Appendix B

Fourier Series for Common Waveforms

INTRODUCTION

This appendix gives the Fourier components for waveforms commonly encountered in power electronic circuits. This appendix does not attempt to repeat the detailed treatment given in Chapter 3, but rather is meant to provide the reader with basic techniques to obtain Fourier components for selected square and sinusoidal waveforms without the need to rely on tedious and complex mathematics.

B.1 SQUARE WAVEFORMS

Let us first consider waveforms that have the general shape of a squarewave. Figure B.1 shows a common squarewave function with amplitudes $\pm V$.

Since $f_1(\omega t)$ is odd and half-wave symmetric, only odd harmonics for the sine components are present. The b_n coefficients are given by

$$\begin{aligned}
b_n &= \frac{1}{\pi}\left[\int_0^{\pi} V\ \sin(n\omega t)\ d(\omega t) + \int_{\pi}^{2\pi}(-V)\ \sin(n\omega t)\ d(\omega t)\right] \\
&= \frac{V}{\pi}\left\{\left[-\frac{1}{n}\cos(n\omega t)\right|_0^{\pi} + \left[\frac{1}{n}\cos(n\omega t)\right|_{\pi}^{2\pi}\right\} \\
&= \frac{V}{\pi}[-\cos(n\pi) + \cos(0) + \cos(2n\pi) - \cos(n\pi)] \\
&= \frac{2V}{n\pi}[1 - \cos(n\pi)]
\end{aligned}$$

Hence, $f_1(\omega t)$ may be expressed as

$$f_1(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\sin n\omega t \tag{B.1}$$

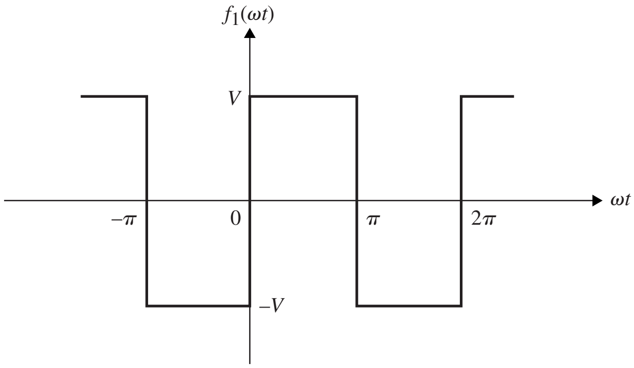


Figure B.1 Square waveform with no dc component.

Figure B.2(a) and (b) shows forms of Fig. B.1 shifted by $\pi/2$ and $\pi/3$ to the left, respectively. The wave of Fig. B.2(a) is even and is half-wave symmetric with only the odd harmonics of the cosine terms present as follows:

$$a_n = \frac{2}{\pi}\left[\int_{-\pi/2}^{\pi/2} V\cos(n\omega t)\, d(\omega t)\right]$$

$$= \frac{2V}{n\pi}\left[\sin(n\omega t)\Big|_{-\pi/2}^{\pi/2}\right]$$

$$= \frac{2V}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{4V}{n\pi}\sin\left(\frac{n\pi}{2}\right) \quad \text{for } n = 1, 3, 5, \ldots$$

Hence, the harmonics are given by

$$f_2(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\sin\frac{n\pi}{2}\cos n\omega t \tag{B.2}$$

It can be shown that shifting $f_1(\omega t)$ by $\pi/2$ will produce the same result shown in Eq. (B.2). This is illustrated as follows:

$$f_2(\omega t) = f_1(\omega t + \pi/2)$$

Therefore, $f_2(\omega t)$ is given by

$$f_2(\omega t) = \frac{4V}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\sin\left[n\omega\left(t + \frac{\pi}{2}\right)\right]$$

$$= \frac{4V}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\left[\cos(n\omega t)\sin\left(\frac{n\pi}{2}\right) + \sin(n\omega t)\cos\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{4V}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\cos(n\omega t)\sin\left(\frac{n\pi}{2}\right)$$

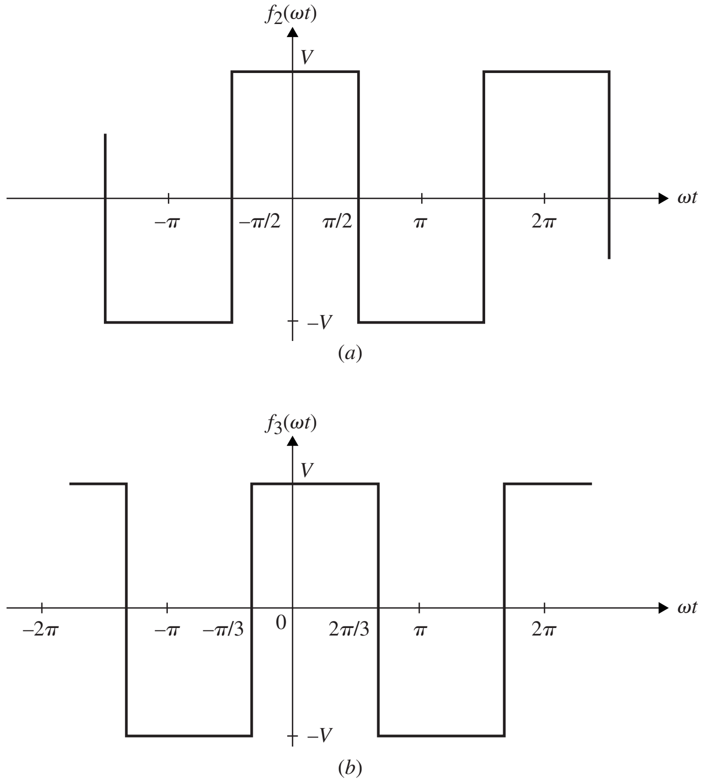


Figure B.2 Versions of Fig. B.1 shifted by (*a*) $\pi/2$ and (*b*) $\pi/3$.

$$f_2(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi} \sin\frac{n\pi}{2} \cos n\omega t \tag{B.3}$$

Similarly, the Fourier series for $f_3(\omega t)$ of Fig. B.2(*b*) is given by

$$f_3(\omega t) = f_1\left(\omega t + \frac{\pi}{3}\right)$$

$$= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi} \sin\ n\left(\omega t + \frac{\pi}{3}\right) \tag{B.4}$$

We should point out that many of the Fourier components can be obtained simply by manipulating the waveforms of Figs. B.1 and B.2.

Three-Level Square Waveform

Next we consider a square waveform with symmetric dead angle, α, as shown in Fig. B.3. The waveform of Fig. B.3 is odd and half-wave symmetric with only odd harmonics of the sine terms as follows:

$$b_n = \frac{2}{\pi}\left[\int_0^{\pi} f(\omega t) \sin(n\omega t)\ d(\omega t)\right]$$

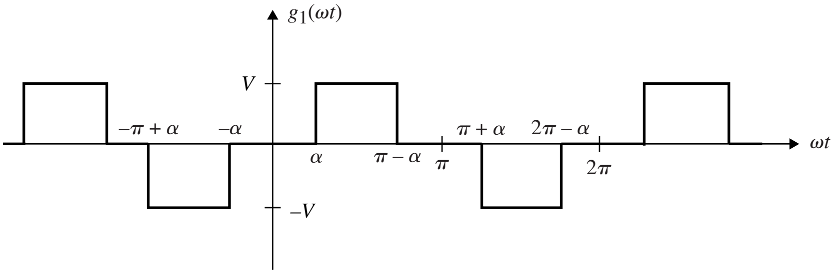


Figure B.3 Squarewave function with symmetric dead angle, α.

$$= \frac{2}{\pi}\left[\int_{\alpha}^{\pi-\alpha} V \sin(n\omega t)\, d(\omega t)\right]$$

$$= \frac{2V}{n\pi}\left[-\cos(n\omega t)\Big|_{\alpha}^{\pi-\alpha}\right]$$

$$= \frac{2V}{n\pi}[-\cos n(\pi-\alpha) + \cos(n\alpha)]$$

$$= \frac{4V}{n\pi}\cos n\alpha$$

The harmonic expression for $g_1(\omega t)$ is given by

$$g_1(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\cos n\alpha \sin n\omega t \tag{B.5}$$

Figure B.4(a) and (b) shows versions of Fig. B.3 shifted by α to the right and left, respectively.

The function $g_2(\omega t)$ is obtained by shifting $g_1(\omega t)$ by α as follows:

$$\begin{aligned} g_2(\omega t) &= g_1(\omega t - \alpha) \\ &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\cos n\alpha \sin n(\omega t - \alpha) \end{aligned} \tag{B.6}$$

Similarly, $g_3(\omega t)$ is expressed as follows:

$$g_3(\omega t) = g_1(\omega t + \alpha)$$

$$= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\cos n\alpha \sin n(\omega t + \alpha) \tag{B.7}$$

Multi-Pulse Waveforms

Let us consider multi-level pulse square waveforms normally encountered in three-phase circuits. One way to analyze such waveforms is to consider the addition of several square waveforms with different dc levels.

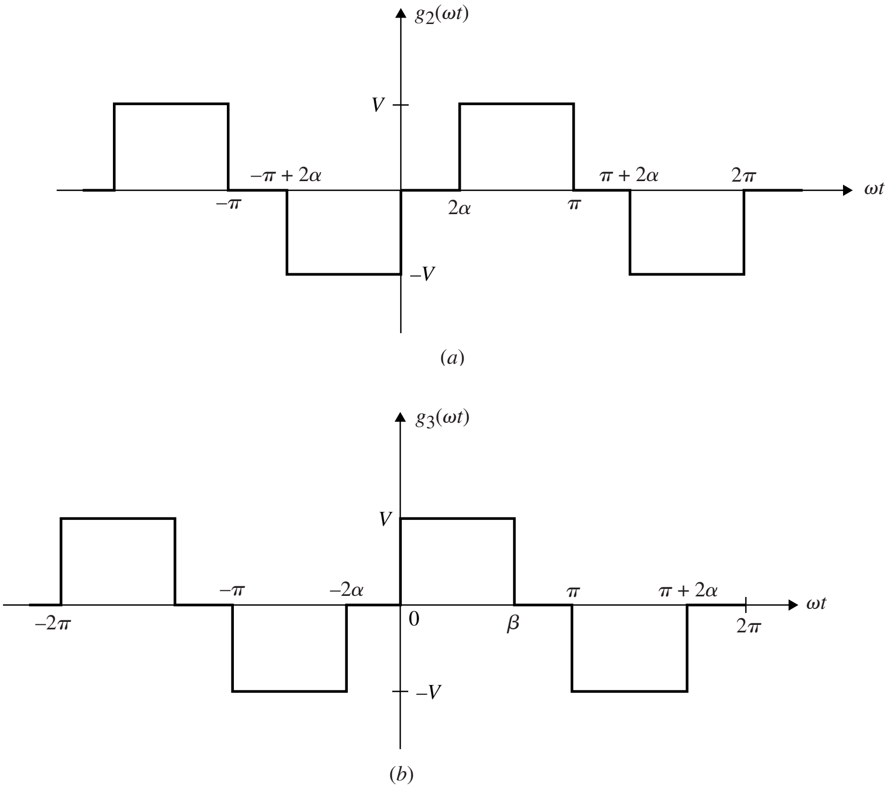


Figure B.4 Versions of Fig. B.3 shifted by angle α. (*a*) Right shift. (*b*) Left shift.

Figure B.5 is the same as Fig. B.2 but includes a dc component. The function $h_1(\omega t)$ is expressed as follows:

$$h_1(\omega t) = \frac{1}{2} f_2(\omega t) + \frac{V}{2}$$

where the dc value of $h_1(\omega t)$ is $V / 2$. As a result, Eq. (B.2) yields the following Fourier series for $h_1(\omega t)$:

$$h_1(\omega t) = \frac{V}{2} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V}{n\pi} \sin\frac{n\pi}{2} \cos n\omega t \tag{B.8}$$

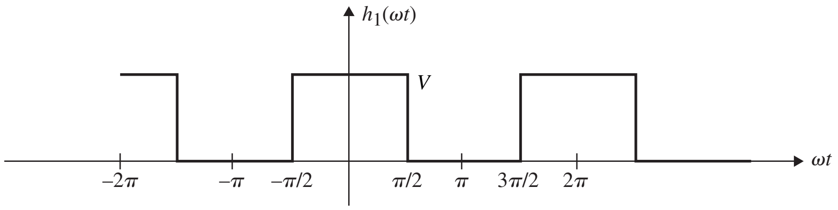


Figure B.5 Single-pulse waveform with width π.

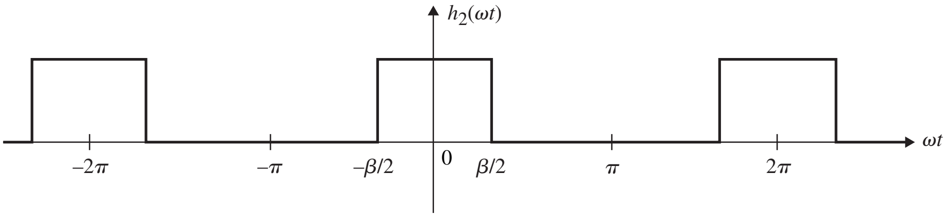


Figure B.6 Single-pulse waveform with pulse width β.

Figure B.6 shows a pulse square waveform with a general pulse width of β. The waveform of $h_2(\omega t)$ has the following coefficients:

$$a_o = \frac{1}{2\pi}\int_{-\beta/2}^{\beta/2} V\, d(\omega t) = \frac{\beta V}{2\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\beta/2}^{\beta/2} V\, \cos n\omega t\, d(\omega t) = \frac{2V}{n\pi}\sin(n\beta)$$

Therefore, the function $h_2(\omega t)$ is expressed as

$$h_2(\omega t) = a_o + a_n \cos n\omega t$$

$$= \frac{\beta V}{2\pi} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V}{n\pi}\sin n\beta\, \cos n\omega t \tag{B.9}$$

Figure B.7 is obtained by shifting Fig. B.6 to the right by $\beta/2$. Its Fourier components are given as follows:

$$h_3(\omega t) = h_2\left(\omega t - \frac{\beta}{2}\right)$$

$$= \frac{\beta V}{2\pi} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V}{n\pi}\sin n\beta\, \cos n\left(\omega t - \frac{\beta}{2}\right) \tag{B.10}$$

Next we consider multi-pulse square waveforms such as the two-pulse waveform shown in Fig. B.8(*a*). It can be shown that its harmonics may be obtained by adding $h_5(\omega t)$ and $h_6(\omega t)$ of Fig. B.8(*b*) and (*c*), respectively. Figure B.8(*d*) is the same as Fig. B.8(*a*) but shifted to the left by $\alpha/2$.

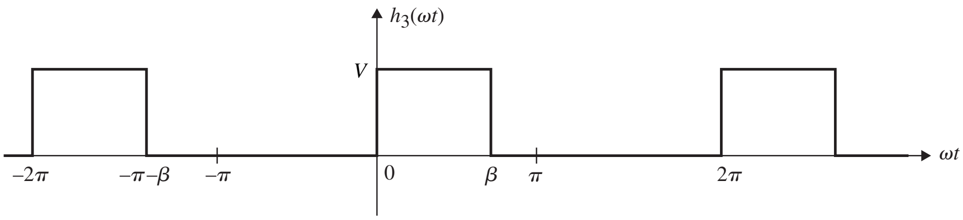


Figure B.7 Version of Fig. B.6 shifted to the right by $\beta/2$.

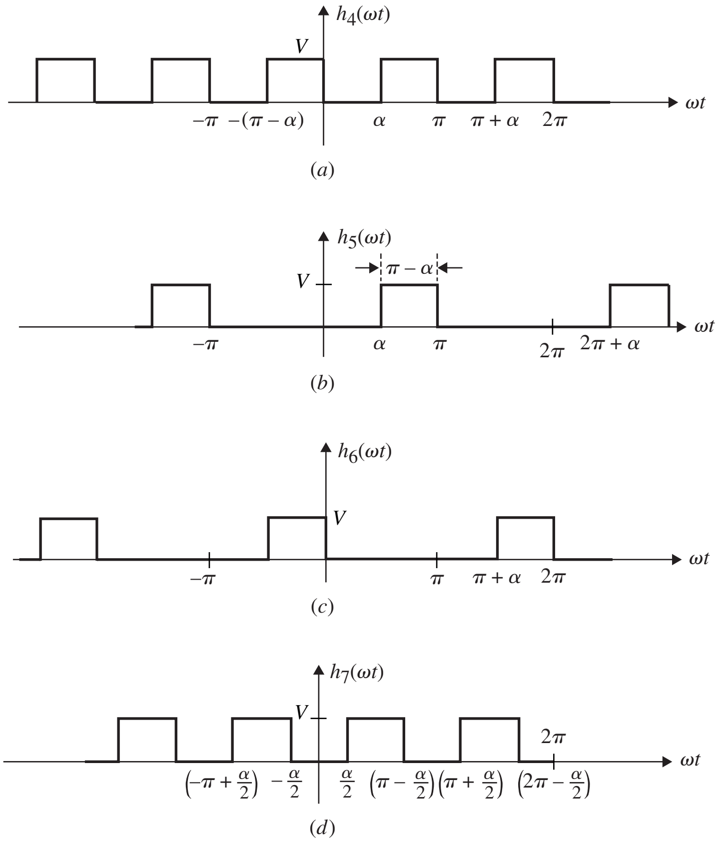


Figure B.8
(*a*) Two-pulse waveform with equal pulse widths.
(*b*) Single pulse with width $\pi - \alpha$.
(*c*) Part (*b*) shifted to the right by π.
(*d*) Part (*a*) shifted to the left by $\alpha/2$.

Figure B.8(*d*) shows the waveform $h_7(\omega t)$ obtained by shifting $h_4(\omega t)$ by $\alpha/2$ to the left. Fourier series for $h_4(\omega t)$ and $h_7(\omega t)$ are given by

$$\begin{aligned} h_4(\omega t) &= h_5(\omega t) + h_6(\omega t) \\ &= \frac{V}{\pi}(\pi - \alpha) + \sum_{n=2,4,6,\ldots}^{\infty} \frac{-4V}{n\pi} \sin\left(\frac{n\alpha}{2}\right) \cos n\left(\omega t - \frac{\alpha}{2}\right) \end{aligned} \tag{B.11}$$

where

$$h_5(\omega t) = V\left(\frac{\pi - \alpha}{2\pi}\right) + \frac{2V}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n} \sin n\left(\frac{\pi - \alpha}{2}\right) \cos n\left[\omega t - \left(\frac{\pi}{2} + \frac{\alpha}{2}\right)\right]$$

$$h_6(\omega t) = V\left(\frac{\pi - \alpha}{2\pi}\right) + \frac{2V}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n} \sin n\left(\frac{\pi - \alpha}{2}\right) \cos n\left[\omega t - \left(\frac{3\pi}{2} + \frac{\alpha}{2}\right)\right]$$

and

$$\begin{aligned} h_7(\omega t) &= h_4\left(\omega t + \frac{\alpha}{2}\right) \\ &= \frac{V}{\pi}(\pi - \alpha) + \sum_{n=2,4,6,\ldots}^{\infty} \frac{-4V}{n\pi} \sin\left(\frac{n\alpha}{2}\right) \cos n\omega t \end{aligned} \tag{B.12}$$

Two- and Three-step Waveforms

Figure B.9(*a*) is a two-step square waveform whose harmonic contents are difficult to obtain by applying the original Fourier series equations. Instead, we simply add two previously analyzed waveforms shown again in Figs. B.9(*b*) and (*c*).

The harmonics of $f(\omega t)$ are obtained from the following relation:

$$
\begin{aligned}
f(\omega t) &= f_1(\omega t) + g_1(\omega t) \\
&= \frac{4V}{\pi}\left[\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\sin n\omega t\left(1+\cos\frac{n\pi}{3}\right)\right] \\
&= \frac{4V}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\left(1+\cos\frac{n\pi}{3}\right)\sin n\omega t \\
&= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\left(1+\cos\frac{n\pi}{3}\right)\sin n\omega t
\end{aligned}
\tag{B.13}
$$

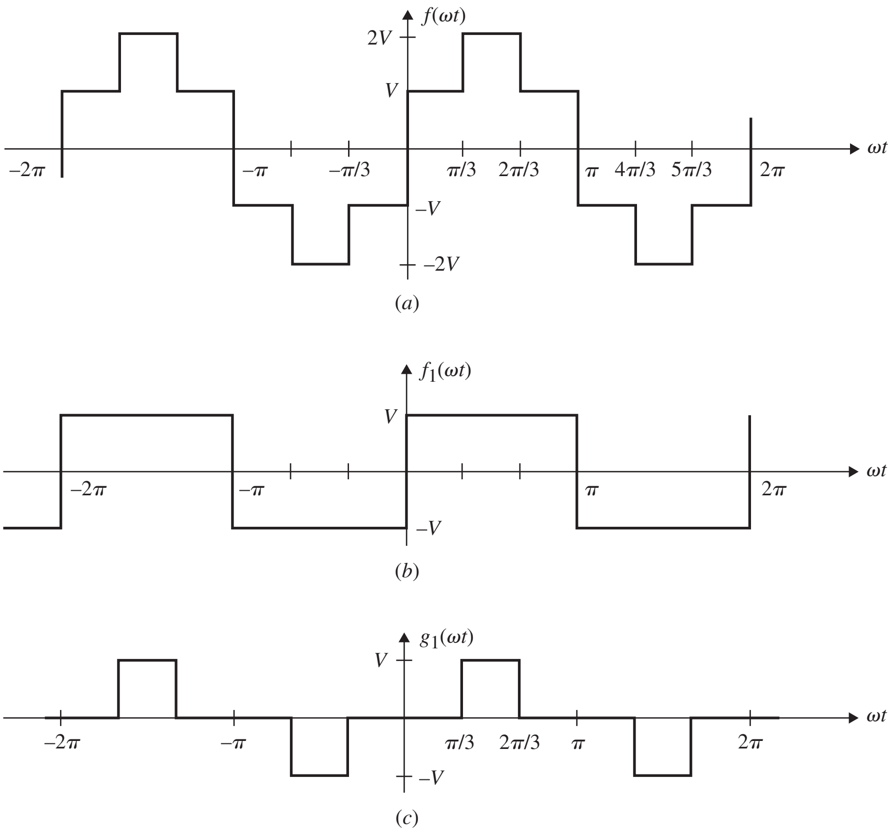


Figure B.9 (*a*) Two-step waveform obtained by adding parts (*b*) and (*c*).

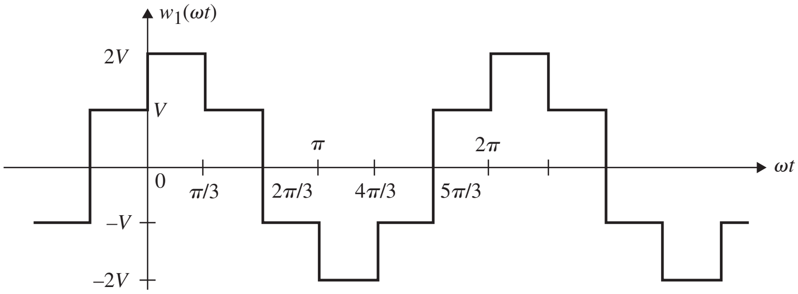


Figure B.10 Two-step waveform of Fig. B.9(a) shifted to the left by $\pi/3$.

where $f_1(\omega t)$ and $g_1(\omega t)$ are given by

$$f_1(\omega t) = \frac{4V}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin n\omega t$$

$$g_1(\omega t) = \frac{4V}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \cos\frac{n\pi}{3} \sin n\omega t$$

Figure B.10 is obtained by shifting $f(\omega t)$ of Fig. B.9(a) to the left by $\pi/3$, giving the following Fourier series:

$$\begin{aligned} w_1(\omega t) &= f\left(\omega t + \frac{\pi}{3}\right) \\ &= \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\left(1 + \cos\frac{n\pi}{3}\right)\sin n\left(\omega t + \frac{\pi}{3}\right) \end{aligned} \tag{B.14}$$

Finally, we consider the three-step waveform shown in Fig. B.11(a), obtained by adding Fig. B.11(b), (c), and (d).

The waveform $\omega_2(\omega t)$ of Fig. B.11(a) is obtained by adding the harmonics of Fig. B.11(b), (c), and (d), which results in the following harmonic series:

$$w_2(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{3\pi}\frac{1}{n}\left(1 + \cos\frac{n\pi}{3} + \cos\frac{2n\pi}{5}\right)\sin n\omega t \tag{B.15}$$

B.2 SINUSOIDAL WAVEFORMS

In this section, we will consider sinusoidal waveforms frequently encountered in power electronic circuits. These include half- and full-wave diode and SCR sinusoidal waveforms.

B.2.1 Diode Sinusoidal Waveforms

Figure B.12 shows a half-wave rectifier waveform, with its Fourier components given in Eq. (B.16).

The waveform is an even function, with its harmonics containing only cosine terms and a dc value, as follows:

$$a_o = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} V\cos(\omega t)\; d(\omega t) = \frac{V}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} V\cos(\omega t)\cos n(\omega t)\; d(\omega t) = \frac{2V}{\pi(1-n^2)}\cos\frac{n\pi}{2}$$

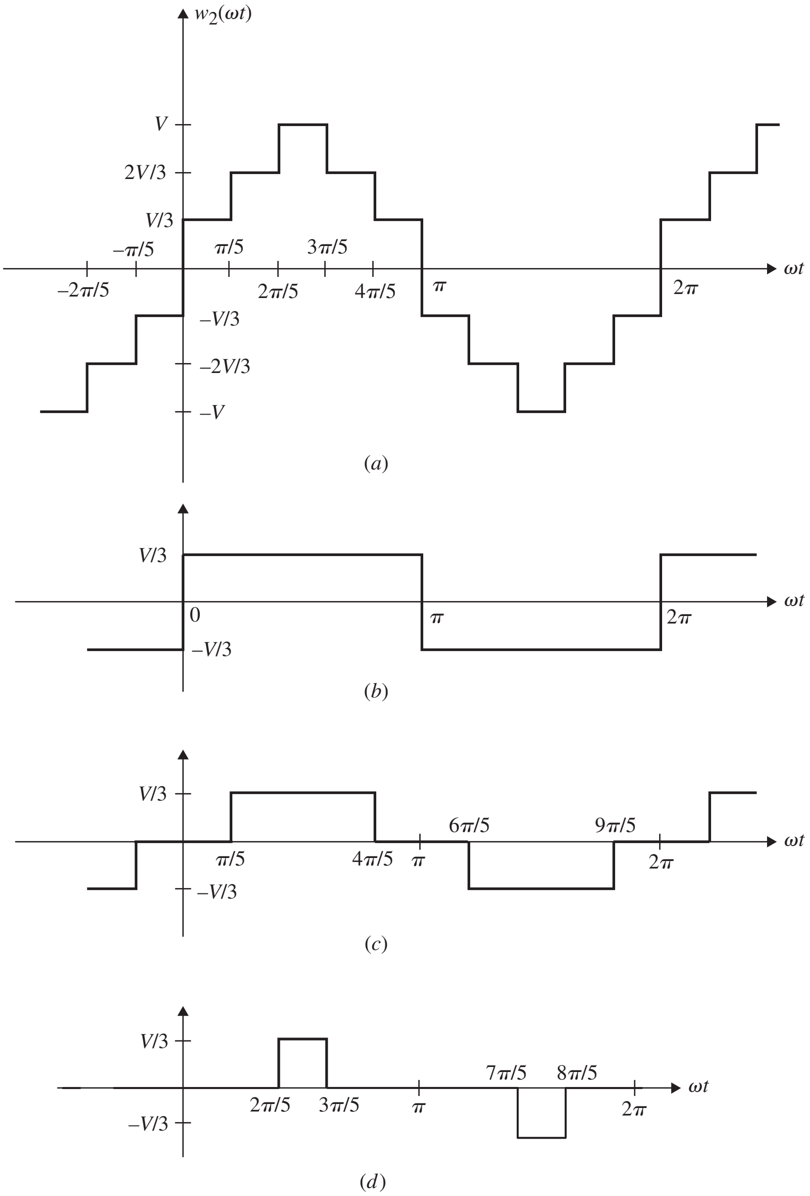


Figure B.11 (*a*) Three-step waveform obtained by adding parts (*b*), (*c*), and (*d*).

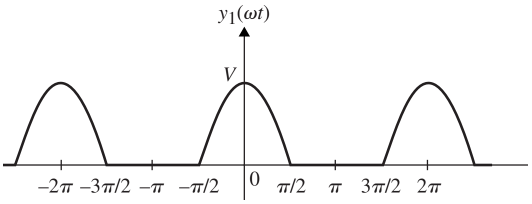


Figure B.12 Half-wave sinusoidal waveform.

The Fourier series for Fig. B.12 is given by

$$y_1(\omega t) = \frac{V}{\pi} + \frac{V}{2}\cos\omega t + \sum_{n=2,4,6,\ldots}^{\infty} \frac{2V}{\pi(1-n^2)}\cos\frac{n\pi}{2}\cos n\omega t \qquad \text{(B.16)}$$

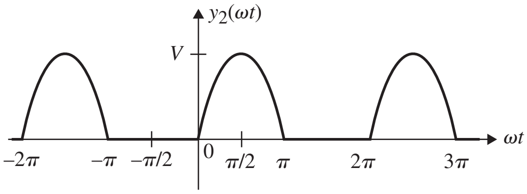


Figure B.13 Shifted half-wave sinusoidal waveform.

Figure B.13 is the same as Fig. B.12 but shifted to the right by $\pi/2$. Hence, $y_2(\omega t) = y_1(\omega t - \pi/2)$, and its harmonic components are given by

$$y_2(\omega t) = \frac{V}{\pi} + \frac{V}{2}\sin\omega t + \sum_{n=2,4,6,\ldots}^{\infty} \frac{2V}{\pi(1-n^2)}\cos\frac{n\pi}{2}\cos n\left(\omega t - \frac{\pi}{2}\right) \tag{B.17}$$

Another way to obtain the Fourier series for Fig. B.12 is to use the product of two known waveforms. The product of $\sin(\omega t)$ and $h_3(\omega t)$ for $\beta = \pi$, redrawn in Fig. B.14, will produce the waveform $y_2(\omega t)$.

The harmonic function of Fig. B.14(*a*) is expressed as

$$y_2(\omega t) = \sin(\omega t)\cdot h_3(\omega t)$$

where $h_3(\omega t)$ is obtained from Eq. (B.10) by setting $\beta = \pi$, to yield

$$h_3(\omega t) = \frac{V}{2} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V}{n\pi}\sin(n\pi)\cos n\left(\omega t - \frac{\pi}{2}\right)$$

$$h_3(\omega t) = \frac{V}{2} + \frac{2V}{\pi}\left[\cos\left(\omega t - \frac{\pi}{2}\right) - \frac{1}{3}\cos 3\left(\omega t - \frac{\pi}{2}\right) + \frac{1}{5}\cos 5\left(\omega t - \frac{\pi}{2}\right)\cdots\right]$$

$$= \frac{V}{2} + \frac{V}{\pi}\left[\sin\ \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t\cdots\right]$$

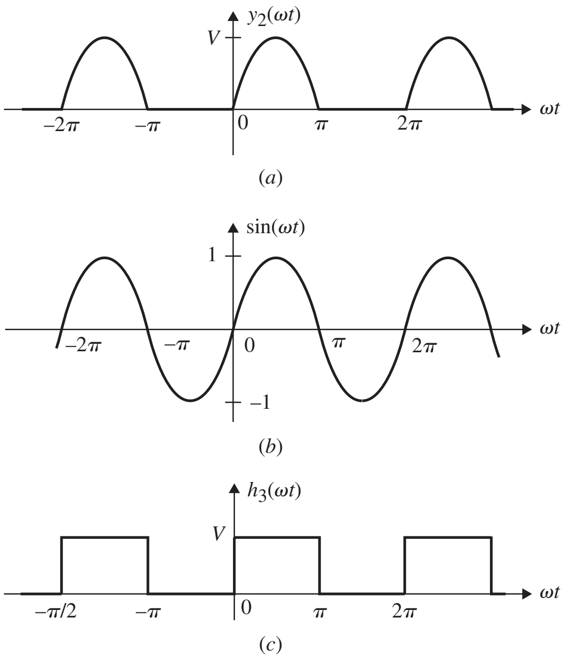


Figure B.14 (*a*) Sinusoidal waveform obtained by the product of parts (*b*) and (*c*).

Hence,

$$y_2(\omega) = \sin\omega t \cdot h_3(\omega t)$$
$$= \frac{V}{\pi}\left[1 + \frac{\pi}{2}\sin(\omega t) - \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t - \frac{2}{35}\cos 6\omega t \cdots\right] \qquad \text{(B.18)}$$

which is similar to Eq. (B.17).

Similarly, the harmonics of Fig. B.15(*a*) are obtained by multiplying the waveforms of Fig. B.15(*b*) and (*c*).

Another common waveform is the full-wave rectifier waveform shown in Fig. B.16(*a*). The harmonic coefficients of Fig. B.16(*a*) can be written as follows:

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} y_3(\omega t)\, d(\omega t) = \frac{1}{\pi}\int_0^{\pi} V\,\sin(\omega t)\, d(\omega t) = \frac{2V}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{T/2} V\,\sin\omega t\,\cos n\omega t\, d(\omega t) = \frac{4V}{\pi}\int_0^{\pi/2} \sin\omega t\,\cos n\omega t\, d(\omega t)$$
$$= \frac{2V}{\pi}\left[\int_0^{\pi/2} \sin(1+n)\omega t\, d(\omega t) + \int_0^{\pi/2} \sin(1-n)\omega t\, d(\omega t)\right]$$
$$= -\frac{2V}{\pi}\left[\frac{\cos(1+n)\omega t}{1+n}\bigg|_0^{\pi/2} + \frac{\cos(1-n)\omega t}{1-n}\bigg|_0^{\pi/2}\right]$$
$$= -\frac{2V}{\pi}\left[\frac{\cos(1+n)\frac{\pi}{2}}{1+n} - \frac{1}{1+n} + \frac{\cos(1-n)\frac{\pi}{2}}{1-n} - \frac{1}{1-n}\right]$$

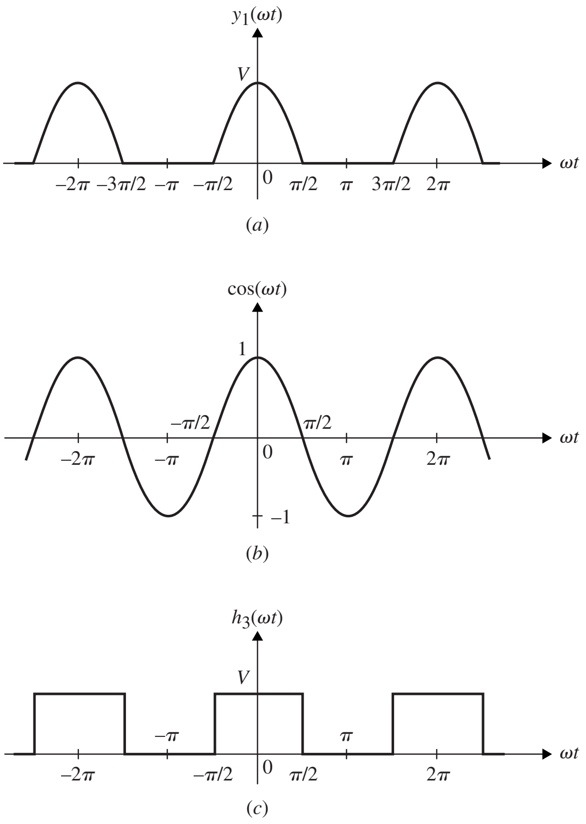


Figure B.15 (*a*) Waveform obtained as the product of parts (*b*) and (*c*).

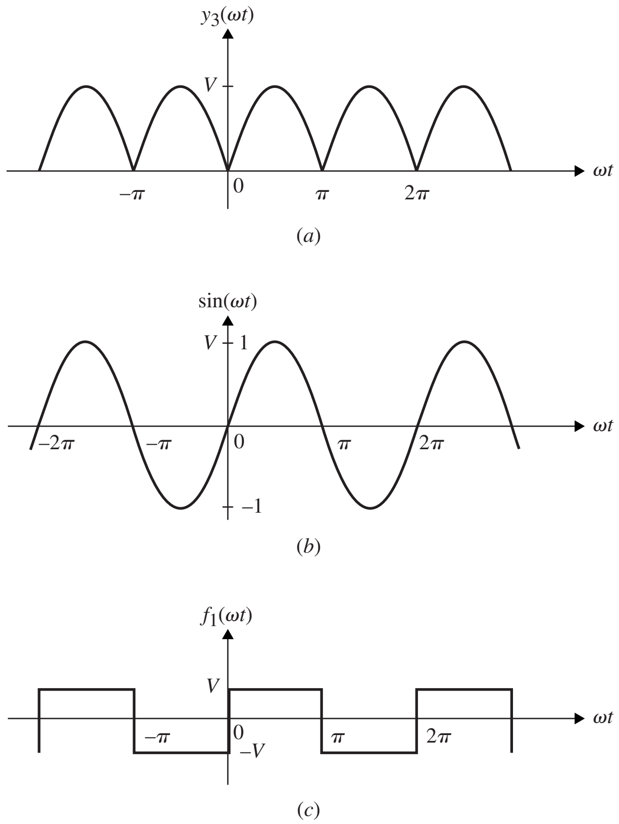


Figure B.16 (*a*) Full-wave sinusoidal waveform obtained by multiplying parts (*b*) and (*c*).

$$a_n = \frac{2V}{\pi}\left[\frac{1}{1+n} + \frac{1}{1-n}\right] = \frac{4V}{(1-n^2)\pi}$$

Hence, the harmonics of $y_3(\omega t)$ are given by

$$y_3(\omega t) = \frac{2V}{\pi} + \sum_{n=2,4,6,\ldots}^{\infty} \frac{4V}{(1-n^2)\pi} \cos n\omega t \tag{B.19}$$

It can be shown that $y_3(\omega t)$ of Fig. B.16(*a*) may be obtained by multiplying the waveforms of Fig. B.16(*b*) and (*c*).

B.2.2 SCR Sinusoidal Waveforms

Figure B.17(*a*) shows a typical half-wave SCR voltage waveform. Its harmonic contents were studied in Chapter 8. The same harmonic relationship can be obtained by multiplying the waveforms $\sin(\omega t)$ and $g_1(\omega t)$ shown in Fig. B.17(*b*) and (*c*), respectively. Therefore, $x_1(\omega t)$ is given by

$$x_1(\omega t) = \sin(\omega t) \cdot g_1(\omega t)$$

$$= V \sin \omega t\left[\frac{(\pi-\alpha)}{2\pi} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2}{n\pi} \sin n\left(\frac{\pi-\alpha}{2}\right)\cos n\left(\omega t - \left(\frac{\pi}{2} + \frac{\alpha}{2}\right)\right)\right] \tag{B.20}$$

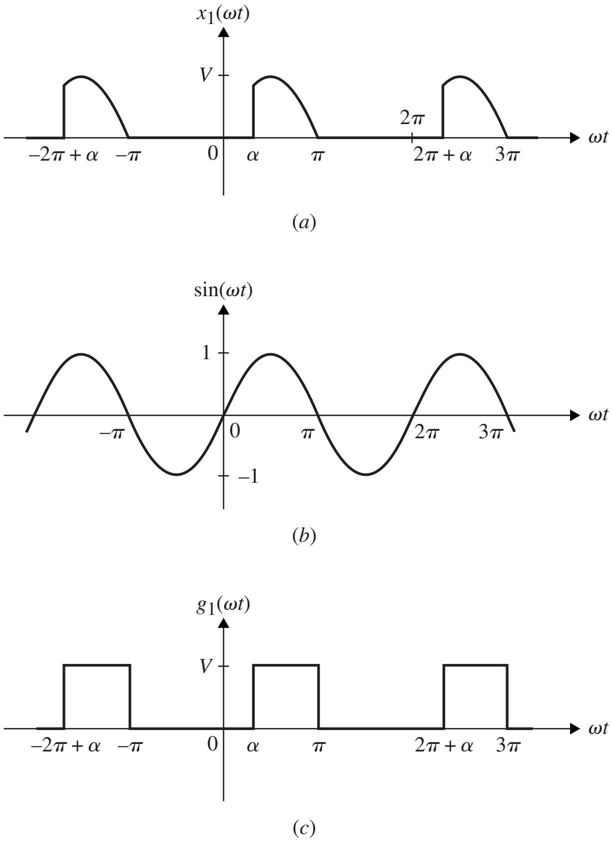


Figure B.17 (*a*) Typical half-wave SCR waveform, whose harmonics are obtained by multiplying parts (*b*) and (*c*).

Figure B.18(*a*) shows a typical voltage waveform produced by a bidirectional SCR circuit or triac. Its harmonic contents are obtained by multiplying $\sin(\omega t)$ and $h_4(\omega t)$ of Fig. B.18(*b*) and (*c*).

The Fourier series for $x_2(\omega t)$ is given by

$$x_2(\omega t) = \sin(\omega t) \cdot h_4(\omega t)$$

$$= V \sin \omega t \left[\frac{(\pi - \alpha)}{\pi} - \sum_{n=2,4,6,\ldots}^{\infty} \frac{4}{n\pi} \sin \frac{n\alpha}{2} \cos n\left(\omega t - \frac{\alpha}{2} \right) \right] \qquad \text{(B.21)}$$

Finally, Fig. B.19(*a*) shows a full-wave rectified SCR voltage waveform whose Fourier series can be obtained by multiplying Fig. B.19(*b*) and (*c*). The harmonics for Fig. B.19(*a*) are given by

$$x_3(\omega t) = \sin(\omega t) \cdot g_2(\omega t)$$

$$= V \sin \omega t \left[\sum_{n=1,3,5,\ldots}^{\infty} \frac{4}{n\pi} \cos \frac{n\alpha}{2} \sin n\left(\omega t - \frac{\alpha}{2} \right) \right] \qquad \text{(B.22)}$$

Table B.1 is a summary of common waveforms with their harmonic spectrum plots.

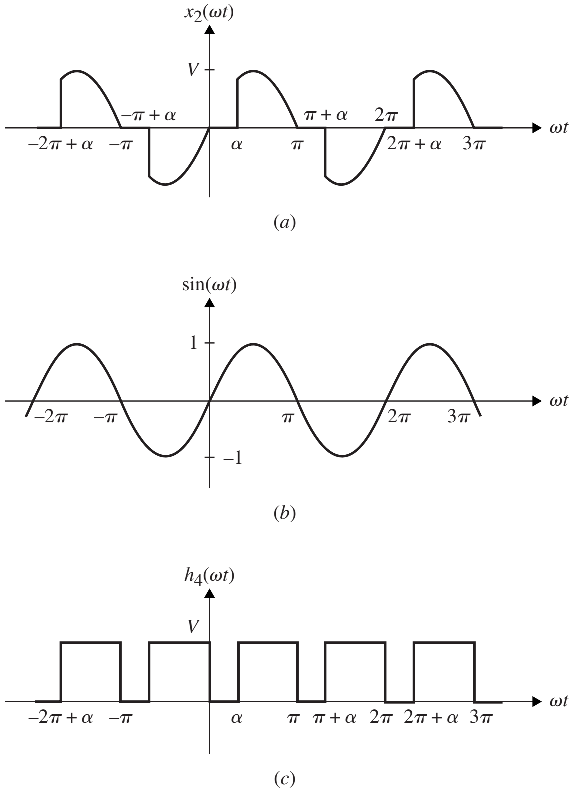


Figure B.18 (*a*) Typical bidirectional SCR voltage circuit waveform obtained by multiplying parts (*b*) and (*c*).

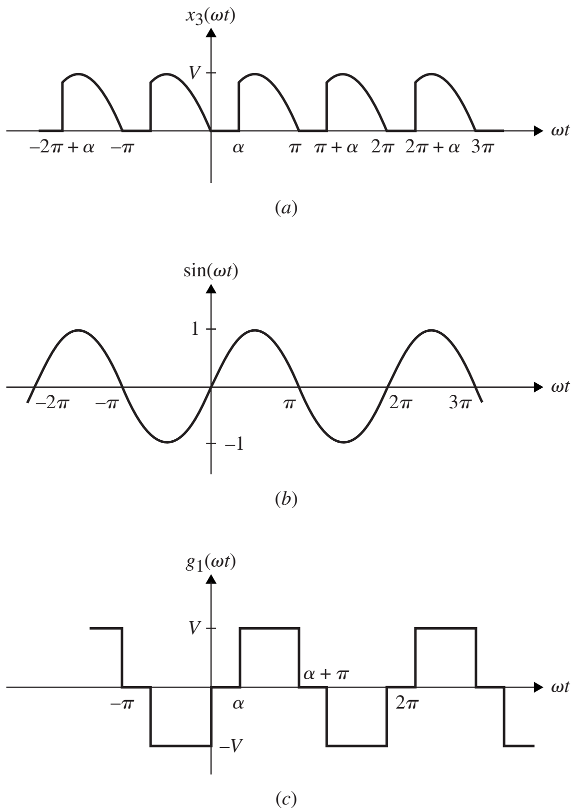


Figure B.19 (*a*) Typical rectified full-wave SCR voltage waveform obtained by multiplying waveforms of parts (*b*) and (*c*).

Table B.1 Summary of Common Waveforms with Their Harmonic Spectrum Plots

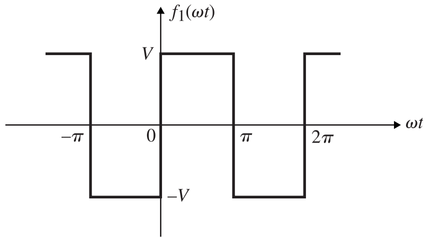
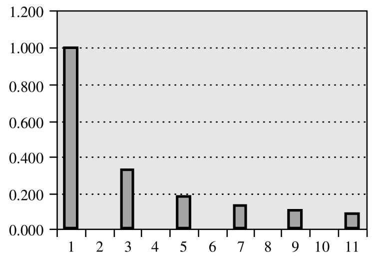
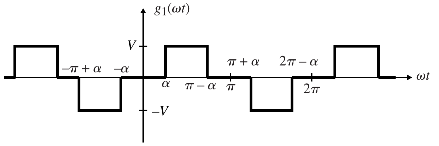
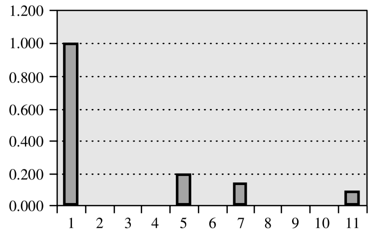
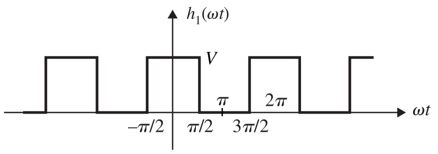
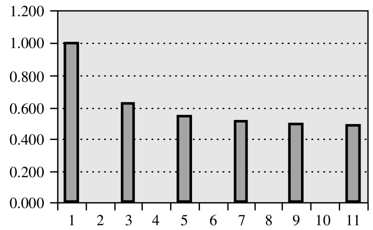
Waveform	Fourier series	THD = $\sqrt{\frac{1}{k_{dist}^2} - 1}$	Harmonic spectrum (normalized to fundamental component)
	$f_1(\omega t) = \sum_{n=1,3,5,...}^{\infty} \frac{4V}{n\pi} \sin n\omega t$	0.438	
	$g_3(\omega t) = \sum_{n=1,3,5,...} \frac{4V}{n\pi} \cos n\alpha \ \sin n\omega t$	0.262 ($\alpha = \pi/6$)	
	$h_1(\omega t) = \frac{V}{2} + \sum_{n=1,3,5,...}^{\infty} \frac{2V}{n\pi} \sin \frac{n\pi}{2} \cos n\omega t$	1.194	

Table B.1 (*continued*)

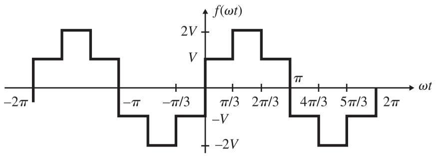
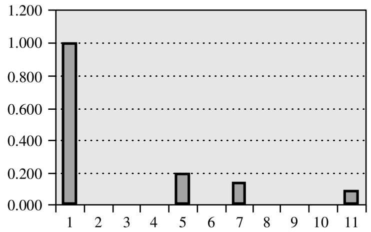
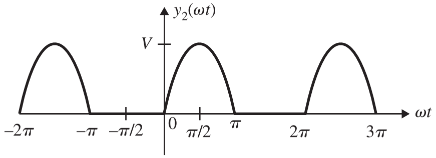
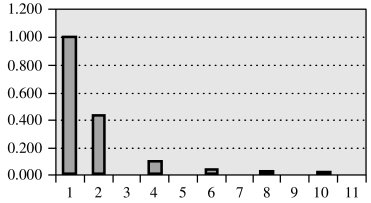
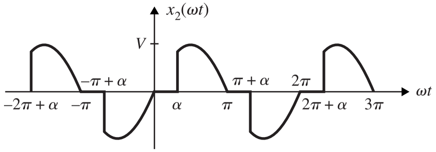
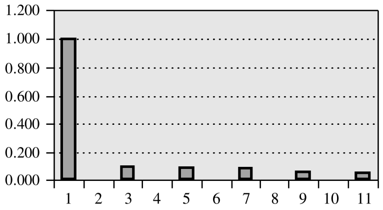
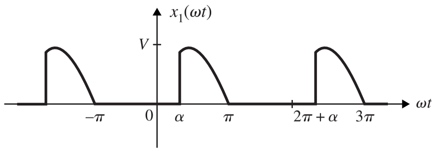
Waveform	Fourier series	THD = $\sqrt{\frac{1}{k_{\text{dist}}^2} - 1}$	Harmonic spectrum (normalized to fundamental component)
	$f(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi}\left(1 + \cos\frac{n\pi}{3}\right)\sin n\omega t$	0.262	
	$y_2(\omega t) = \frac{V}{\pi} + \frac{V}{2}\sin\omega t + \sum_{n=2,4,6,\ldots}^{\infty} \frac{2V}{\pi(1-n^2)}\cos\frac{n\pi}{2}\cos n\left(\omega t - \frac{\pi}{2}\right)$	1	

Table B.1 (*continued*)

Waveform	Fourier series	THD = $\sqrt{\frac{1}{k_{\text{dist}}^2} - 1}$	Harmonic spectrum (normalized to fundamental component)
	$x_2(\omega t) = V\sin\omega t\left[\frac{(\pi-\alpha)}{\pi} - \sum_{n=2,4,6,\ldots}^{\infty} \frac{4}{n\pi}\sin n\left(\frac{\alpha}{2}\right)\cos n\left(\omega t - \frac{\alpha}{2}\right)\right]$	0.136 ($\alpha = \pi/6$)	
	$x_1(\omega t) =$ $V\sin\omega t\left[\frac{(\pi-\alpha)}{2\pi} + \sum \frac{2}{n\pi}\sin n\left(\frac{\pi-\alpha}{2}\right)\cos n\left(\omega t - \left(\frac{\pi}{2} + \frac{\alpha}{2}\right)\right)\right]$	1.018 ($\alpha = \pi/6$)