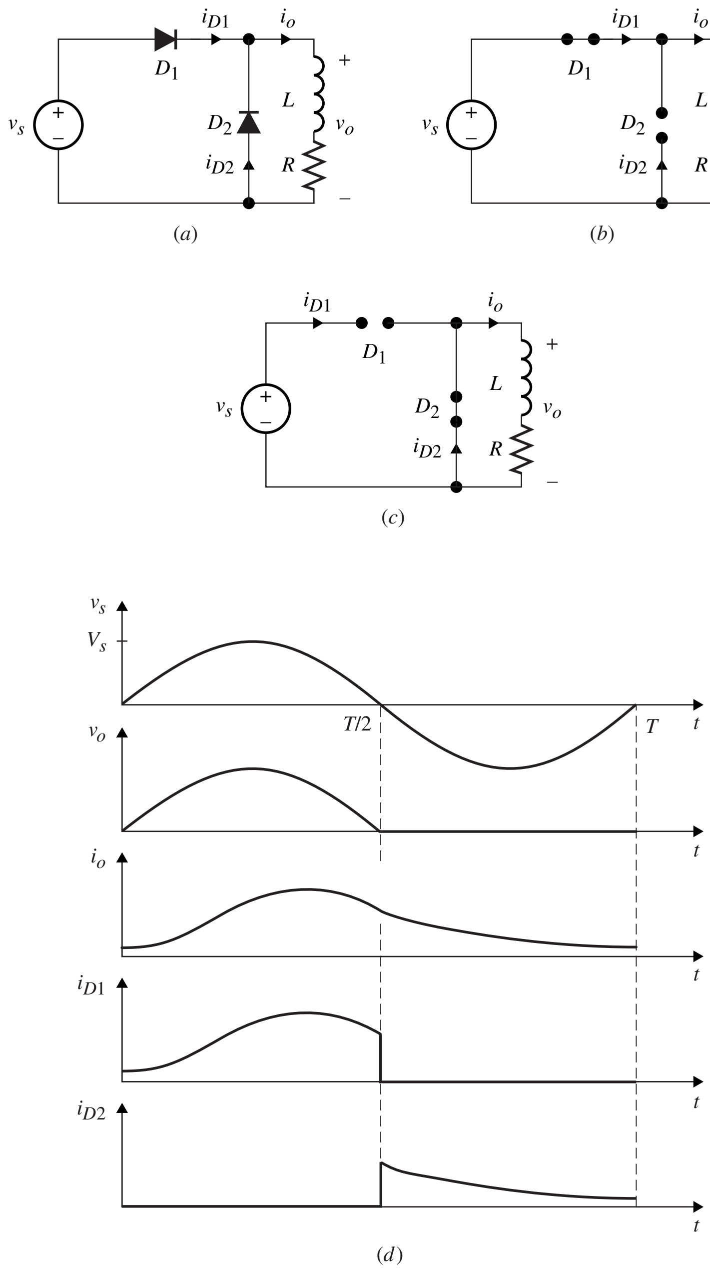


Figure 7.6 (*a*) Half-wave rectifier with a flyback diode. (*b*) Mode 1: D_1 is on and D_2 is off. (*c*) Mode 2: D_1 is off and D_2 is on. (*d*) Voltage and current waveforms.

Mode 2: D_1 Is Off and D_2 Is On Mode 2 occurs for $v_s < 0$, at which D_1 becomes reverse biased and D_2 turns on to pick up the load current. This way the load current commutates between D_1 and D_2 to maintain its continuity. The equivalent circuit is shown in Fig. 7.6(c), and the governing equation for this topology is given by

$$\frac{di_o}{dt} + \frac{R}{L} i_o = 0$$

The solution of this equation consists of only the transient response as shown in Eq. (7.18).

$$i_{o2}(t) = I_2 e^{-(t-T/2)/\tau} \tag{7.18}$$

where I_2 is constant and can be solved for, along with I_1, from the following relations that must hold in the steady state:

$$i_{o1}(0) = i_{o2}(T) \tag{7.19a}$$

$$i_{o1}(T/2) = i_{o2}(T/2) \tag{7.19b}$$

The left-side currents of Eqs. (7.19) are obtained from Eq. (7.17), and the right-side currents are obtained from Eq. (7.18). Consequently, we obtain the solution for I_1 and I_2 as follows:

$$I_1 = \frac{V_s}{|Z|}\sin\theta\frac{1+e^{-T/2\tau}}{1-e^{-T/\tau}} \tag{7.20}$$

$$I_2 = \frac{V_s}{|Z|}\sin\theta\frac{1+e^{-T/2\tau}}{1-e^{-T/\tau}} \tag{7.21}$$

The waveforms for i_o, i_{D1}, i_{D2}, and v_o are shown in Fig. 7.6(*d*).

If the load inductance is assumed infinitely large, then there will be no ripple in the load current, $i_o(t)$, Eqs. (7.17) and (7.18) become equal, and a constant load current is obtained as shown in the waveforms of Fig. 7.7.

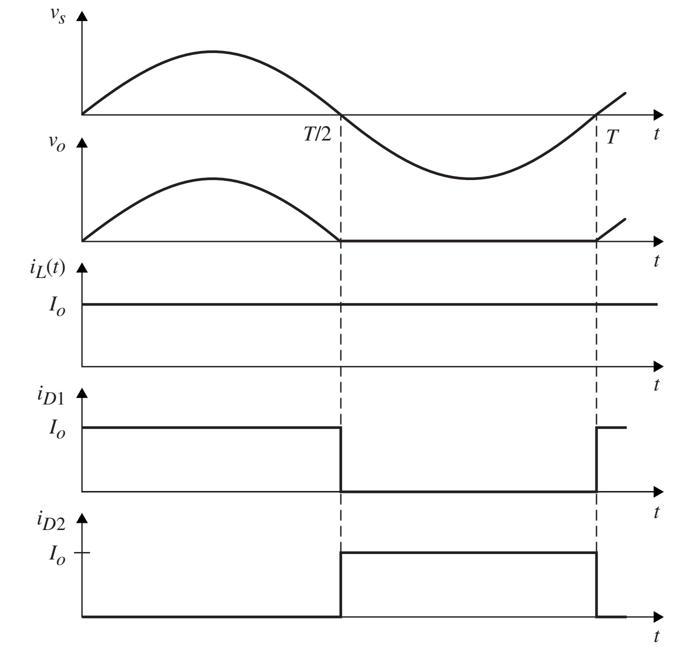


Figure 7.7 Waveform for Fig. 7.6(*a*) with an infinite load inductance.

The load resistance according to the following relation limits the value of the dc output current:

$$I_o = \frac{V_o}{R} = \frac{V_s}{\pi R} \tag{7.22}$$

One important observation from Fig. 7.7 is that the source current is a pulsating squarewave, an unattractive feature in power electronic circuits. The above circuit is also known as a *current doubler* since the average output current is twice the average input current.

EXERCISE 7.4

Consider the circuit shown in Fig. 7.6(a). Determine V_o, I_o, and the inductor currents at $t = 0$ and at $t = T/2$ in the steady state by using $v_s = 220 \sin 377t$, $R = 15\ \Omega$. Use (a) $L = 20$ mH, (b) $L = 200$ mH, and (c) L infinitely large.

ANSWER 70 V, 4.67 A, 16 mA, 5.9 A, 3.2 A, 6 A, 4.67 A, 4.67 A

EXERCISE 7.5

Determine the rms currents of diodes D_1 and D_2 in Fig. 7.3(a).

ANSWER 3.4 A rms

A final observation to be made about the half-wave inductive-load rectifier circuit is that unlike in Fig. 7.4(a), the inductor current exists for the entire period. As a result, the rectifier circuits of Fig. 7.4(a) and Fig. 7.6(a) are said to operate in the discontinuous and continuous conduction modes, respectively.

7.1.3 Capacitive Load

In some applications in which a constant output voltage is desirable, a series inductor is replaced by a parallel capacitor as shown in the half-wave rectifier circuit of Fig. 7.8(a).

Like the single-diode inductive-load half-wave rectifier, the circuit of Fig. 7.8(a) can be analyzed by considering the two topological modes resulting from the conducting states of the diode. Let mode 1 correspond to the time interval in which the diode conducts, resulting in the equivalent circuit shown in Fig. 7.8(b). Figure 7.8(c) shows the equivalent circuit when the diode is off, representing mode 2.

The circuit will be in mode 1 as long as the capacitor voltage is lower than the input voltage, v_s. If we assume the diode starts conducting at $t = t_1$, then the next time it will turn off to enter mode 2 of operation is when $t = T/4$, at which the input voltage reaches its peak. The diode will stay off until the capacitor and the input voltages become equal again at $t = T + t_1$. Figure 7.9 shows the steady-state waveforms for the output voltage and the source current.

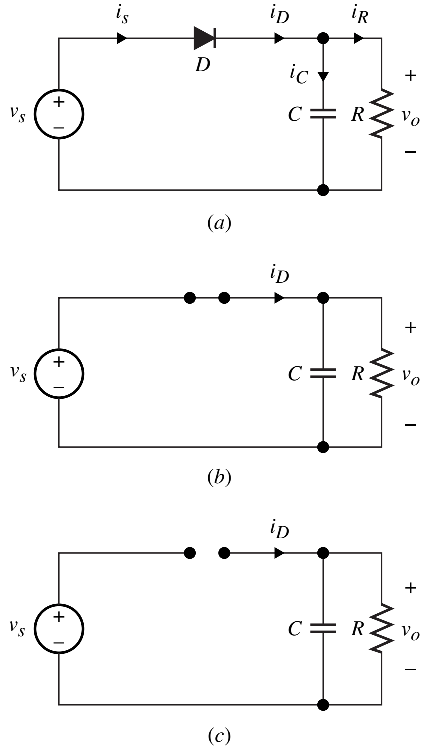


Figure 7.8 (*a*) Half-wave rectifier with capacitive load. (*b*) Equivalent circuit mode when the diode is on. (*c*) Equivalent circuit mode when the diode is off.

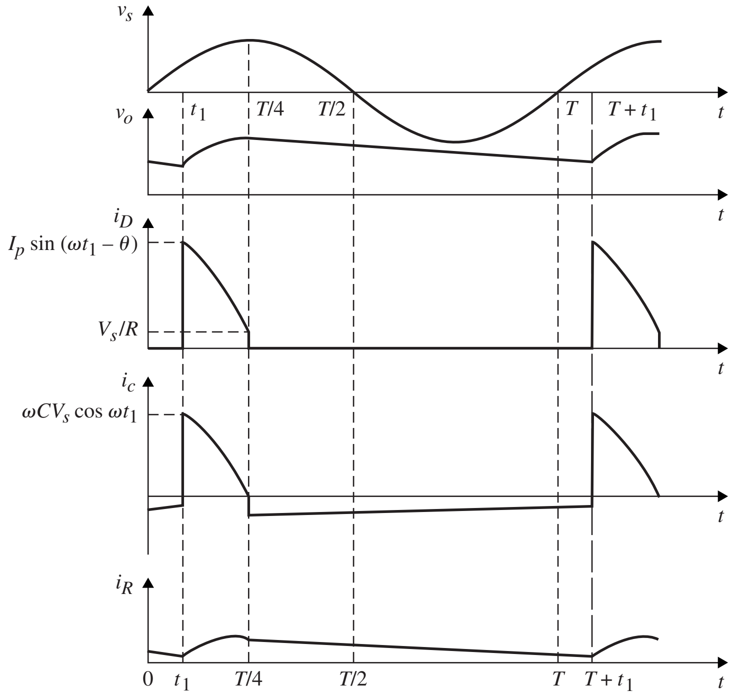


Figure 7.9 Steady-state waveforms for Fig. 7.8(*a*).

The analytical expressions for v_o and i_s are given as follows. For $t_1 \le t < T/4$,

$$v_o = V_s \sin \omega t \tag{7.23}$$

$$i_s(t) = \frac{V_s}{R} \sin \omega t + \omega C V_s \cos \omega t \tag{7.24}$$

The peak input current occurs at t_1, i.e., $i_s(t_1) = I_p$, where

$$I_p = \omega C V_s \cos \omega t_1 + \frac{V_s}{R} \sin \omega t_1$$

For $T/4 \le t < T + t_1$,

$$v_o(t) = V_s e^{-(t - T/4)/\tau} \tag{7.25}$$

$$i_s(t) = 0 \tag{7.26}$$

where the time constant is given by $\tau = RC$.

The value of $t = t_1$ can be determined by equating Eqs. (7.23) at $t = t_1$ and (7.25) at $t = t_1 + T$. This yields the following relation:

$$\sin \omega t_1 - e^{(t_1 + 3T/4)/\tau} = 0 \tag{7.27}$$

Equation (7.27) may be evaluated iteratively to obtain t_1. The average value of the output voltage is determined from the following:

$$\begin{aligned}
V_o &= \frac{1}{T}\int_{t_1}^{T+t_1} v_o(t)\, dt \\
&= \frac{1}{T}\left[\int_{t_1}^{T/4} V_s \sin \omega t \; dt + \int_{T/4}^{T+t_1} V_s e^{-(t - T/4)/\tau} dt\right] \\
&= \frac{V_s}{2\pi}\left[\cos \omega t_1 + \tau\omega(1 - e^{-(t_1 + 3T/4)/\tau})\right] \\
&= \frac{V_s}{2\pi}\left[\cos \omega t_1 + \tau\omega(1 - e^{-(t_1 + 3T/4)/\tau})\right]
\end{aligned}$$

EXAMPLE 7.3

Consider the half-wave rectifier circuit of Fig. 7.8(*a*) with $v_s(t) = 20 \sin 2\pi 60t$, $R = 10\ \text{k}\Omega$, and $C = 47\ \mu\text{F}$.

(a) Sketch the waveforms for v_o, i_R, i_D, and i_c.

(b) Determine the output voltage ripple.

SOLUTION

(a) First we need to find the time t_1 at which the diode turns on. Using $\tau = RC = 470$ ms and $T = 16.67$ ms, through several iterations, Eq. (7.27) gives $t_1 \approx 3.48$ ms. At this time, the output voltage is 19.3 V and the resistor current is 1.93 mA.

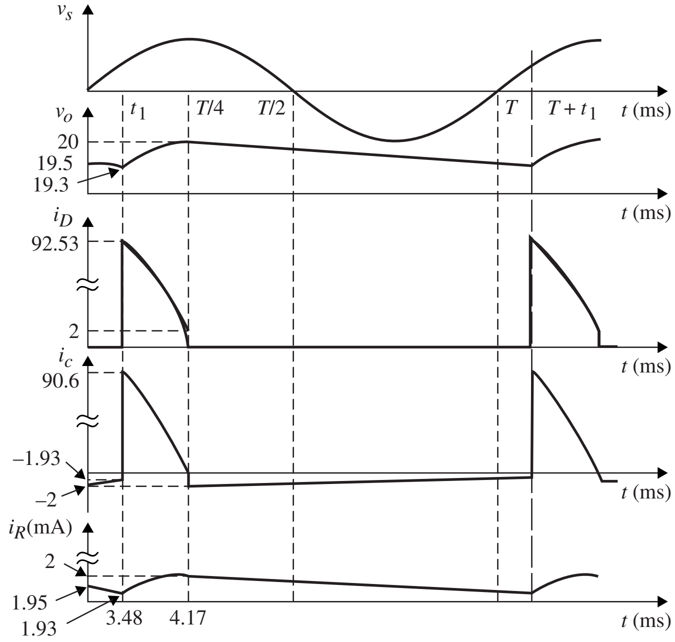


Figure 7.10 Steady-state waveforms for Example 7.3.

At $t = T/4 = 4.17$ ms, $v_o = 20$ V and $i_R = 2$ mA. When the diode turns off, $i_c = -i_R = -2$ mA. However, while the diode is conducting, i_c is given by

$$i_c(t) = \omega C V_s \cos \omega t$$
$$= 354 \cos \omega t \text{ mA}$$

At $t_1 \approx 3.48$ ms, $i_c = 354 \cos 377(3.48 \text{ ms}) = 90.6$ mA; hence, $i_D = i_R + i_c = 1.93 + 90.6 = 92.53$ mA. Finally, the output voltage at T or at $t = 0$ is obtained from Eq. (7.25) as

$$v_o = V_s e^{-(t - T/4)/\tau}$$
$$= 20e^{-3T/4\tau}$$
$$= 20e^{-3(16.67)/(4)(470)}$$
$$= 19.5 \text{ V}$$

The waveforms are shown in Fig. 7.10.

(b) The output voltage ripple, V_r, is given by

$$V_r = v_{o,\max} - v_{o,\min}$$
$$= v_o(T/4) - v_o(t_1)$$
$$= 20 - 19.3$$
$$= 0.7 \text{ V}$$

EXERCISE 7.6

Calculate the average output voltage for Example 7.3.

ANSWER 19.68 V

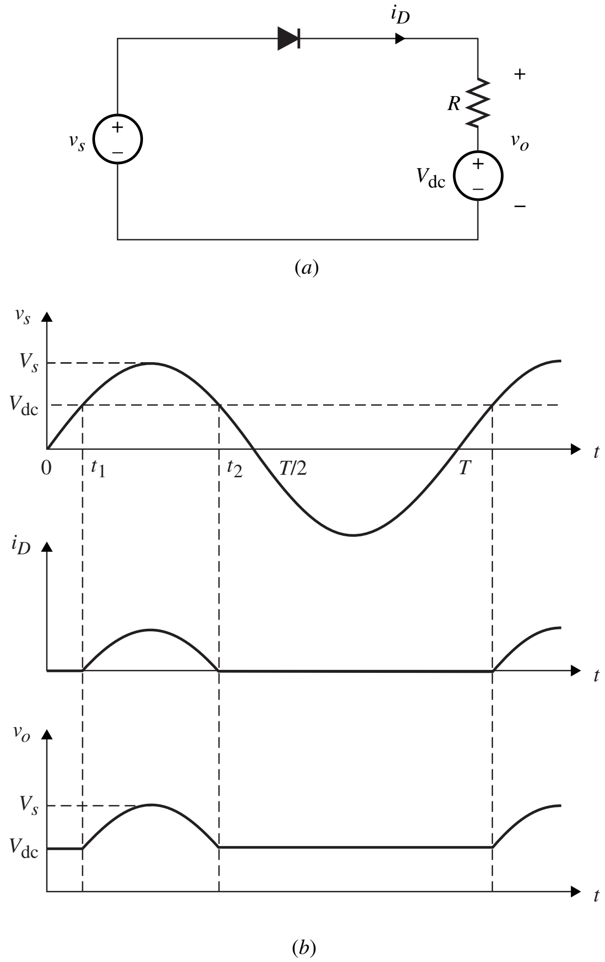


Figure 7.11 (*a*) Half-wave rectifier with voltage source in the dc side. (*b*) Voltage and current waveforms.

7.1.4 Voltage Source in the dc Side

Figure 7.11(*a*) shows a half-wave rectifier circuit with a voltage source in series with the load resistance. This constant voltage may be considered as an emf source located in the load side, as in the case of dc machine circuit modeling.

Let us assume that the diode starts conducting at $t = t_1$. This time occurs when $v_s = V_{dc}$. Furthermore, the diode stops conducting at $t = t_2$, when the current i_o becomes zero. Typical waveforms are shown in Fig. 7.11(*b*). See Problem 7.19 to derive the equation for the load current during the conduction period $t_1 \le t \le t_2$.

7.2 THE EFFECT OF AC-SIDE INDUCTANCE

7.2.1 Half-Wave Rectifier with Inductive Load

Let us consider the half-wave diode rectifier circuit by including an ac-side inductance, which is inevitable in all ac systems. Studying the effect of the line inductance on the behavior of diode circuits is important since in practical rectifier systems connected directly to the line voltage, the transformer leakage inductance and the line parasitic inductance might cause line voltage and current disturbances and could affect the average power drawn from the mains. Moreover, an external inductance might also be added in some applications in order to limit di/dt or dv/dt. For simplicity, we normally assume that the ac-side inductance is finite and may be represented by a discrete value, L_s, in series with the source voltage given in the block diagram representation in Fig. 7.1. Figure 7.12(*a*) shows the circuit for the half-wave rectifier including L_s and under an inductive load. In the following analysis, we will assume that $L/R \gg T/2$ so that the load current, i_o, is constant. This assumption is valid since in many applications the load inductance is very much larger than the ac-side inductance.

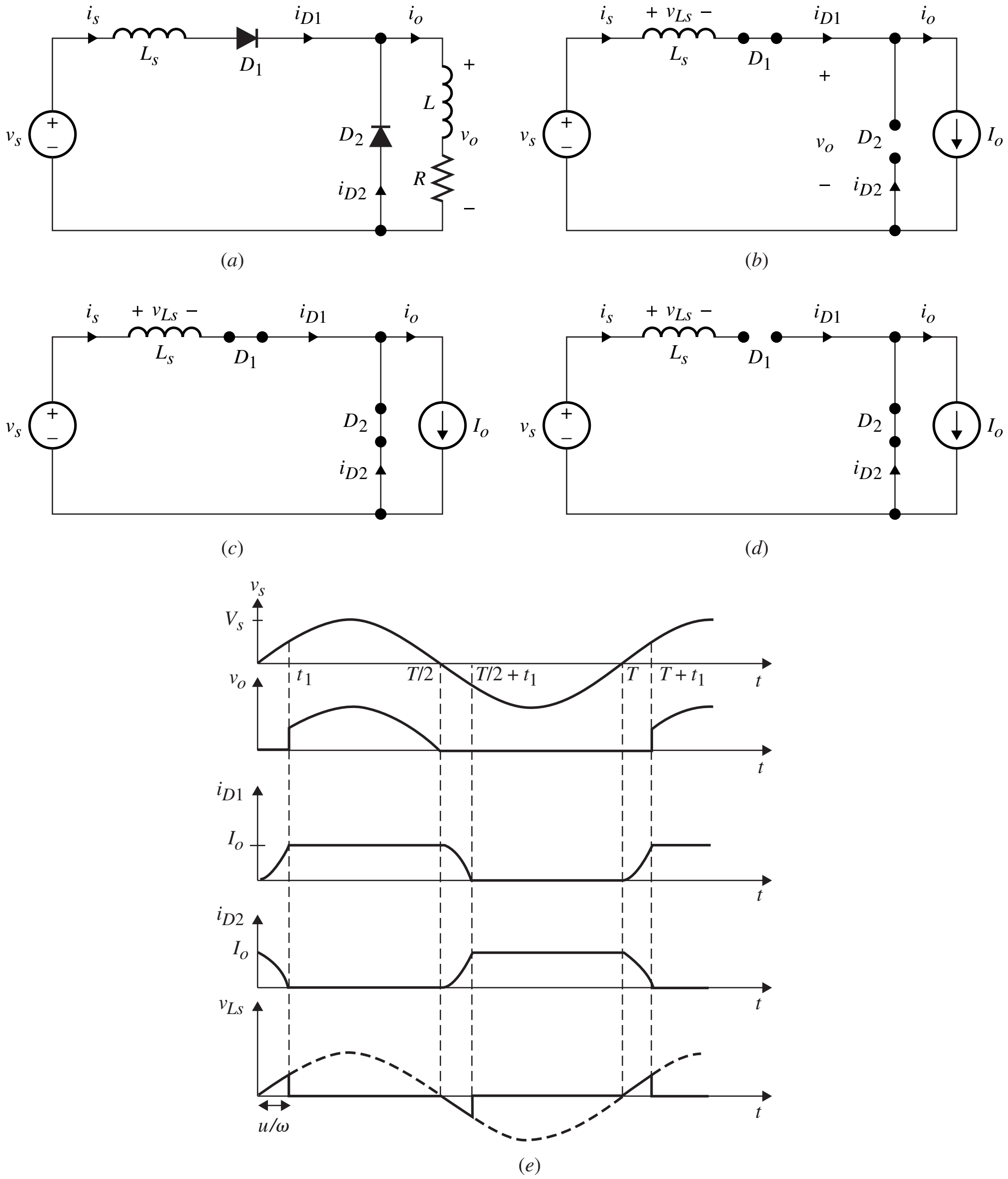


Figure 7.12 (*a*) Half-wave rectifier with ac-side inductance. (*b*) Mode 1: D_1 is on and D_2 is off during $v_s(t) > 0$. (*c*) Mode 2 (commutation mode): D_1 and D_2 are on while $v_s(t) < 0$. (*d*) Mode 3: D_1 is off and D_2 is on while $v_s(t) < 0$. (*e*) The waveforms for v_s, v_o, i_{D1}, i_{D2}, and v_{Ls}.

The behavior of the circuit can be easily analyzed by assuming that one of the diodes, D_1, is conducting for some time during the positive cycle of $v_s(t)$, while D_2 is off. Since the current in D_1 is constant, then the voltage across L_s is zero and the voltage across D_2 is positive. Hence, our assumption is valid. The equivalent circuit under this mode is shown in Fig. 7.12(*b*). During this mode, we have the following current and voltage values:

$$i_{D1} = i_s(t) = I_o \qquad i_{D2} = 0 \qquad v_{Ls} = 0 \qquad v_o = v_s$$

At $t = T/2$, $v_s(t)$ starts to become negative, causing D_1 to stop conducting. However, since the current in D_1 is the same as the inductance current, which is not allowed to change instantaneously, D_2 turns on in order to maintain the inductor current's continuity. During this overlapping time, when both diodes are conducting, $i_s(t)$ changes from $+I_o$ to zero, while $i_{D2}(t)$ changes from zero to $+I_o$. This is known as a *commutation process*, during which currents are moved from one branch of the circuit to another branch. This is why the ac-side inductance, L_s, is known as the *commutation inductance*. This circuit mode of operation is referred to as commutation mode, as shown in Fig. 7.12(*c*). The waveforms for v_s, v_o, i_{D1}, i_{D2}, and v_{Ls} are shown in Fig. 7.12(*e*).

During mode 2, the following equations hold:

$$v_{Ls} = v_s(t) = L_s\frac{di_s}{dt}$$

$$i_{D1} = i_s(t)$$

$$i_{D2} = I_o - i_{D1}$$

$$v_o = 0$$

The initial condition for $i_s(t)$ at $t = T/2$ is I_o. Using the above v_{Ls} equation with the given initial condition, we obtain the following input current integral relation:

$$i_s(t) = \frac{1}{L_s}\int_{T/2}^{t} v_{Ls}(t)\ dt + I_o \tag{7.28}$$

Substituting for $v_{Ls}(t) = v_s(t) = V_s \sin\omega t$ in the integral and solving for $i_s(t)$, we obtain

$$i_s(t) = I_o - \frac{V_s}{L_s\omega}(1 + \cos\omega t) \quad T/2 \le t \le t_1 + T/2 \tag{7.29}$$

At the end of the commutation period, $t = t_1 + T/2$; $i_s(t)$ becomes zero, forcing D_1 to turn off at zero current; and D_2 remains forward biased, carrying the load current as shown in the circuit of mode 3 given in Fig. 7.12(*d*). In this mode we have the following current and voltage equations:

$$i_{D1} = i_s(t) = 0 \quad i_{D2} = I_o \quad v_{Ls} = 0 \quad v_o = 0$$

To solve for the commutation time, t_1, or the commutation angle, $u = \omega t_1$, we evaluate $i_s(t)$ at $t = T/2 + t_1$ and set it to zero. The load current and t_1 are given by

$$I_o = \frac{V_s}{L_s\omega}(1 - \cos\omega t_1) \tag{7.30}$$

$$t_1 = \frac{1}{\omega}\cos^{-1}\left(1 - \frac{L_s\omega I_o}{V_s}\right) \tag{7.31}$$

The average output voltage is expressed by

$$\begin{aligned} V_o &= \frac{1}{T}\int_{t_1}^{T/2} v_s(t)\,dt \\ &= \frac{V_s}{\pi}\left(1 - \frac{\omega L_s I_o}{2V_s}\right) \end{aligned} \tag{7.32}$$

If we normalize the output voltage by V_s and the load current by Z_o, where $Z_o = \omega L_s$ is the characteristic impedance of the ac line, we have the following normalized output quantities:

$$\begin{aligned} V_{no} &= \frac{V_o}{V_s} \\ &= \frac{1}{\pi}\left(1 - \frac{I_{no}}{2}\right) \end{aligned} \tag{7.33}$$

$$\begin{aligned} I_{no} &= \frac{Z_o I_o}{V_s} \\ &= 1 - \cos u \end{aligned} \tag{7.34}$$

or u is given by

$$u = \cos^{-1}(1 - I_{no}) \tag{7.35}$$

The maximum voltage across L_s, $V_{Ls,\max}$, occurs at $t = t_1$ and is given by

$$\begin{aligned} V_{Ls,\max} &= v_s(t_1) \\ &= V_s \sin \omega t_1 \end{aligned}$$

Normalizing $V_{Ls,\max}$ by V_s and substituting the normalized current I_{no} into the above relation, we obtain

$$V_{nLs,\max} = \sqrt{(2 - I_{no})I_{no}} \tag{7.36}$$

Figure 7.13 shows the curve for $V_{nLs,\max}$ vs. I_{no}, and Fig. 7.14(a) and (b) shows I_{no} vs. u and V_{no} vs. I_{no}, respectively.

EXERCISE 7.7

Consider the circuit of Fig. 7.12(a) with $L_s = 0.05$ mH, $R = 20\ \Omega$, $I_o = 1$ A, and $v_s(t) = 100 \sin 2\pi 60t$. Find: ($a$) the average output voltage, (b) the commutation intervals in

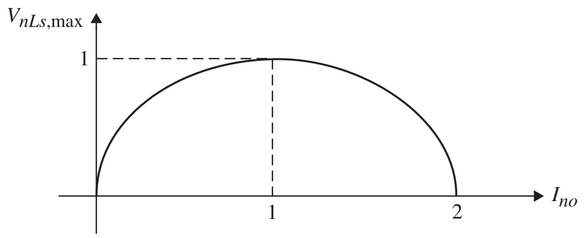


Figure 7.13 Normalized maximum inductor voltage versus normalized output current.

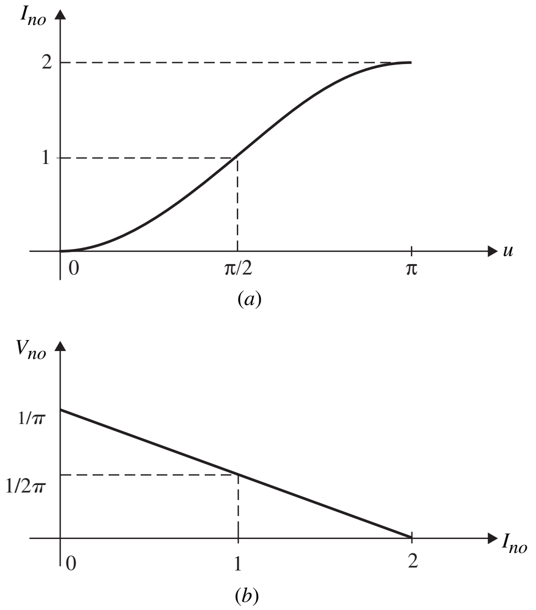


Figure 7.14 (*a*) Normalized output current versus commutation angle. (*b*) Normalized output voltage versus normalized output current.

seconds and degrees, (*c*) the rms diode currents, (*d*) the peak inverse diode voltages, and (*e*) the peak voltage across the inductor L_s.

ANSWER 31.83 V, 51.5 μs, 1.1°, 585.26 A rms, 100 V, 1.94 V

7.2.2 Half-Wave Rectifier with Capacitive Load

Consider the half-wave rectifier circuit with a capacitive load including an ac-side inductance as shown in Fig. 7.15(*a*). Again, to simplify the analysis we assume $RC \gg T/2$ so the output voltage is constant. Its equivalent circuit is given in Fig. 7.15(*b*).

Let us assume first that the diode is off while the input voltage is negative. As shown in Fig. 7.15(*a*), for the diode to conduct, its voltage must be positive. At $t = t_1$, $v_s = V_o$ and the diode turns on. The waveforms are shown in Fig. 7.15(*c*), and the voltage across L_s is given by

$$\begin{aligned} v_{Ls} &= L_s \frac{di_s}{dt} \\ &= v_s(t) - V_o \end{aligned}$$

Since the initial inductor current value at $t = t_1$ is zero, the solution for $i_s(t)$ is given by

$$i_s(t) = \frac{V_s}{\omega L_s}(\cos \omega t_1 - \cos \omega t) - \frac{V_o}{L_s}(t - t_1) \tag{7.37}$$

The peak inductor current occurs when $v_{Ls} = 0$, i.e., when $v_s = V_o$. Let us assume this occurs at $t = t_2$.

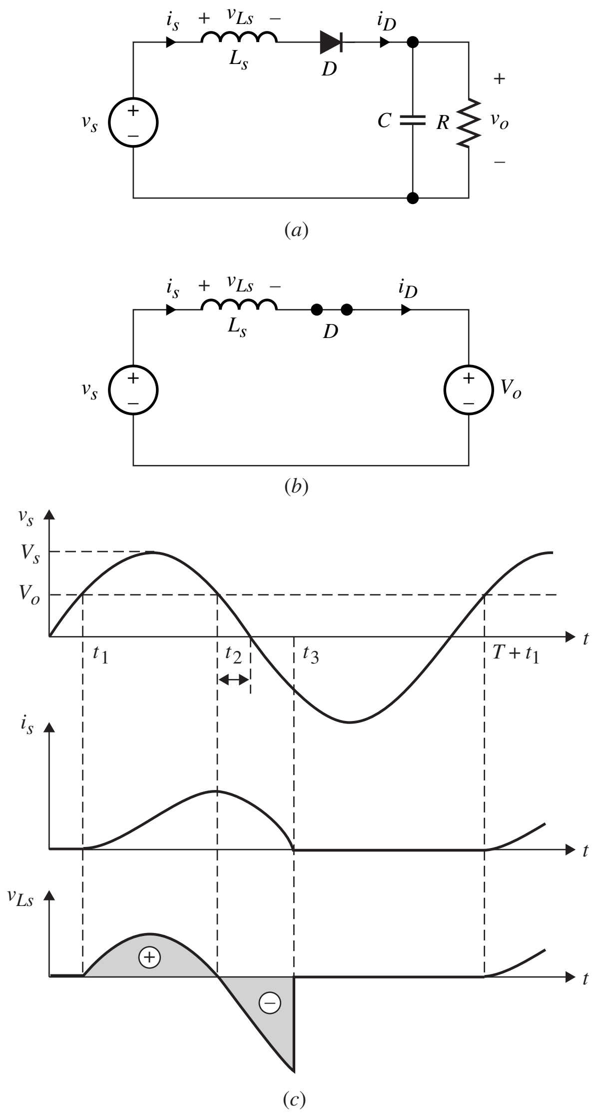


Figure 7.15 (*a*) Half-wave capacitive-load circuit with commutative inductance. (*b*) Equivalent circuit with $RC \gg T/2$. (*c*) Current and voltage waveforms.

$$I_{s,\text{peak}} = i_s(t_2)$$
$$= \frac{V_s}{\omega L_s}(\cos\omega t_1 - \cos\omega t_2) - \frac{V_o}{L_s}(t_2 - t_1) \tag{7.38}$$

where t_2 is obtained from

$$V_o = V_s \sin\omega t_2 \tag{7.39}$$

Also at $t = t_1$, when the diode first turns on, we have

$$V_o = V_s \sin\omega t_1 \tag{7.40}$$

where $t_1 + t_2 = T/2$.

As long as $i_s(t)$ is positive, D remains on, and at $t = t_3$, i_s reaches zero, causing D to turn off. The current at $t = t_3$ is given by

$$\begin{aligned} i_s(t_3) &= \frac{V_s}{\omega L_s}(\cos\omega t_1 - \cos\omega t_3) - \frac{V_o}{L_s}(t_3 - t_1) \\ &= 0 \end{aligned} \tag{7.41}$$

Equations (7.39), (7.40), and (7.41) can be used to solve for t_1, t_2, and t_3 numerically. The average output current is given by

$$\begin{aligned} I_o &= \frac{1}{T}\int_{t_1}^{t_3} i_s(t)\ dt \\ &= \frac{V_s}{2\pi L_s}\left((t_3 - t_1)\cos\omega t_1 - \frac{\sin\omega t_3}{\omega} + \frac{\sin\omega t_1}{\omega}\right) - \frac{V_o}{2L_s T}(t_3 - t_1)^2 \end{aligned}$$

Normalizing I_o and V_o, and using the relation (7.39) for $t = t_2$, we obtain

$$\begin{aligned} I_{no} &= \frac{\omega L_s I_o}{V_s} \\ &= \frac{1}{2\pi}\left(\alpha\cos\omega t_1 - \sin\omega t_3 + \sin\omega t_1 - \frac{\alpha^2}{2}V_{no}\right) \end{aligned} \tag{7.42}$$

where $\alpha = \omega(t_3 - t_1)$, which is the diode conducting time. Figure 7.16 shows I_{no} vs. α, and Fig. 7.17 shows V_{no} vs. α.

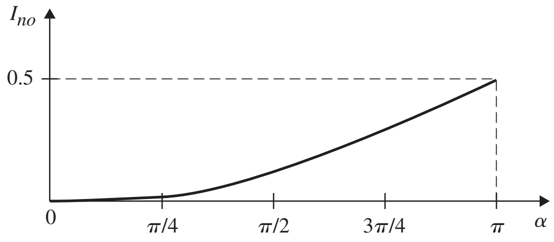


Figure 7.16 Normalized load current versus conduction angle.

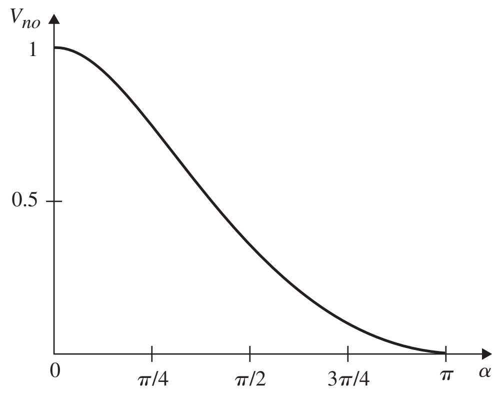


Figure 7.17 Normalized load voltage versus conduction angle.

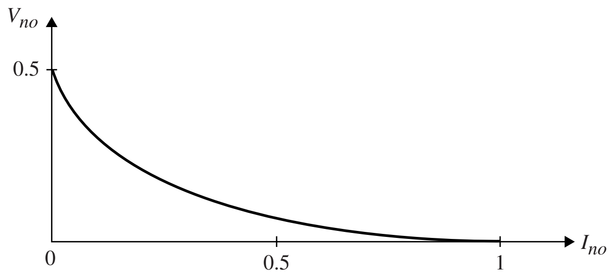


Figure 7.18 Normalized load voltage versus normalized load current.

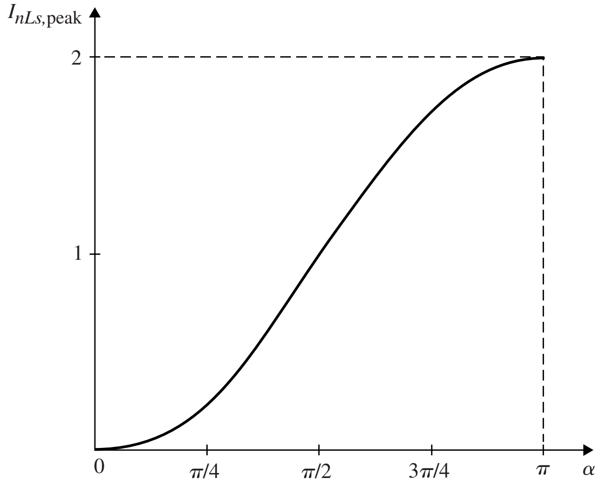


Figure 7.19 Normalized peak inductor current versus conduction angle.

The effect of L_s on the variation of the average output voltage is shown in Fig. 7.18.

The normalized inductor peak current is given by

$$I_{nLs,\text{peak}} = \frac{\omega L_s I_{Ls,\text{peak}}}{V_s}$$
$$= (\cos \omega t_1 - \cos \omega t_2) - \alpha V_{no}$$

A plot of the normalized peak inductor current vs. α under different load conditions is shown in Fig. 7.19.

EXAMPLE 7.4

Consider the half-wave rectifier of Fig. 7.15(a) with $v_s(t) = 25 \sin 377t$, $V_o = 20$ V, and $L_s = 2$ mH. Determine V_{no}, I_{no}, α, and $I_{nLs,\text{peak}}$.

SOLUTION First find the value of t_1:

$$t_1 = \frac{1}{\omega}\sin^{-1}\left(\frac{V_o}{V_s}\right) = 2.46 \text{ ms}$$

Through some numerical iterations we find that $t_3 = 7.66$ ms. The normalized average output current can be obtained theoretically as follows:

$$I_o = \frac{V_s}{2\pi L_s}\left((t_3 - t_1)\cos\omega t_1 - \frac{\sin\omega t_3}{\omega} + \frac{\sin\omega t_1}{\omega}\right) - \frac{V_o}{2L_sT}(t_3 - t_1)^2$$
$$= 0.99 \text{ A}$$

Hence,

$$I_{no} = \frac{\omega L_s I_o}{V_s} = 0.03$$

The other parameters are given as

$$\alpha = \omega(t_3 - t_1) = 1.96 \text{ rad/s}$$
$$V_{no} = \frac{V_o}{V_s} = 0.8 \text{ V}$$
$$I_{nLs,\text{peak}} = (\cos\omega t_1 - \cos\omega t_2) - \alpha V_{no} = 0.17$$

EXERCISE 7.8

Determine the diode rms current for Example 7.4.

ANSWER 2.078 A

7.2.3 Full-Wave Rectifier with Inductive Load

The effect of the commutation inductance on the waveforms and parameters of the full-wave rectifier shown in Fig. 7.20(a) is investigated next. As before, we will assume $L/R \gg T/2$, resulting in a constant load current, I_o. The typical waveforms are shown in Fig. 7.20(b). The behavior of the circuit can be discussed similarly to the case for the half-wave rectifier circuit of Fig. 7.6(a). We first assume that D_1 and D_4 are conducting during the positive cycle of $v_s(t)$. Since I_o is constant, the voltage across L_s is zero, and $v_o(t) = v_s(t)$ as shown in Fig. 7.21(a), which is represented as mode 1. Notice that the voltages across D_2 and D_3 are negative and equal to $-v_s(t)$. As long as $v_s(t)$ is positive, D_2 and D_3 remain off. The second topological mode starts at $t = T/2$, when $v_s(t)$ becomes negative. At this time, D_2 and D_3 start conducting while D_1 and D_4 remain on in order to maintain the continuity of the inductor current. Figure 7.21(b) shows this mode, when all diodes are conducting. The following current and voltage relations apply in this mode:

$$\begin{aligned} v_{Ls} &= L_s\frac{di_s}{dt} \\ v_o &= 0 \\ i_{D1} = i_{D4} &= \frac{I_o + i_s}{2} \\ i_{D2} = i_{D3} &= \frac{I_o - i_s}{2} \end{aligned} \tag{7.43}$$

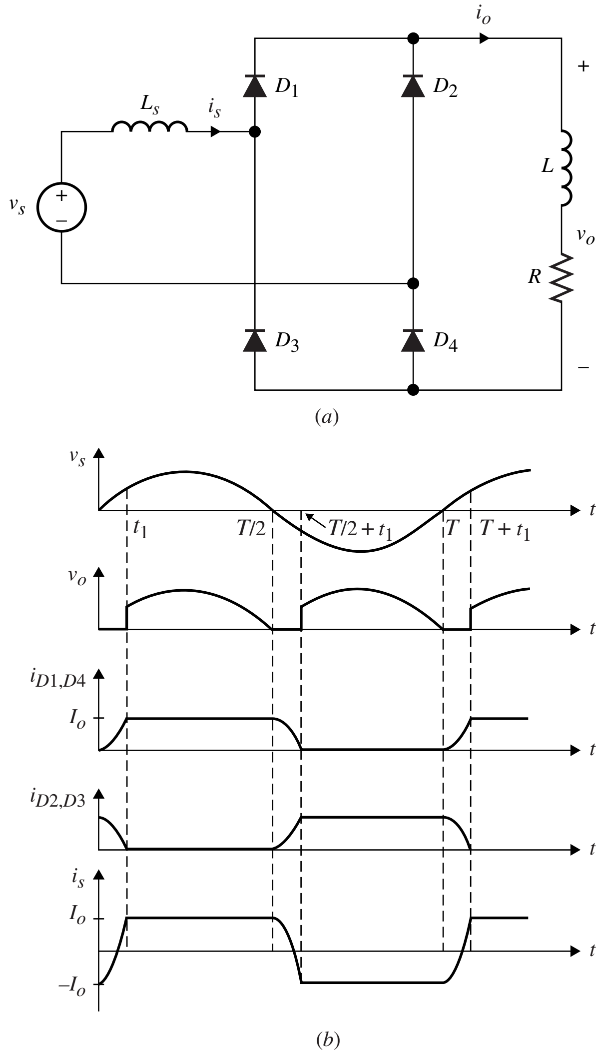


Figure 7.20 (*a*) Full-wave rectifier with ac-side inductance. (*b*) Current and voltage waveforms.

Using the initial inductor value of $i_s(T/2) = +I_o$ in solving the above equation for $i_s(t)$, we obtain

$$i_s(t) = I_o - \frac{V_s}{L_s\omega}(1 + \cos\omega t) \tag{7.44}$$

At $t = t_1 + T/2$, we have $i_s(t_1 + T/2) = -I_o$. This gives the following relation for the commutation period:

$$\cos\omega t_1 = 1 - \frac{2L_s\omega I_o}{V_s} \tag{7.45}$$

In terms of normalized values, $u = \omega t_1$ is given by

$$u = \cos^{-1}(1 - 2I_{no})$$

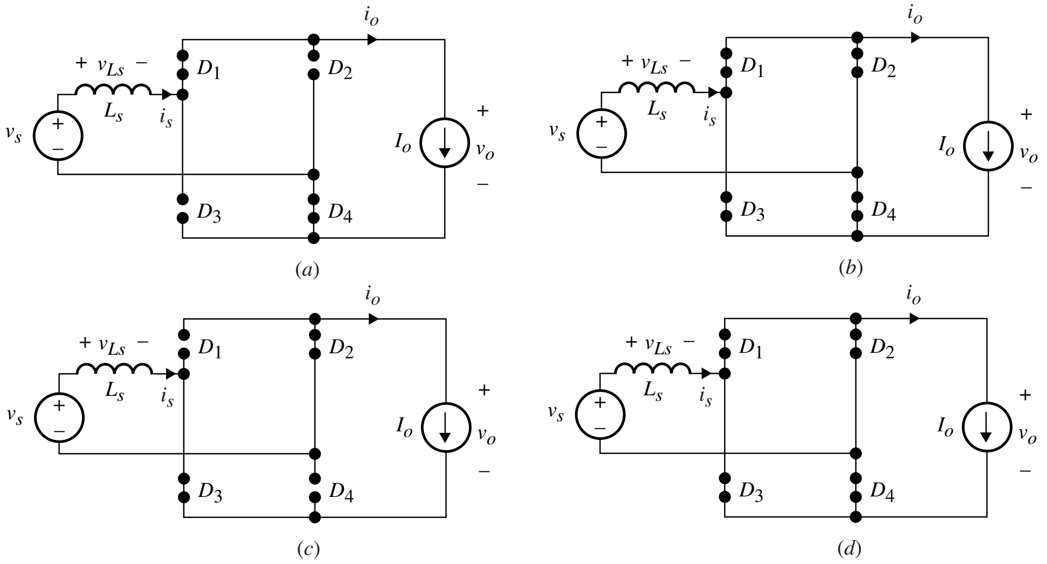


Figure 7.21 Circuit modes of operation for Fig. 7.20. (*a*) Mode 1: equivalent circuit for $v_s(t) > 0$. (*b*) Mode 2: equivalent circuit for $v_s(t) < 0$ and during diode commutation period. (*c*) Mode 3: equivalent circuit for $v_s(t) < 0$. (*d*) Mode 4: equivalent circuit for $v_s(t) > 0$ and during diode commutation period.

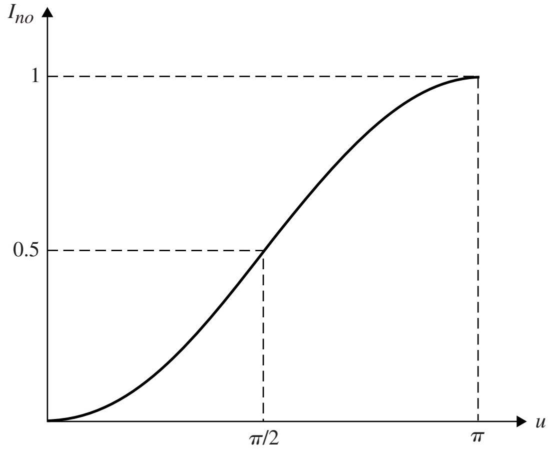


Figure 7.22 Normalized output current vs. commutation angle.

Figure 7.22 shows the curves for the normalized output current as a function of the commutation angle. The larger the commutation inductance, the less the average output current. At $t = T/2 + t_1$, the currents in D_1 and D_4 become zero and the currents in D_2 and D_3 reach I_o, as shown in the topology of mode 3 given in Fig. 7.21(*c*). The output voltage is equal to $v_s(t)$. This mode remains until $t = T$, when the voltage becomes positive again. Since $v_s(t) > 0$, D_1 and D_4 turn on and D_2 and D_3 remain on until the current commutates from D_2 and D_3 to D_1 and D_4 at the end of the commutation period u, and the circuit enters mode 4 as shown in Fig. 7.21(*d*). The current and voltage equations are the same as those for mode 2. The initial condition for $i_s(t)$ is $i_s(T) = -I_o$, and the solution for $i_s(t)$ is given by

$$i_s(t) = -I_o + \frac{V_s}{L_s\omega}(\cos\omega t - 1) \tag{7.46}$$

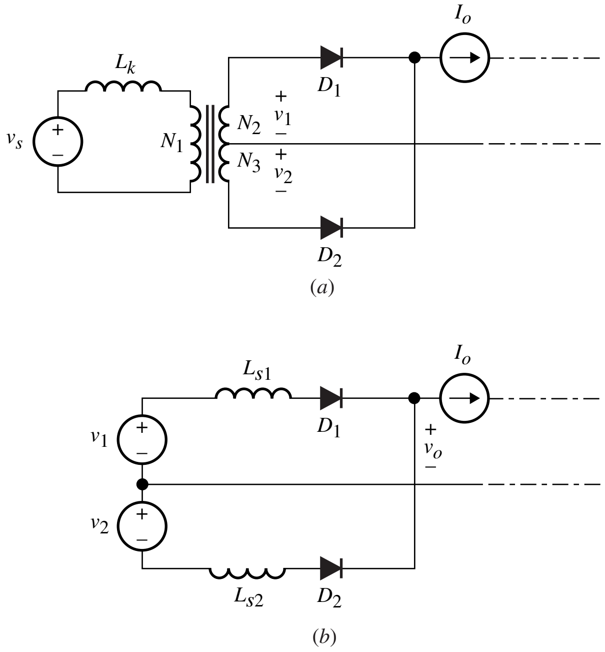


Figure 7.23 (*a*) Full-wave center-tap transformer with leakage inductance. (*b*) Simplified equivalent circuit.

We notice from the above relations that the average output voltage, inductor peak current, and rms value of $i_s(t)$ are directly related to the normalized load current.

To illustrate the source of L_s in the full-wave rectifier, we refer to Fig. 7.23. This is a center-tap transformer including a leakage inductance L_k reflected to the primary side. The equivalent circuit for Fig. 7.23(*a*) can be drawn as shown in Fig. 7.23(*b*), where the inductance is reflected to the primary side. The voltage relations are given by

$$v_1 = \frac{N_2}{N_1} v_s$$

$$v_2 = \frac{N_3}{N_1} v_s$$

where $L_{s1} = (N_1/N_2)^2 L_k$ and $L_{s2} = (N_1/N_3)^2 L_k$.

EXERCISE 7.9

Consider the center-tap full-wave rectifier of Fig. 7.23(*a*) with $N_1 = 10$, $N_2 = 5$, $N_3 = 15$, $L_k = 10\ \mu\text{H}$, $I_o = 20$ A, and $v_s = 100 \sin 2\pi(60)t$. Assume the diodes have no forward voltage drops.

(a) Determine the average output voltage.

(b) Repeat part (*a*) by assuming each diode has a 1 V forward drop.

ANSWER 63.61 V, 62.63 V

7.3 THREE-PHASE RECTIFIER CIRCUITS

Many industrial applications require high power that a single-phase system is unable to provide. Three-phase diode rectifier circuits are used widely in high-power applications with low output ripple. It is important that we understand the basic concepts of a three-phase rectifier circuit. In this section, we will cover both half- and full-wave rectifier circuits under resistive and high inductive loads.

7.3.1 Three-Phase Half-Wave Rectifier

Figure 7.24 shows the general configuration for an *m*-phase half-wave rectifier connected to a single load. The explanation of the circuit is quite simple since all diode cathodes are connected to the same point, creating a diode-OR arrangement. At any given time, the highest anode voltage will cause its corresponding diode to conduct, with all other diodes in the reverse-bias state. In other words, the output voltage will ride on the peak voltage at all times. Figure 7.25 shows four random sine functions and the output voltage.

The half-wave three-phase resistive-load rectifier circuit is shown in Fig. 7.26(*a*). We assume that the three-phase voltage source is a Δ configuration with the three balance voltages given by

$$v_1 = V_s \sin \omega t \qquad v_2 = V_s \sin(\omega t - 120°) \qquad v_3 = V_s \sin(\omega t - 240°)$$

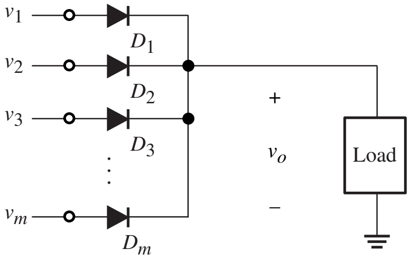


Figure 7.24 *m*-phase half-wave rectifier connected to a single load.

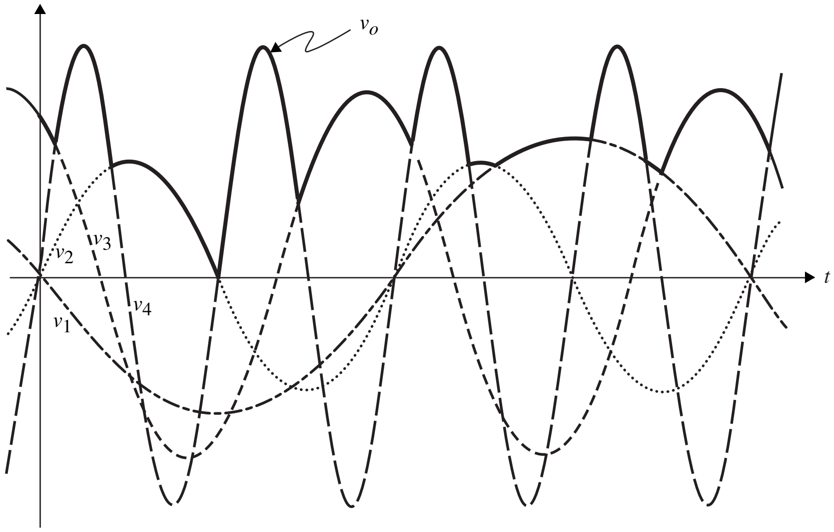
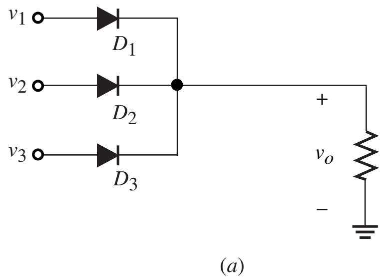


Figure 7.25 Example of random four-phase sinusoidal input voltages and the output voltage.



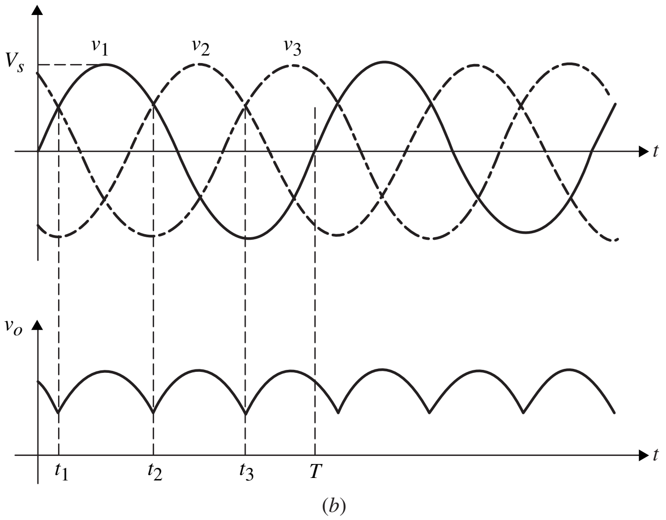


Figure 7.26 (*a*) Half-wave three-phase resistive-load rectifier. (*b*) Waveforms for the output voltage.

Figure 7.26(*b*) shows the output voltage waveform. This circuit is also known as a *three-pulse rectifier* circuit. Here, the number of pulses refers to the number of voltage peaks in a given cycle.

The circuit works as follows. At $t = t_1$, we have $v_1 = v_3$. This occurs when $t_1 = T/12$, and at $t_2 = 5T/12$ we have $v_2 = v_1$. The average output voltage is given by

$$\begin{aligned} V_o &= \frac{1}{T}\int_0^T v_o(t)\,dt \\ &= \frac{3}{T}\int_{t_1}^{t_2} v_1(t)\,dt \\ &= \frac{3}{T}\int_{T/12}^{5T/12} v_1(t)\,dt \\ &= \frac{3\sqrt{3}}{2\pi}V_s \end{aligned} \tag{7.47}$$

The source voltage arrangement of v_1, v_2, and v_3 can be a wye (Y) or a delta (Δ) connection, as shown in Fig. 7.27(*a*) and (*b*), respectively. The voltages v_a, v_b, and v_c are the balanced, positive-sequence phase voltages given by

$$v_a = V_s \sin \omega t \qquad v_b = V_s \sin(\omega t - 120°) \qquad v_c = V_s \sin(\omega t - 240°)$$

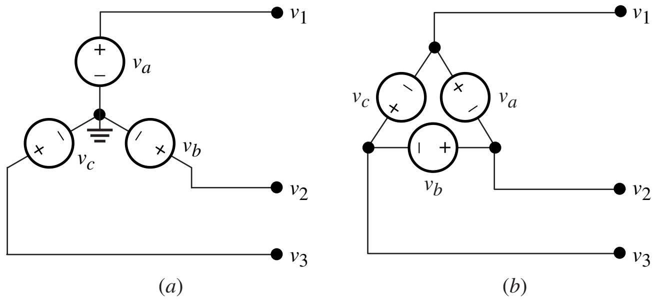


Figure 7.27 Three-phase connections: (*a*) Y and (*b*) Δ connections.

The voltage set v_1, v_2, and v_3 are line voltages. In the Δ configuration, the line-to-line voltages are the same as the phase voltages, whereas the line-to-line voltages in the Y configuration are given by

$$\begin{aligned} v_{12} &= v_1 - v_2 \\ &= v_a - v_b \\ &= v_{ab} \\ &= \sqrt{3}V_s \sin(\omega t + 30°) \end{aligned}$$

$$\begin{aligned} v_{23} &= v_2 - v_3 \\ &= v_b - v_c \\ &= v_{bc} \\ &= \sqrt{3}V_s \sin(\omega t - 90°) \end{aligned}$$

$$\begin{aligned} v_{31} &= v_3 - v_1 \\ &= v_c - v_a \\ &= v_{ca} \\ &= \sqrt{3}V_s \sin(\omega t + 150°) \end{aligned}$$

The phase diagram representation for the phase and the line-to-line voltages is given in Fig. 7.28.

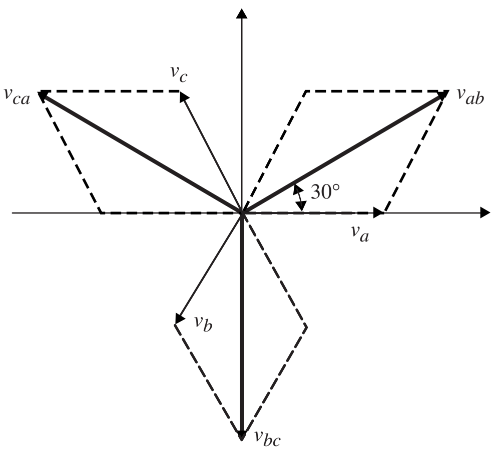
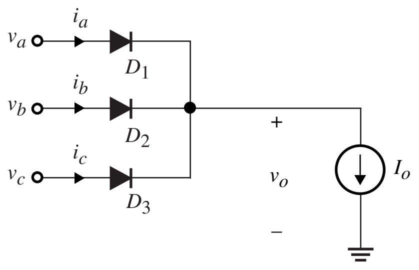
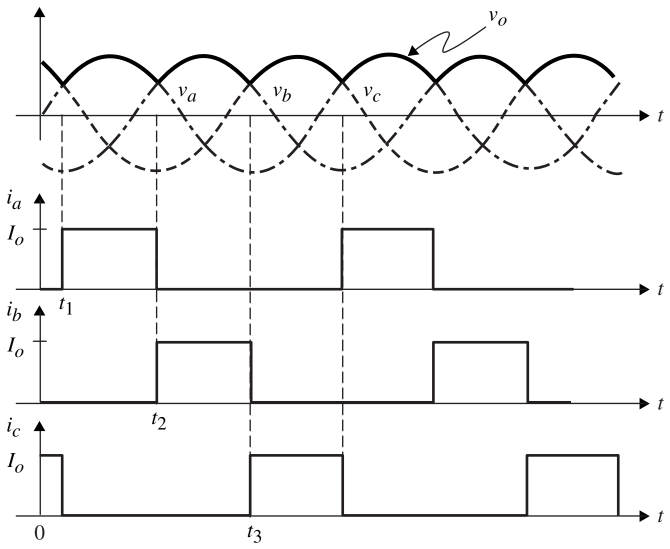


Figure 7.28 Phasor diagram representation for phase and line-to-line three-phase voltages.



(*a*)



(*b*)

Figure 7.29 (*a*) Half-wave three-phase rectifier with infinite load inductance. (*b*) Current and voltage waveforms.

Figure 7.29(*a*) and (*b*) shows the equivalent circuit for a half-wave circuit under a highly inductive load and its waveforms, respectively.

The half-wave circuit with a capacitive load is shown in Fig. 7.30(*a*) and its output waveform is shown in Fig. 7.30(*b*). To obtain an expression for the output voltage ripple of Fig. 7.30(*b*), we will make the assumption that the load time constant, RC, is much larger than $T/3$. Let $t = t_1$ be the time at which the diode D_1 turns on when v_a is the most positive. At $t = T/4$, $v_a = V_s$ and diode D_1 stops conducting; since the capacitor voltage at this instant is equal to the peak voltage V_s, all diodes remain off and the capacitor starts discharging through R. At $t = t_1 + T/3$, v_o becomes equal to v_b and D_2 turns on. Once again, the capacitor voltage increases with v_b until it equals V_s, at which the cycle repeats. For $T/4 < t < T/3 + t_1$, the output voltage is given by

$$v_o = V_s e^{-(t - T/4)/RC}$$

At $t = t_1 + T/3$, the output voltage equals v_b, i.e.,

$$\begin{aligned} v_b(t_1 + T/3) &= V_s \sin(\omega(t_1 + T/3) - 120°) \\ &= V_s e^{-(t_1 + T/12)/RC} \end{aligned}$$

This equation can be rewritten as follows:

$$\sin \omega t_1 = e^{-(t_1 + T/12)/RC}$$

As in the case of the single-phase rectifier circuit, we can solve for t_1 numerically. The ripple voltage, V_r, is obtained from the following relation:

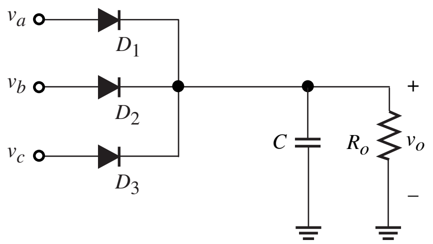
$$\begin{aligned} V_r &= V_s - v_b(t_1 + T/3) \\ &= V_s(1 - \sin \omega t_1) \end{aligned} \tag{7.48}$$

Using the approximation $t_1 \approx T/4$, the above equation becomes

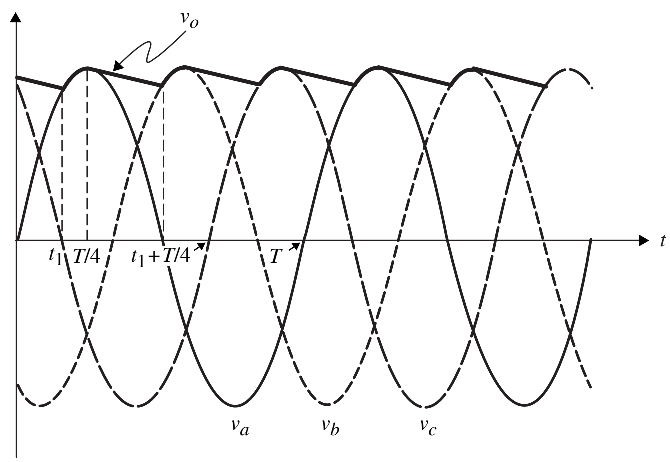
$$\begin{aligned} v_o(t_1 + T/3) &= V_s e^{-(T/4 + T/12)/RC} \\ &= V_s e^{-T/3RC} \\ &\cong V_s\left(1 - \frac{T}{3RC}\right) \end{aligned}$$

so the ripple is given by

$$\begin{aligned} v_r &= V_s - v_o(t_1 + T/3) \\ &= \frac{T}{3RC} V_s \\ &= \frac{1}{3fRC} V_s \end{aligned} \tag{7.49}$$



(a)



(b)

Figure 7.30 (*a*) Three-phase half-wave capacitive-load rectifier. (*b*) Output voltage waveform.

It can be shown that this ripple is one-third of the voltage ripple for the single-phase circuit.

EXERCISE 7.10

Consider the three-phase rectifier shown in Fig. 7.30(a) with $f = 60$ Hz and $R_o = 10\ \text{k}\Omega$. Design for C so that the output voltage ripple doesn't exceed 1% of the peak voltage.

ANSWER $C \geq 55.6\ \mu\text{F}$

It can be shown that the diode conduction time can be approximated by

$$\frac{T}{2} - t_1 - \sqrt{\frac{2V_r}{V_s}} \tag{7.50}$$

Using this expression it is possible to obtain approximate average and rms current values for the diodes.

7.3.2 Three-Phase Full-Wave Rectifier

Let us now consider the full-bridge rectifier circuit including the commutation inductance. The full-bridge rectifier is more common since it provides a high output voltage and less ripple. First let us consider the full-bridge circuit under a resistive load as shown in Fig. 7.31. Let us assume that v_a, v_b, and v_c are the three phase voltages in Fig. 7.32. The easiest way to approach the full-bridge rectifier circuit of Fig. 7.31 is to consider it as a combination of a positive commutating diode group D_1, D_2, and D_3, and a negative commutating diode group D_4, D_5, and D_6. Since no commutating inductance is included, at any given time only two diodes are conducting simultaneously—one from the positive group and the other from the negative group. To show that this is the case, let us redraw Fig. 7.31 as shown in Fig. 7.33. It is clear from Fig. 7.32 that only one diode from the upper group can be on during the positive cycle, and only one diode among the lower group can be on during the negative cycle. The output voltage, v_o, is given by

$$v_o = v_{o1} - v_{o2}$$

where v_{o1} and v_{o2} are the output voltages of the positive and negative commuting diode groups to ground, respectively.

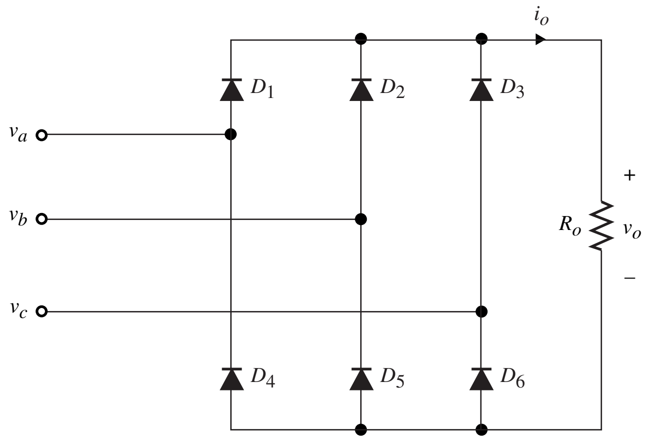


Figure 7.31 Full-bridge rectifier circuit under resistive load.

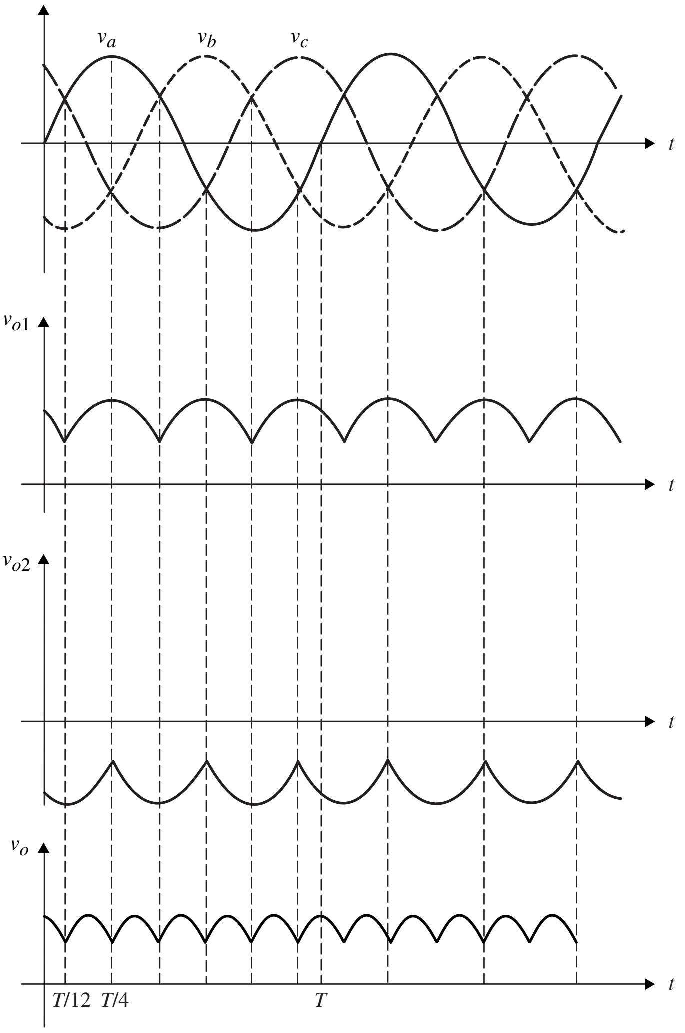


Figure 7.32 Output voltage waveforms for Fig. 7.31.

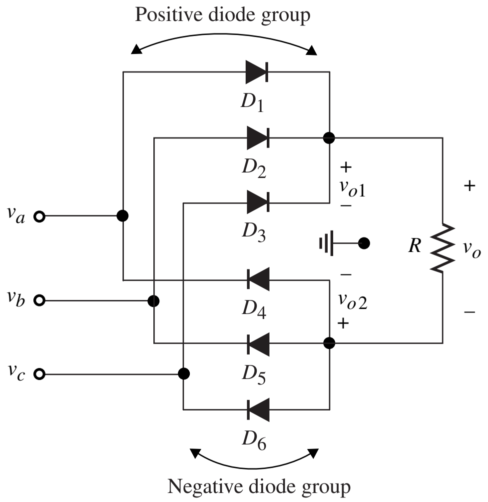


Figure 7.33 Equivalent circuit for Fig. 7.31.

The average output voltage is given by

$$V_o = \frac{1}{T/6}\int_{T/12}^{T/4} V_a\,dt$$
$$= \frac{3\sqrt{3}}{\pi}V_s \tag{7.51}$$

Let us consider the full-bridge rectifier under a highly inductive load using a Y-connected voltage source with $L/R \gg T/6$ as shown in Fig. 7.34(a). Table 7.1 shows all six modes of operation with the corresponding diode conduction angles and currents, where $\theta = \omega t$.

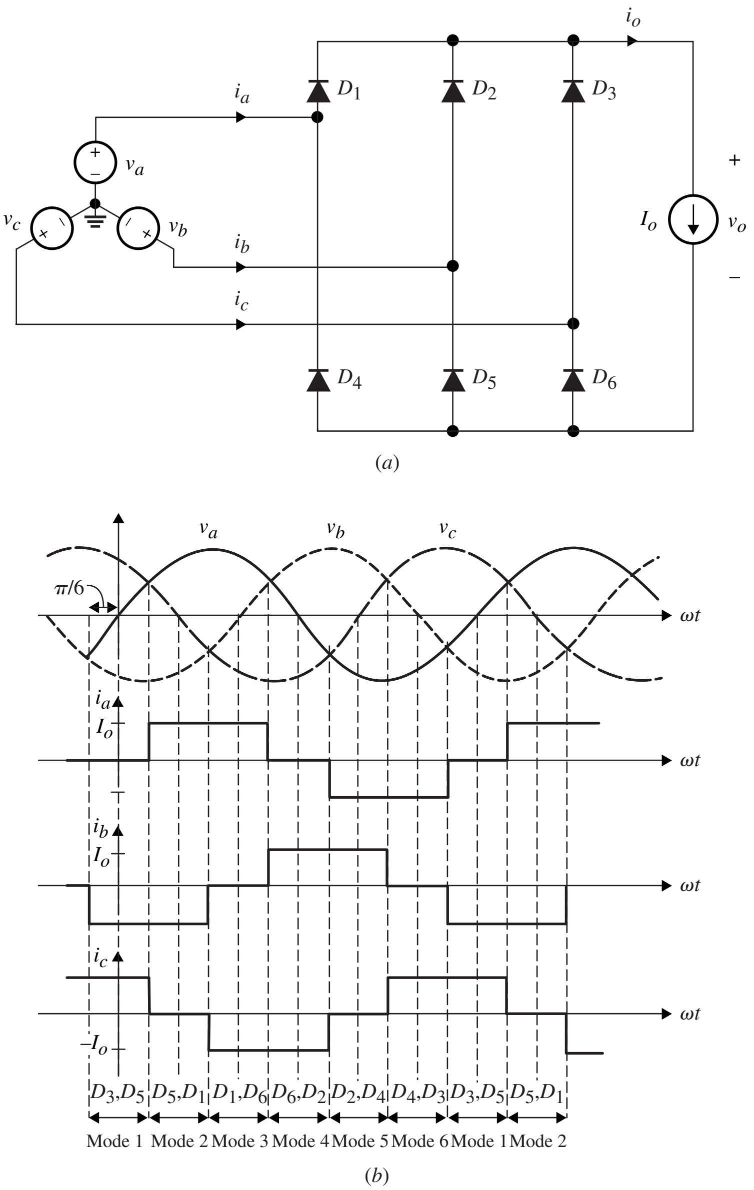


Figure 7.34 (a) Three-phase bridge rectifier with highly inductive load. (b) Phase current waveforms.

Table 7.1 Conduction Modes for Fig. 7.34

Mode	Conduction angle	Diodes conducting	i_a	i_b	i_c	Most positive absolute voltage
I	$-\pi/6 < \theta < \pi/6$	D_3, D_5	0	$-I_o$	I_o	$\lvert v_b\rvert, v_c$
II	$\pi/6 < \theta < \pi/2$	D_5, D_1	I_o	$-I_o$	0	$v_a, \lvert v_b\rvert$
III	$\pi/2 < \theta < 5\pi/6$	D_1, D_6	I_o	0	$-I_o$	$v_a, \lvert v_c\rvert$
IV	$5\pi/6 < \theta < 7\pi/6$	D_6, D_2	0	I_o	$-I_o$	$v_b, \lvert v_c\rvert$
V	$7\pi/6 < \theta < 9\pi/6$	D_2, D_4	$-I_o$	I_o	0	$\lvert v_a\rvert, v_b$
VI	$9\pi/6 < \theta < 11\pi/6$	D_4, D_3	$-I_o$	0	I_o	$\lvert v_a\rvert, v_c$

The waveforms for the output voltage and the diode and line currents are shown in Fig. 7.34(*b*). The output voltage is the same as that in the resistive case given in Fig. 7.31.

7.4 AC-SIDE INDUCTANCE IN THREE-PHASE RECTIFIER CIRCUITS

7.4.1 Half-Wave Rectifiers

Figure 7.35(*a*) shows a three-phase, half-wave rectifier circuit including an ac-side commutating inductance under a highly inductive load. The phase voltages v_a, v_b, and

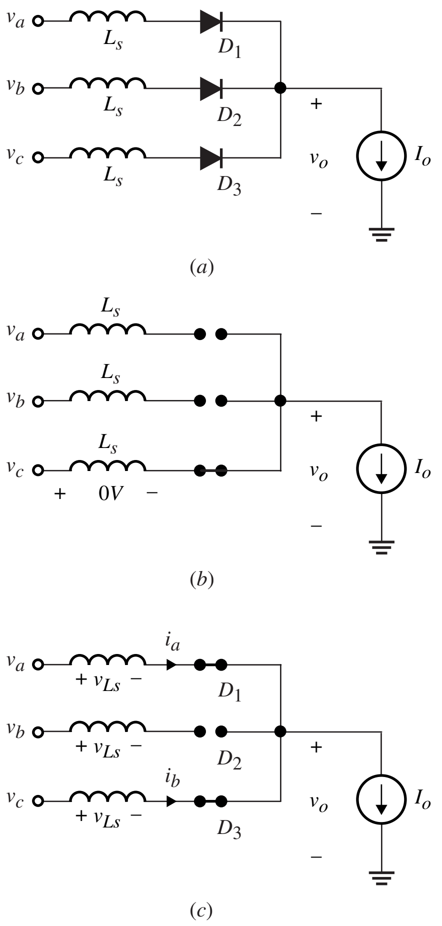


Figure 7.35 (*a*) Three-phase, half-wave rectifier with constant load current. (*b*) Equivalent circuit with v_c most positive. (*c*) Equivalent circuit during current commutation from D_3 to D_1.

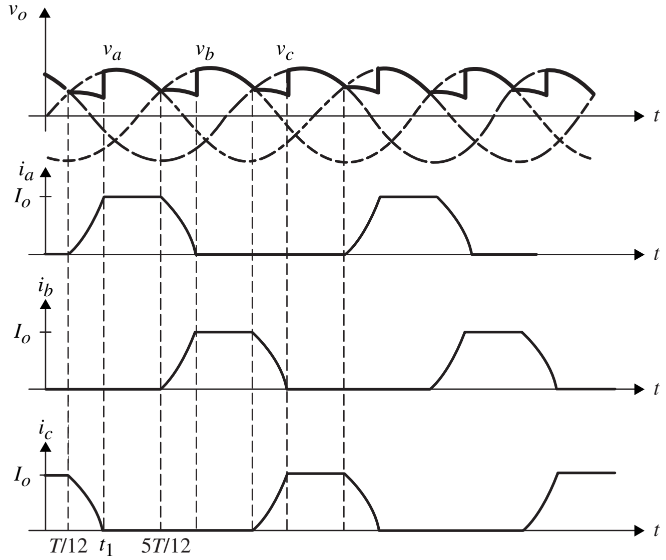


Figure 7.36 Waveforms for v_o, i_a, i_b, and i_c and the three phase voltages.

v_c are shown in Fig. 7.36. The operation of the circuit can be analyzed as follows. Prior to $\pi/12$, v_c is the most positive and D_3 is conducting, with D_1 and D_2 off, as shown in the equivalent circuit of Fig. 7.35(b). During this interval, $i_c = +I_o$ and $v_o = v_c$.

At $t = T/12$, v_a and v_c become equal and D_1 starts conducting, as in the single-phase case. Since i_c cannot change to zero instantaneously, D_3 remains on while the load current is being moved from D_3 to D_1, as shown in Fig. 7.35(c). At this instant, $i_a = 0$, $i_c = +I_o$, and v_o is the average value between v_a and v_c, to be shown next.

Assuming equal ac-side inductances, the following relations are obtained:

$$v_a = L_s\frac{di_a}{dt} + v_o \tag{7.52a}$$

$$v_c = L_s\frac{di_c}{dt} + v_o \tag{7.52b}$$

Since $i_a + i_c = I_o$, then $di_c/dt = -di_a/dt$, resulting in the following relation for v_o:

$$v_o = \frac{v_c + v_a}{2} \tag{7.53}$$

Substituting for v_o in Eq. (7.53), we obtain the following differential equation for i_a:

$$\frac{di_a}{dt} = \frac{1}{2L_s}(v_a - v_c)$$

Substitute for $v_a = V_s\sin(\omega t)$ and $v_c = V_s\sin(\omega t - 240°)$ and use the initial values of $i_c(T/12) = I_o$, $i_a(T/12) = 0$. The solution for i_c is then given by

$$i_c(t) = \frac{V_s\sqrt{3}}{2L_s\omega}[1 - \cos(\omega t - 30°)]$$

The waveforms for v_o, i_a, i_b, and i_c are given in Fig. 7.36.

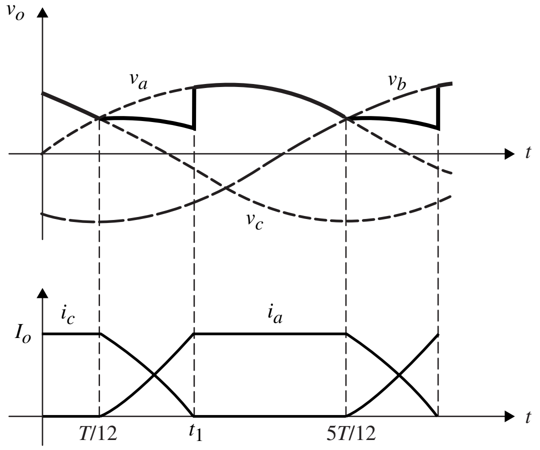


Figure 7.37 Commutation waveforms from phase *c* to phase *a* in Fig. 7.35(*a*).

The commutation angle is obtained by evaluating $i_a(t)$ at t_1 when the current reaches I_o, as shown in the commutating waveform from phase *c* to phase *a* in Fig. 7.37.

$$i_a(t) = \frac{V_s\sqrt{3}}{2L_s\omega}[1 - \cos(\omega t_1 - T/12)] = I_o \tag{7.54}$$

$$i_c(t) = I_o - i_a = I_o - \frac{V_s\sqrt{3}}{2L_s\omega}[1 - \cos(\omega t - 30°)] \tag{7.55}$$

Hence, u can be found from the following expression:

$$u = \cos^{-1}\left(1 - \frac{2L_s\omega I_o}{V_s\sqrt{3}}\right) \tag{7.56}$$

Since the pulse width of the phase current is smaller than that of the single-phase case, the power factor for a three-phase is lower. Finally, the output voltage is given by the following integral:

$$V_o = \frac{3}{T}\left[\int_{T/12}^{t_1} v_o\, dt + \int_{t_1}^{5T/12} v_a\, dt\right] \tag{7.57}$$

Substituting for v_o in the above equation, Eq. (7.57) can be rewritten as

$$\begin{aligned} V_o &= \frac{3}{T}\left[\int_{T/12}^{t_1} \frac{v_a + v_c}{2}\, dt + \int_{t_1}^{5T/12} v_a\, dt\right] \\ &= \frac{3}{T}\left[\int_{T/12}^{t_1} v_a\, dt - \int_{T/12}^{t_1} \frac{v_a - v_c}{2}\, dt + \int_{t_1}^{5T/12} v_a\, dt\right] \\ &= \frac{3}{T}\left[\int_{T/12}^{5T/12} v_a\, dt - \int_{T/12}^{t_1} \frac{v_a - v_c}{2}\, dt\right] \end{aligned} \tag{7.58}$$

We recognize that the first integral is the average output voltage for the half-wave three-phase converter with $L_s = 0$. The second integral is the average voltage loss due to the commutation. Substituting $L_s di_a/dt$ for $(v_a - v_c)/2$ in Eq. (7.58), we have

$$\begin{aligned} V_o &= V_o\Big|_{L_s=0} - \frac{3}{T}\int_{T/12}^{t_1} L_s \frac{di_a}{dt}\,dt \\ &= V_o\Big|_{L_s=0} - \frac{3}{T}\int_{\pi/6}^{\pi/6+u} L_s \frac{di_a}{dt}\,d\theta \\ &= V_o\Big|_{L_s=0} - \frac{3}{T}L_s I_o \end{aligned} \tag{7.59}$$

Recall that the average value of the voltage when $L_s = 0$ is given by

$$V_o\Big|_{L_s=0} = \frac{3\sqrt{3}V_s}{2\pi}$$

Then V_o of Eq. (7.59) may be written as

$$V_o = \frac{3\sqrt{3}}{2\pi}V_s\left(1 - \frac{\omega L_s I_o}{\sqrt{3}V_s}\right)$$

Normalizing the output voltage by V_s and I_o by $V_s/(\omega L_s)$ gives

$$V_{no} = \frac{3\sqrt{3}}{2\pi}\left(1 - \frac{I_{no}}{\sqrt{3}}\right) \tag{7.60}$$

7.4.2 Full-Wave Bridge Rectifiers

Let us consider the full-wave, three-phase rectifier circuit including an ac-side inductance. The unidirectional source current in the half-wave circuit has a significant effect on the input power factor. This is why the bridge connection is widely used. Figure 7.38 shows the full-bridge with an ac-side commutation inductance from a three-phase Y-connected voltage source. Again, we assume a large inductive load.

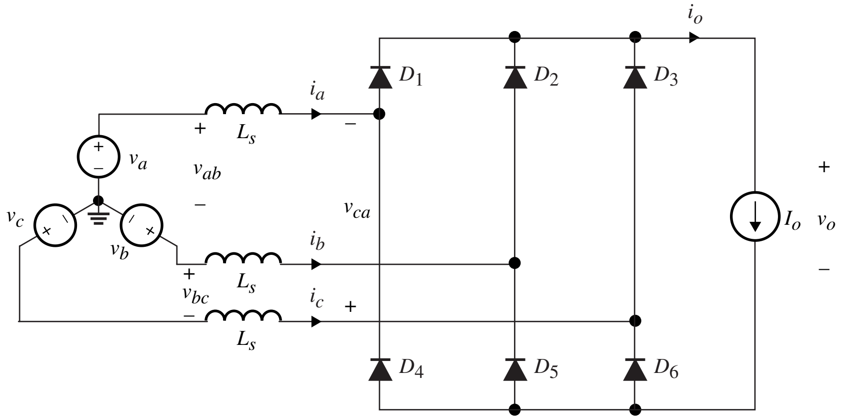


Figure 7.38 Three-phase bridge rectifier with ac-side inductance.

The three source line-to-line voltages are

$$v_{ab} = \sqrt{3}V_s \sin(\omega t + 30°) \quad v_{bc} = \sqrt{3}V_s \sin(\omega t - 90°) \quad v_{ca} = \sqrt{3}V_S \sin(\omega t + 150°)$$

The operation of this circuit can be discussed in a similar way to the case with $L_s = 0$, except in this case the commutation inductance will present an additional commutation angle, u, during which three diodes are on simultaneously. From mode I in Table 7.1, for $\omega t < \pi/6$, $|v_{bc}|$ is the maximum voltage with D_3 and D_5 conducting, as shown in Fig. 7.39(a). In this mode, we have

$$i_a = 0 \quad i_b = -I_o \quad i_c = I_o \quad v_o = -v_{bc} \quad v_{Ls} = 0$$

From Table 7.1, the next mode is when D_5 and D_1 turn on, since the next most positive voltage in this mode is v_{ab}. However, the current will commutate from D_3 to D_1 during a commutation period, u, during which D_3 remains on while D_1 and D_5 are conducting. Figure 7.39(b) shows the equivalent circuit during this commutation period. From mode I to mode II the currents commutate from zero to $+I_o$ in i_a; hence, D_3 remains on for a short time until its current goes to zero at $t = t_1$. In this time interval we have

$$-v_b - v_o - L_s\frac{di_c}{dt} + v_c = 0 \tag{7.61a}$$

$$v_b - v_o - L_s\frac{di_a}{dt} + v_a = 0 \tag{7.61b}$$

Since $i_a + i_c = I_o$, then $di_c/dt = -di_a/dt$. Hence, the above relation becomes

$$v_o = \frac{v_{ab} + v_{cb}}{2} = \frac{v_{ab} + v_{bc}}{2} \tag{7.62}$$

From Eqs. (7.61) and (7.62), we have the following first-order differential equations in terms of the line-to-line voltages and currents:

$$-v_{cb} - v_o - L_s\frac{di_c}{dt} = 0 \tag{7.63a}$$

$$v_{ab} - v_o - L_s\frac{di_a}{dt} = 0 \tag{7.63b}$$

Substituting for v_o in the above equations, we obtain

$$\frac{v_{cb} - v_{ab}}{2L_s} = \frac{di_c}{dt} = -\frac{di_a}{dt}$$

$$\frac{v_{ab} - v_{cb}}{2L_s} = \frac{di_a}{dt}$$

Substituting for $v_{cb} - v_{ab} = v_{ca} = \sqrt{3}V_s \sin(\omega t + 150°)$, and since at $t = 0$, $i_c(\pi/6) = I_o$, we have

$$i_c(t) = \frac{1}{2L_s}\int_{\pi/6}^{\omega t} \sqrt{3}V_s \sin(\omega t + 150°)\, d\omega t + I_o$$

At $\omega t = u + \pi/6$, we have $i_c(u + \pi/6) = 0$; hence, u is given by

$$u = \cos^{-1}\left(1 - \frac{2L_s \omega I_o}{V_s\sqrt{3}}\right) \tag{7.64}$$

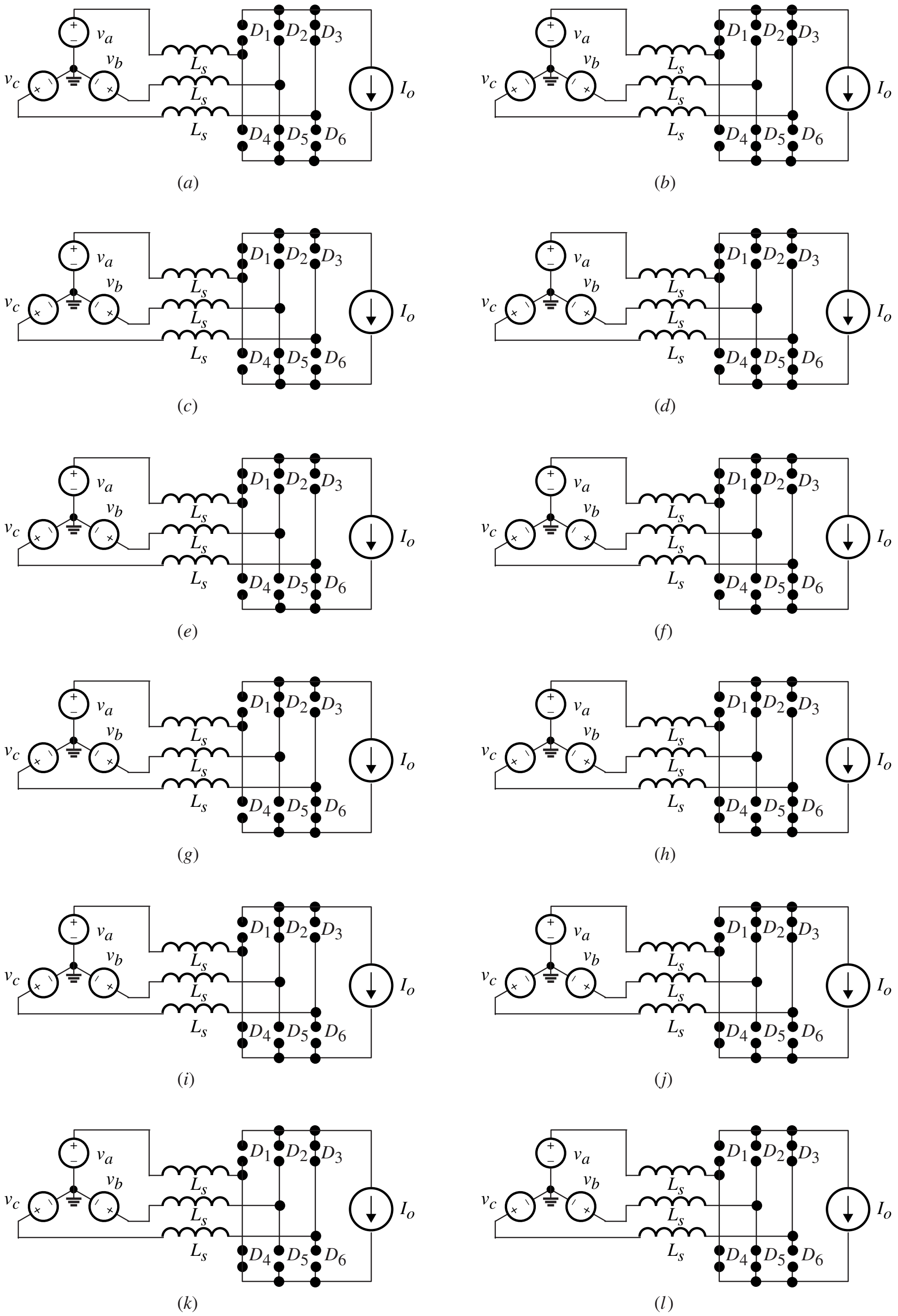


Figure 7.39 Equivalent circuit modes for mode I to mode VI. (*a*) Mode I: D_3 and D_5 are conducting. (*b*) Mode I and mode II commutation. (*c*) Mode II: D_1 and D_5 are conducting. (*d*) Mode II and mode III commutation. (*e*) Mode III: D_1 and D_6 are conducting. (*f*) Mode III and mode IV commutation. (*g*) Mode IV: D_2 and D_6 are conducting. (*h*) Mode IV and mode V commutation. (*i*) Mode V: D_2 and D_4 are conducting. (*j*) Mode V and mode VI commutation. (*k*) Mode VI: D_3 and D_4 are conducting. (*l*) Mode VI and mode I commutation.

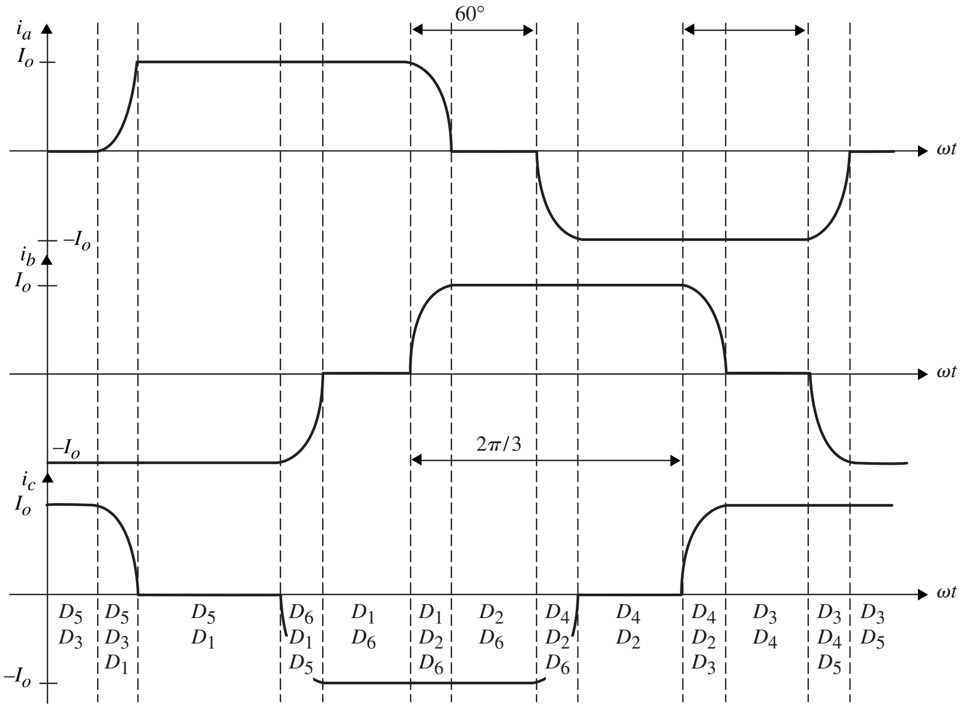


Figure 7.40 Current waveforms for i_a, i_b, and i_c for Fig. 7.38.

At the end of the commutation period u, i_c becomes zero and D_3 stops conducting. Diodes D_1 and D_5 remain on until mode III starts at $\pi/2$, when $|v_{ca}|$ becomes the most positive, turning diodes D_1 and D_6 on and in turn turning D_5 off. In this mode, the current i_b will commutate from I_o to 0 through D_5, and i_c goes from 0 to $-I_o$ through D_6. Diodes D_5 and D_6 will overlap during the commutation period u. The remaining modes and their corresponding mode-to-mode transition are shown in Fig. 7.39(a)–(l). The current waveforms of i_a, i_b, and i_c and the output voltage are shown in Fig. 7.40.

PROBLEMS

In all the problems assume ideal diodes unless stated otherwise.

Single-Phase Rectifiers

Half-Wave Rectifiers

7.1 Determine the power factor and the THD for the waveforms of Example 7.1.

7.2 Consider the half-wave rectifier in Fig. P7.2 with $v_s = 110 \sin \omega t$ V and $R = 1$ kΩ. Assume an ideal diode except the forward resistance (r_d) is 10 Ω. Sketch v_o and determine:

(a) The average output voltage

(b) The rms current in the diode

(c) The peak current in the diode

(d) The input power factor

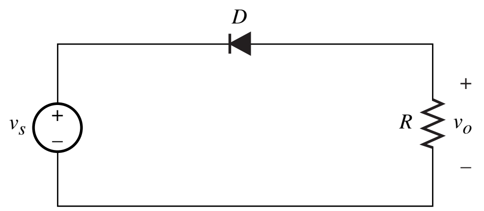


Figure P7.2

7.3 Derive Eqs. (7.9) and (7.11).

7.4 Consider the half-wave inductive-load rectifier circuit of Fig. 7.4(a) with $v_s = 110\sqrt{2} \sin \omega t$ V, $\omega = 60$ Hz, $L = 800$ mH, and $R = 100$ Ω. Find:

(a) The expression for $i_L(t)$

(b) The time t_1 when i_L becomes zero

(c) The average load current

(d) The peak inductor current

(e) The rms current in the inductor

(f) The average power delivered to the load

7.5 Repeat Problem 7.4 with the diode reversed.

7.6 Repeat Problem 7.4 by placing a free-wheeling diode in parallel with the load as shown in Fig. 7.6(*a*).

7.7 Repeat Example 7.2 for the circuit of Fig. 7.6(*a*).

D7.8 Design for L and R for the half-wave rectifier of Fig. 7.6(*a*) to provide a ripple current not to exceed 5% of its average value at $V_o = 20$ V. Assume $v_s = 40\sin 120\pi t$ V.

D7.9 The single-phase, half-wave rectifier of Fig. 7.6(*a*) is required to supply the load with an output ripple current not to exceed 5% of its dc value.

(a) Design for L to meet this requirement. Use $R = 20\ \Omega$, $v_s = 150\sin 120\pi t$ V.

(b) What is the average load voltage?

7.10 Derive Eqs. (7.20) and (7.21) for the half-wave rectifier with an *L-R* load.

D7.11 Design the half-wave rectifier of Fig. 7.4(*a*) with an *L-R* load to supply 25 W to the load with a minimum output ripple current of 10% and an average output of 28 V. Use $v_s = 120\sin(120\pi t)$.

D7.12 Design a dc power supply with an average dc output of 20 V with a 2% ripple. The power supply should be capable of providing 500 mA to the load. Use the rectifier of Fig. P7.12 for your design with $v_s = 110\sin(2\pi 60t)$ V, and assume the diode's forward voltage drop is 1 V. Specify the diode's ratings, including its peak reverse voltage and rms values.

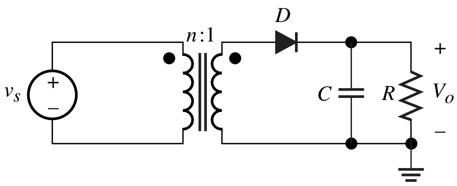


Figure P7.12

D7.13 Design the half-wave rectifier with an *R-C* load with a 10% output ripple voltage and that provides 10 W to the load. Assume $v_s = 20\sin(120\pi t)$ V.

7.14 Sketch the waveforms for i_s, i_c, i_R, and v_o for Fig. 7.8(*a*) with $R = 10\ \text{k}\Omega$, $C = 100\ \mu\text{F}$, and $v_s = 10\sin 377t$ V.

Full-Wave Rectifiers

7.15 Determine the power factor and the THD for the waveforms of Fig. 7.3(*b*).

7.16 Consider the full-wave rectifier with a resistive load of Fig. 7.3(*a*) with $v_s = 120\sin\omega t$ and $R = 20\ \Omega$. Find:

(a) The average output voltage

(b) The average output current

(c) The rms output current

(d) The input power factor

(e) The load power

7.17 Consider a full-wave rectifier with an infinite inductive load.

(a) Show that the dc and harmonic components of the output voltage are given by

$$v_o(t) = \frac{2V_s}{\pi} - \frac{4V_s}{\pi}\sum_{n=2}^{\infty}\frac{1}{(n-1)(n+1)}\cos n\omega t$$

for $n = 2, 4, 6, \ldots$.

(b) Show that the nth harmonic of the source current is given by

$$i_s(t) = \frac{4I_o}{\pi}\sum_{n=1}^{\infty}\frac{\sin n\omega t}{n}$$

D7.18 It is required that the circuit shown in Fig. P7.18 provide an output power of 2 kW at an average load current of 20 A. Design for the transformer turns ratio and R to meet this requirement. Assume $L/R >> (2\pi/\omega)$. Use $v_s = 100\sin(2\pi 60t)$ V.

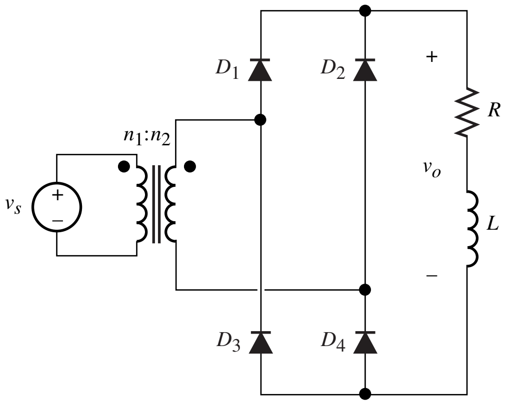


Figure P7.18

7.19 Consider the generalized-type load with a dc source representing a dc-motor-load full-wave rectifier as shown in Fig. P7.19. Assume $v_s = V_s \sin \omega t$ V and $V_{dc} < V_s$. Derive:

(a) The expression for $i_o(t)$

(b) The rms value of $i_o(t)$

(c) The average load current

(d) The ripple load current

(e) Calculate the above values for $V_s = 25$ V, $\omega = 377$ rad/s, $R = 2\ \Omega$, and $L = 5$ mH.

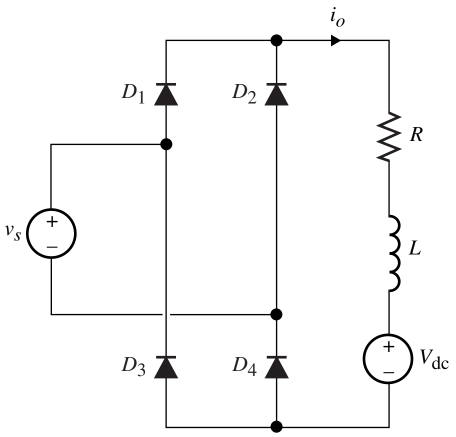


Figure P7.19

7.20 Consider the center-tap full-wave rectifier of Fig. P7.20.

(a) Sketch v_o.

(b) Determine the average output voltage.

(c) Repeat parts (*a*) and (*b*) by including each diode drop V_D.

(d) Determine the peak inverse voltage (PIV) for the two diodes.

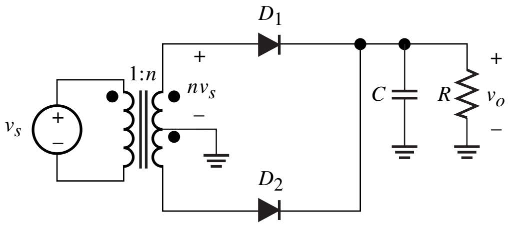


Figure P7.20

D7.21 The full-wave rectifier of Problem 7.20 is to provide an average output voltage of 48 V with an output power of 25 W. Design for the required transformer turns ratio n. Assume $v_s = v_s = 110\sqrt{2}\sin 377t$ V.

D7.22 Redesign Problem 7.21 by assuming $v_s(t)$ fluctuates by ±20%. Design for the worst case.

7.23 Consider the full-wave, center-tap diode rectifier circuit shown in Fig. P7.23.

(a) Sketch v_{o1} and v_{o2}.

(b) Determine the peak inverse voltage for each diode.

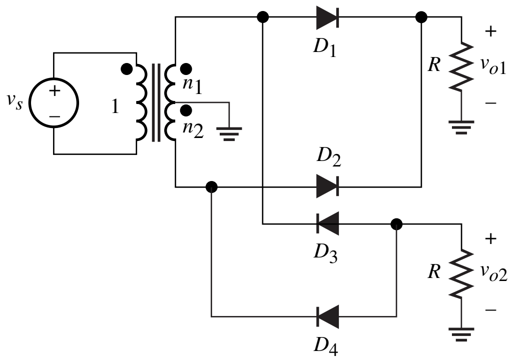


Figure P7.23

D7.24 Design the dual center-tap full-wave rectifier of Fig. P7.24 to provide ±15 VDC from a line voltage $v_s = 155\sin\omega t$ V, where $\omega = 377$ rad/s.

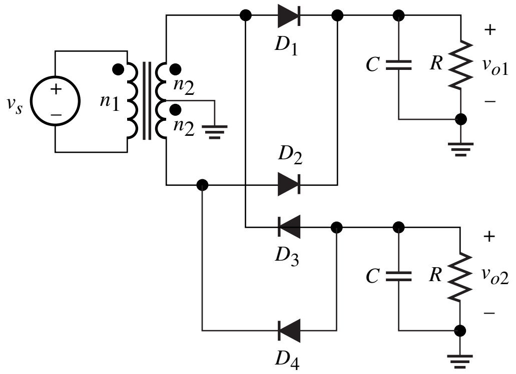


Figure P7.24

7.25 Repeat Problem 7.9 by using the full-bridge rectifier given in Fig. 7.20(*a*) with $L_s = 0$.

7.26 **(a)** Calculate the output ripple voltage for the full-wave rectifier of Fig. P7.26. Use $R = 10$ kΩ and $C = 47\ \mu$F, with $v_s = 20\sin 377\omega t$.

(b) Show that the expression for the average output voltage is given by

$$V_o = \frac{V_s}{2\pi}\left[\cos\omega t_1 + \tau\omega\left(1 - e^{-(t_1 + 3T/4)/\tau}\right)\right]$$

where $\tau = RC$, V_s is the peak voltage, and t_1 $(0 \le t_1 \le T/4)$ is the time at which the diode starts conducting.

(c) If it is assumed that the ripple voltage is much smaller than the peak input voltage, i.e., $t_1 = T/4$, and $RC >> T$, then show that $V_r = V_s/2RCf$. Apply this simplified ripple voltage to part (*a*).

(d) Use the approximation in part (*c*) to show that the average diode current is given by

$$I_{D,\text{ave}} = \frac{V_s}{2R}\left[1 + \frac{1}{\pi}\sqrt{1 - \left(1 - \frac{V_r}{V_s}\right)^2}\right]$$

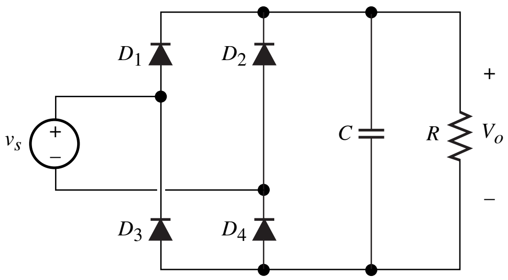


Figure P7.26

7.27 The circuit shown in Fig. P7.27 is called a voltage doubler. It is used when the line voltage of a given system changes from 110 VAC to 220 VAC or vice versa. By changing the switch position, the output voltage can be held constant under both a 110 VAC and a 220 VAC line input. Consider the voltage doubler shown. Determine the voltages v_{o1} and v_{o2} for both cases when the switch is on and off. Assume $v_s = V_s \sin\omega t$ V with $R_1C_1 >> T$ and $R_2C_2 >> T$ $(\omega = 2\pi/T)$.

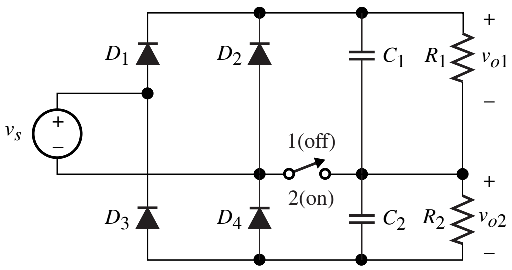


Figure P7.27

7.28 Consider the full-wave rectifier with an *LC* output filter shown in Fig. P7.28.

(a) Draw the equivalent circuit between *a* and *b* by assuming $L = 0$ and $C = \infty$.

(b) Repeat part (*a*) by assuming $L = \infty$ and $C = 0$.

(c) Calculate the power dissipated in the load in parts (*a*) and (*b*). Assume $v_s = V_s \sin\omega t$.

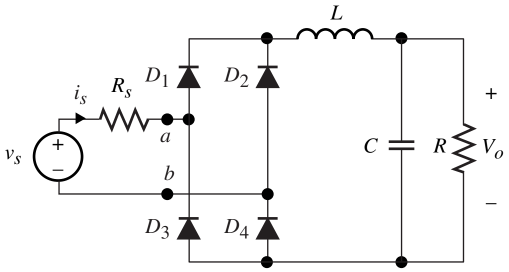


Figure P7.28

D7.29 The single-phase, full-wave rectifier with an *R-C* load shown in Fig. P7.26 is required to supply the load with an output ripple voltage not to exceed 5% of its dc value.

(a) Design for *C* to meet this requirement. Use $R = 20\ \Omega$, $v_s = 150 \sin 120\pi t$ V.

(b) Calculate the average load current.

The Effect of ac-Side Inductance

Half-Wave Circuits

7.30 Consider the half-wave rectifier shown in Fig. P7.30 with a flyback diode including L_s and the diode resistances. Assume $v_s = V_s \sin\omega t$ V.

(a) Derive the expression for the power dissipated in the rectifier.

(b) Sketch i_s, i_{D2}, and v_o.

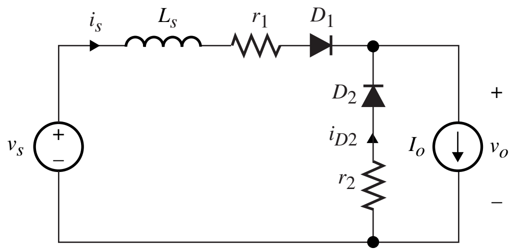


Figure P7.30

D7.31 The circuit of Fig. 7.12(*a*) is to be designed to deliver a 1 kW output power at $I_o = 20$ A. Assume L_s fluctuates in the range of ±20%. Use $L_s = 0.1$ mH and $v_s(t) = 120 \sin 2\pi 60t$ V. Design for the load resistance and inductance.

Full-Wave Circuits

7.32 Derive the rms expression for the input current $i_s(t)$ of a full-wave rectifier circuit with an ac-side inductance, which is redrawn in Fig. P7.32.

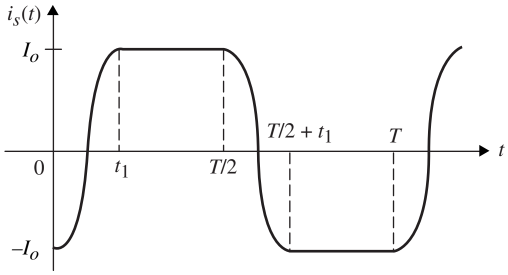


Figure P7.32

7.33 Repeat Problem 7.32 by using the approximation for $i_s(t)$ shown in Fig. P7.33.

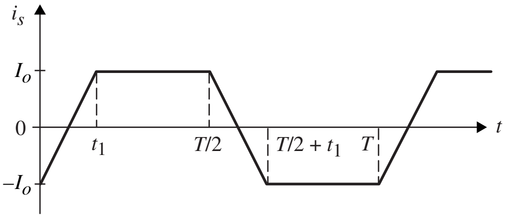


Figure P7.33

7.34 **(a)** Discuss the operation of the circuit in Fig. P7.34 and sketch the waveforms for v_o, i_{s1}, i_{s2}, v_{Ls1}, v_{Ls2}. Assume $v_{s1} = V_{s1} \sin \omega t$ V, $v_{s2} = V_{s2} \sin \omega t$ V.

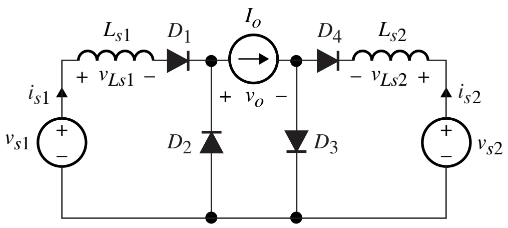


Figure P7.34

(b) Derive the expression for $v_o(t)$.

(c) What is the difference between this circuit and the full-wave bridge circuit of Fig. 7.21?

7.35 **(a)** Find the ratio between the rms currents for the waveforms shown in Fig. P7.35(a) and (b), representing the line currents for half-wave rectifiers with and without an ac-side inductance, respectively.

(b) Sketch the ratios as a function of $\omega t_1 = u$.

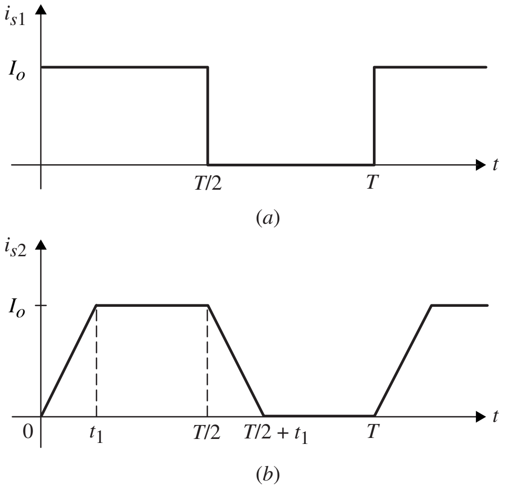


Figure P7.35

Input Power Factor

7.36 Consider the line current for the half-wave rectifier with L_s as shown in Fig. 7.12(e), which is redrawn in Fig. P7.36(a).

(a) Derive the expression for the power factor for $i_s(t)$.

(b) Repeat part (a) by using the trapezoidal approximation shown in Fig. P7.36(b).

(c) Compare the two solutions.

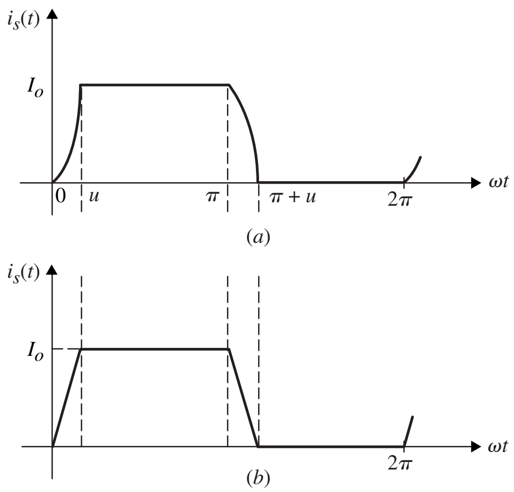


Figure P7.36

7.37 Consider the one-port converter network shown in Fig. P7.37 with $i_s = 5\sin(377t + 30°)$ A, and $v_s = 10\sin(377t - 45°)$ V. Determine:

(a) The instantaneous input power

(b) The input power factor

(c) The average input power

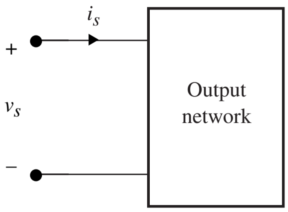


Figure P7.37

7.38 Derive the input power factor and the THD for the input current waveforms shown in Fig. P7.38. Assume the line voltage is pure sinusoidal. (See Appendix B.)

Three-Phase Rectifier Circuit

7.39 **(a)** Show that the average output voltage for the n-phase, half-wave rectifier of Fig. P7.39 is given by

$$V_o = V_s \frac{\sin(\pi/n)}{(\pi/n)}$$

where V_s is the phase peak voltage. What is the value of V_o as $n \longrightarrow \infty$?

(b) Show that the rms and average current in each diode are given by

$$I_{Di,\text{rms}} = \frac{I_o}{\sqrt{n}}$$

$$I_{D,\text{ave}} = \frac{I_o}{n}$$

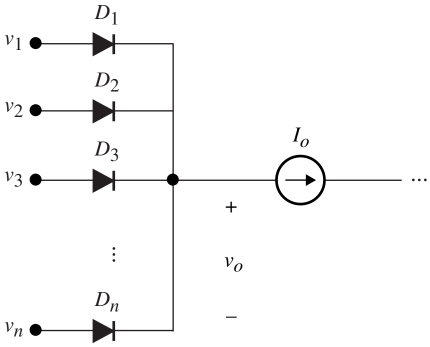


Figure P7.39

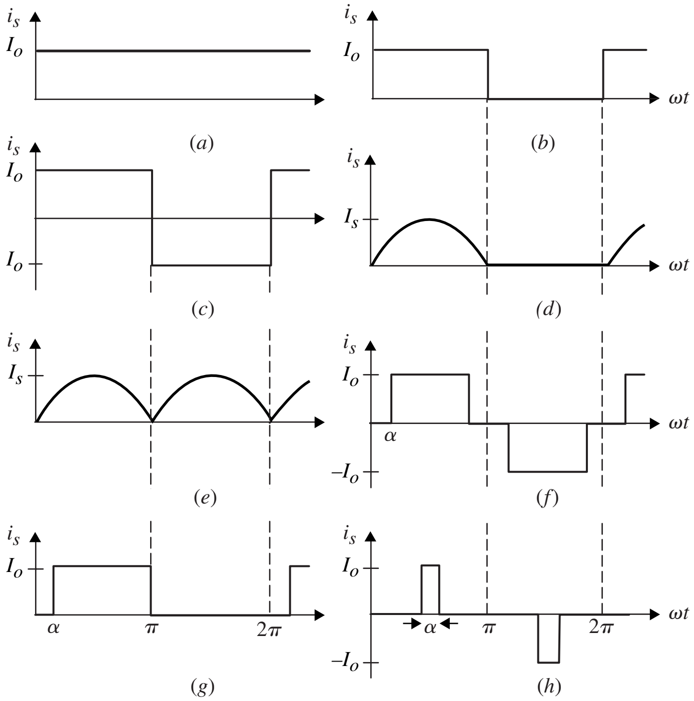


Figure P7.38

7.40 Consider the balanced three-phase rectifier circuit shown in Fig. P7.40 with the delta-connected voltage sources given below:

$$v_a = V_s \sin \omega t \quad v_b = V_s \sin(\omega t - 120°)$$
$$v_c = V_s \sin(\omega t - 240°)$$

(a) Sketch the waveforms for i_1, i_2, i_3, and v_o.

(b) Derive the expression for the average V_o.

(c) Determine the first fundamental component for the line currents.

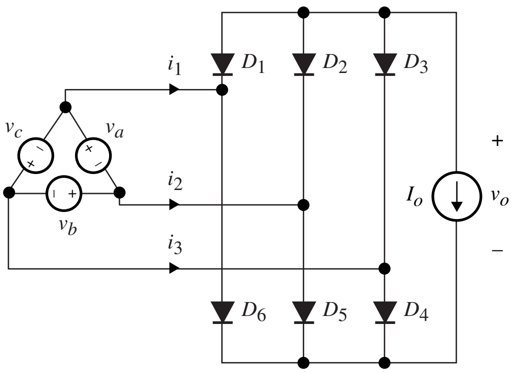


Figure P7.40

7.41 Consider the half-wave, three-phase, capacitive-load circuit of Fig. P7.41.

(a) Determine the rms value for v_o.

(b) Calculate the input power factor for each phase.

(c) Sketch v_o for $RC = T/3$.

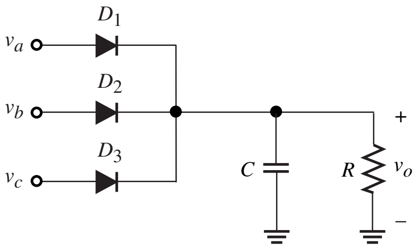


Figure P7.41

Three-Phase Rectifiers with ac-Side Inductance

7.42 Derive Eq. (7.60) for the average output voltage for the half-wave, three-phase converter.

D7.43 Design the transformer turns ratio of the circuit shown in Fig. P7.43 to deliver an average 10 A load current with a 120 V output voltage. Assume the line-to-line voltages are 580 VAC, with $\omega = 2\pi 60$.

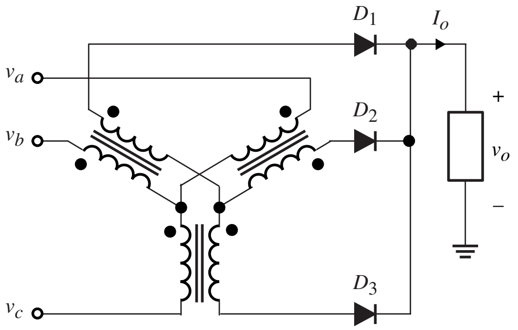


Figure P7.43

7.44 Derive Eq. (7.64) for the commutation angle u for the full-wave converter.

General Problems

7.45 Sketch the input current, i_s, and output voltage, v_o, for each of the circuits shown in Fig. P7.45. Assume $v_s(t) = V_s \sin \omega t$ V.

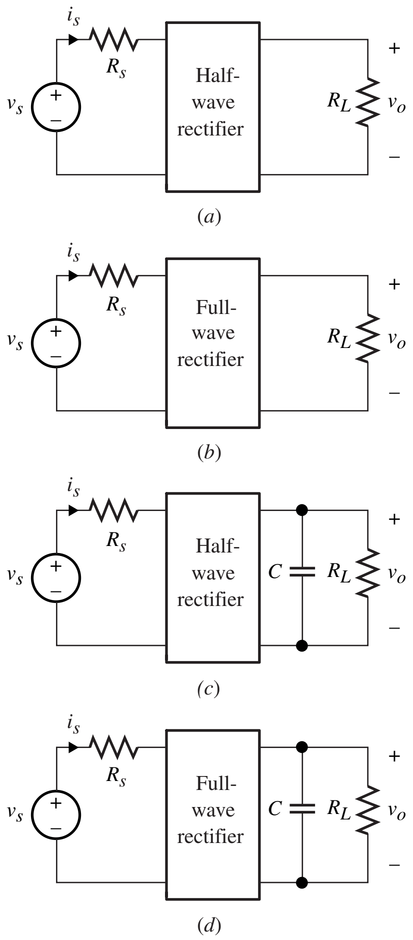


Figure P7.45

7.46 Derive the expression for the average output power and the commutation angle u for the circuit shown in Fig. P7.46. Find the average and rms values of each diode current. Assume $v_s = V_s \sin \omega t$ V.

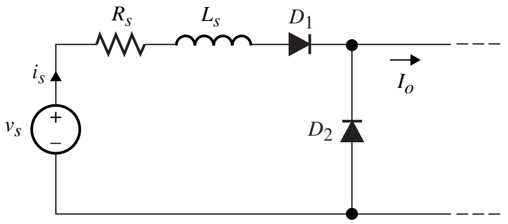


Figure P7.46

7.47 In Problem 7.46, if we assume that D_1 and D_2 have forward resistances r_{D1} and r_{D2}, respectively, show that the commutation angle is given by

$$u = \frac{\omega L_s}{r_{D1} + r_{D2}} \ln \frac{V_s + r_{D2} I_o}{(r_{D1} + r_{D2}) I_o}$$

7.48 Find the average output voltage for the circuit of Fig. P7.48. Determine the percentage change in the average output voltage if the ac-side line inductance doubles. Assume $v_s = 110\sqrt{2} \sin 377t$ V.

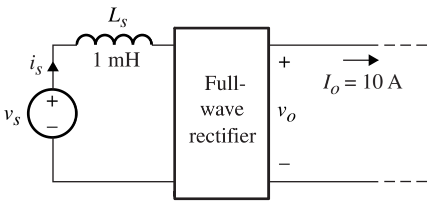


Figure P7.48

7.49 Consider the full-wave rectifier with an ac-side line inductance and a dc source in the load side as shown in Fig. P7.49.

(a) Sketch i_o, i_s, and v_{Ls}.

(b) Calculate the average output power.

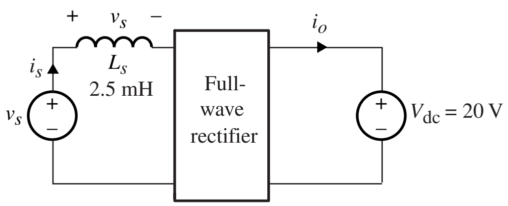


Figure P7.49

(c) Calculate the percentage change in the average output power if the ac-side line inductance doubles in value. Assume $v_s = 50 \sin 2\pi 60t$ V.

7.50 Repeat Problem 7.49 for v_s being a square-wave with ±50 V peaks and $T = 20$ ms.

7.51 Sketch the waveforms for i_{s1}, i_{s2}, and v_s' for the circuit in Fig. P7.51. Find the THD for v_s'.

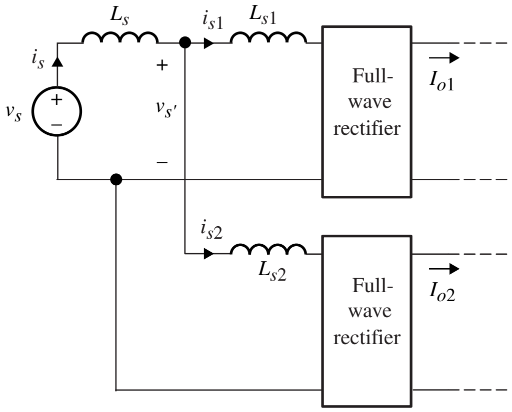


Figure P7.51

7.52 Derive the expression for the average output voltage in Fig. P7.52 in terms of the circuit parameters, assuming $v_s(t) = V_s \sin \omega t$.

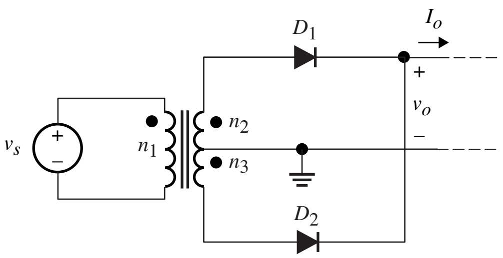


Figure P7.52

7.53 Repeat Problem 7.52 by including diode resistors r_{D1} and r_{D2} as shown in Fig. P7.53.

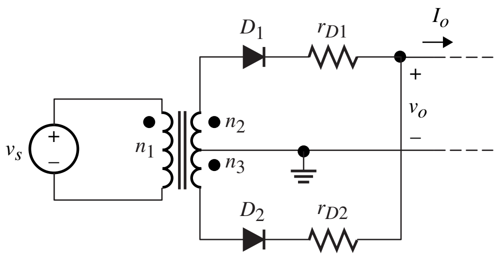


Figure P7.53