Appendix C

Useful Functions

C.1 TRIGONOMETRIC IDENTITES

ωt	$-\theta$	$\theta \mp \frac{\pi}{2}$	$\theta \mp \pi$	$\theta \mp \frac{3\pi}{2}$	$\theta \mp 2\pi$
$\cos(\omega t)$	$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$	$\cos\theta$
$\sin(\omega t)$	$-\sin\theta$	$\cos\theta$	$\pm\sin\theta$	$\cos\theta$	$\mp\sin\theta$

$$\sin(\theta_1 \pm \theta_2) = \sin\theta_1 \cos\theta_2 \pm \cos\theta_1 \sin\theta_2$$

$$\cos(\theta_1 \pm \theta_2) = \cos\theta_1 \cos\theta_2 \mp \sin\theta_1 \sin\theta_2$$

$$\tan(\theta_1 \pm \theta_2) = \frac{\tan\theta_1 \pm \tan\theta_2}{1 \mp \tan\theta_1 \tan\theta_2}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \cos^2\theta - \sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin\theta_1 + \sin\theta_2 = 2\sin\frac{\theta_1 + \theta_2}{2}\cos\frac{\theta_1 - \theta_2}{2}$$

$$\sin\theta_1 - \sin\theta_2 = 2\cos\frac{\theta_1 + \theta_2}{2}\sin\frac{\theta_1 - \theta_2}{2}$$

$$\cos\theta_1 + \cos\theta_2 = 2\cos\frac{\theta_1 + \theta_2}{2}\cos\frac{\theta_1 - \theta_2}{2}$$

$$\cos\theta_1 - \cos\theta_2 = 2\sin\frac{\theta_1 + \theta_2}{2}\sin\frac{\theta_2 - \theta_1}{2}$$

$$\sin\theta_1 \sin\theta_2 = \frac{1}{2}[\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)]$$

$$\cos\theta_1 \cos\theta_2 = \frac{1}{2}[\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)]$$

$$\sin\theta_1 \cos\theta_2 = \frac{1}{2}[\sin(\theta_1 - \theta_2) + \sin(\theta_1 + \theta_2)]$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$a\cos\theta + b\sin\theta = \left[\sqrt{a^2 + b^2}\right]\cos\left[\theta + \tan^{-1}\left(\frac{-b}{a}\right)\right]$$

$$= \left[\sqrt{a^2 + b^2}\right]\sin\left[\theta + \tan^{-1}\left(\frac{a}{b}\right)\right]$$

C.2 SOME LAPLACE TRANSFORMATIONS

$f(t)[t > 0^-]$	$F(s)$
$\delta(t)$	1
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{-\alpha t}$	$\frac{1}{(s+\alpha)}$
$\sin\omega t$	$\frac{\omega}{(s^2+\omega^2)}$
$\cos\omega t$	$\frac{s}{(s^2+\omega^2)}$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos\omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

C.3 DERIVATIVES AND INTEGRALS

Given $f_1(\omega t)$ and $f_2(\omega t)$ with ω and α constants:

$$\frac{d}{dt}(\alpha f_1) = \alpha\frac{df_1}{dt}$$

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$$\frac{d}{dt}(f_1 \cdot f_2) = f_1\frac{df_2}{dt} + f_2\frac{df_1}{dt}$$

$$\frac{d}{dt}(\alpha f_1^n) = n\alpha f_1^{n-1}\frac{df_1}{dt}$$

$$\frac{d}{dt}(\sin\omega t) = \omega\cos\omega t$$

$$\frac{d}{dt}(\cos\omega t) = -\omega\sin\omega t$$

$$\int \alpha\ dt = \alpha t + C$$

$$\int f_1^n(t)\ df_1 = \frac{f_1^{n+1}(t)}{n+1} + C, \quad n \neq -1$$

$$\int e^{\alpha t}\ dt = \frac{1}{\alpha}e^{\alpha t} + C$$

$$\int x e^{\alpha t}\ dt = \frac{e^{\alpha t}}{\alpha^2}(\alpha t - 1) + C$$

$$\int \sin\omega t\ dt = -\frac{1}{\omega}\cos\omega t + C$$

$$\int \cos\omega t\ dt = \frac{1}{\omega}\sin\omega t + C$$

$$\int \sin^2\omega t\ dt = \frac{t}{2} - \frac{\sin 2\omega t}{4\omega} + C$$

$$\int \cos^2\omega t\ dt = \frac{t}{2} + \frac{\sin 2\omega t}{4\omega} + C$$

$$\int \sin\omega_1 t\ \sin\omega_2 t\ dt = \frac{\sin(\omega_1 - \omega_2)t}{2(\omega_1 - \omega_2)} - \frac{\sin(\omega_1 + \omega_2)t}{2(\omega_1 + \omega_2)} + C, \quad \text{for } \omega_1^2 \neq \omega_2^2$$

$$\int \sin\omega_1 t\ \cos\omega_2 t\ dt = -\frac{\cos(\omega_1 - \omega_2)t}{2(\omega_1 - \omega_2)} - \frac{\cos(\omega_1 + \omega_2)t}{2(\omega_1 + \omega_2)} + C, \quad \text{for } \omega_1^2 \neq \omega_2^2$$

$$\int \cos\omega_1 t\ \cos\omega_2 t\ dt = \frac{\sin(\omega_1 - \omega_2)t}{2(\omega_1 - \omega_2)} + \frac{\sin(\omega_1 + \omega_2)t}{2(\omega_1 + \omega_2)} + C, \quad \text{for } \omega_1^2 \neq \omega_2^2$$

$$\int t\sin\omega t\ dt = \frac{1}{\omega^2}(\sin\omega t - \omega t\cos\omega t) + C$$

$$\int t\cos\omega t\ dt = \frac{1}{\omega^2}(\cos\omega t + \omega t\sin\omega t) + C$$

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$$\int e^{\alpha t} \sin \omega t \ dt = \frac{e^{\alpha t}}{\alpha^2 + \omega^2}(\alpha \sin \omega t - \omega \cos \omega t) + C$$

$$\int e^{\alpha t} \cos \omega t \ dt = \frac{e^{\alpha t}}{\alpha^2 + \omega^2}(\alpha \cos \omega t + \omega \sin \omega t) + C$$

C.4 DEFINITE INTEGRALS

For n and m integers:

$$\int_0^{2\pi} \sin nt \ dt = 0$$

$$\int_0^{2\pi} \cos nt \ dt = 0$$

$$\int_0^{\pi} \sin^2 nt \ dt = \int_0^{\pi} \cos^2 nt \ dt = \frac{\pi}{2}$$

$$\int_0^{\pi} \sin nt \ \cos mt \ dt = \begin{cases} 0, & m + n \text{ even} \\ \dfrac{2n}{n^2 - m^2}, & m + n \text{ odd} \end{cases}$$

$$\int_0^{\pi} \sin nt \ \sin mt \ dt = \int_0^{\pi} \cos nt \ \cos mt \ dt = 0; \quad m \neq n$$