Appendix A

Introduction to Magnetic Circuits

INTRODUCTION

Modern power electronic systems are rarely designed without inductors or transformers. Because of the importance of these components, understanding magnetic and magnetic circuits is necessary for the successful design of power electronic systems. Moreover, if not included as discrete components, inductors exist as parasitic components, especially when the power systems are operated at high frequencies. Normally, the magnetic components are the most expensive and difficult to design in power systems.

Inductors are used to accomplish one or more of the following functions:

1. Filter switch waveforms, at both the input and output sides.
2. Form resonant circuits along with capacitors in order to create sinusoidal waveforms for various applications.
3. Limit the rate of change of load currents in switching circuits.
4. Limit transients at power-up of electric systems.

A transformer may be considered as two inductors coupled through a shared magnetic circuit with common flux. Because of this common flux, it is possible to change the ac electric energy at a given voltage level to another voltage level (same form of energy) with high efficiency. Transformers, whether operated at line or higher frequencies, are used to accomplish one or more of the following functions:

1. Step up or step down the voltage to service various needs and applications.
2. Provide isolation between power systems to reduce various EMI problems and for safety considerations.
3. Provide phase shift in multiphase systems to generate systems with three or more phases.
4. Provide a means to store energy to be utilized at later times, especially in high-frequency applications.
5. Provide a coupling mechanism between the gate or base drive circuits of high-power semiconductor switching devices and the power circuits in various power electronic systems.
6. Provide sensing for voltage and/or current in various control feedback systems.

Because of the wide variation of parameters that characterize magnetic circuits (inductors and transformers), it is extremely difficult to standardize the size, values, and ratings of inductors and transformers. As a result, stockpiling such components is highly impractical and normally not done. Among the parameters that characterize magnetic components are current, voltage, and power ratings; energy storage and dissipation; frequency of operation; magnetizing and leakage inductances; turn ratio; fabrication difficulties; size; weight; and cost. An attempt to minimize one or more components will be done at the expense of one or more other parameters. As a result, designing magnetic circuits is a very tricky and challenging engineering problem, and experience plays a very important role.

A.1 TYPES OF MAGNETIC MATERIALS

Just as the types of conductors in electric circuits are identified based on their ability to conduct electric current, the types of magnetic materials are identified based on their degree of receptivity to the magnetic field. A good conductor allows the current to flow freely, and a good magnetic material allows the magnetic field to pass through freely. Even though the two materials experience totally different physical phenomena in the process, many of the concepts applied to electric circuits can be also identified with magnetic circuits. This analogy will be discussed in more detail in Section A.4.

Depending on the degree of magnetization in the presence of an electric field, magnetic materials are divided into three major groups:

1. *Diamagnetics*: These materials experience extremely weak magnetization in the presence of magnetic fields, i.e., they exclude magnetic fields. This property is similar to the property of insulators in electric circuits.
2. *Paramagnetics*: These materials experience slight magnetization. This property is similar to the conductive property of semiconductor materials in electric circuits.
3. *Ferromagnetics*: These materials experience strong magnetization in the presence of magnetic fields. This property is similar to the property of a good conductor material.

A single parameter that characterizes the property of magnetic material, therefore, its type, is known as the permeability, μ, which is expressed in the following constitutive relation:

$$\mathbf{B} = \mu \mathbf{H} \tag{A.1}$$

where

μ is the permeability of the magnetic material

B is the magnetic flux density[1]

H is the magnetic field intensity

Normally, the value of μ is identified relative to the permeability of the vacuum, μ_o, as follows:

$$\mu = \mu_r \mu_o \tag{A.2}$$

where

μ_o is the permeability of the vacuum

μ_r is the *relative permeability* of the magnetic material

Depending on the magnetic material, the value of μ_r can vary from a small fraction of μ_o to thousands of μ_o. The following is a fairly accurate classification of magnetic materials in terms of μ_r:

$\mu_r < 1$ for diamagnetic materials

$\mu_r = 1$ for the vacuum

$\mu_r > 1$ for paramagnetic materials

$\mu_r >> 1$ for ferromagnetic materials

The value of μ_o is equal to $4\pi \times 10^{-7}$ henrys/meter. For many applications, normally it is desired that a large magnetic field be produced with the smallest possible current in the coil. As a result, materials with high μ are desired. This also means that a preponderant portion of the produced flux is confined to the magnetic material.

Ferromagnetic materials are normally obtained from iron with certain alloys such as cobalt (Co), tungsten (Tn), nickel (Ni), aluminum (Al), silicon (Si), manganese (Mn), and zinc (Zn) alloys. Alloys are selected based on several factors, such as maximum saturation flux density, $\mathbf{B}_{sat}$; operating frequency of the magnetic circuit; cost; and resistivity.

Both the conductivity and the frequency of operation affect the total energy losses in the magnetic material, known as *core losses.* For example, the iron-silicon alloy with low silicon content has relatively high losses and high $\mathbf{B}_{sat}$, whereas a high–silicon content material is more expensive and has reduced core losses. As a result, the latter material is used mostly in high-efficiency line-frequency applications, and the former material is used when cost is the determining factor. For high-frequency applications, high-permeability magnetic material is used, such as iron with Ni alloy, with relatively low $\mathbf{B}_{sat}$. However, for higher saturation flux density, Co alloys are used. Compared with the Si alloys, Co and Ni alloys are more expensive.

In power electronic systems, operating frequencies in the several megahertz range are commonplace. A special ceramic known as ferrite, made from various combinations of iron with Mn, Ni, and Zn alloys, has been very popular in the design of magnetic circuits in power electronics. For example Mn-Zn and Ni-Zn ferrites are used in power electronic applications with frequencies in the range of a few to tens of megahertz, respectively.

[1]Boldface letters indicate vector quantities.

A.2 MAGNETIC FIELDS

As you may recall from the study of electromagnetic fields, Maxwell's equations form the basis of the fundamental theory of electromagnetism, governing the relationship between the strength of electric (**E**) and magnetic (**H**) fields, and the electric (**D**) and magnetic (**B**) flux densities. In this section, we focus on only two of Maxwell's equations as they relate to magnetism. Maxwell's equations (A.3)[2] and (A.4) governing the magnetostatic field **B** and **H** are given by

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{A.3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{A.4}$$

where **J** is the current density in amperes per square meter, and the symbol ∇ represents the partial-differential operator; $\nabla \times \mathbf{H}$ is called the *curl of* **H**, and $\nabla \cdot \mathbf{B}$ is called *the divergence of* **B**. Equation (A.3) is known as Ampere's law, and Eq. (A.4) is known as Gauss's magnetic law. Notice that as charge is the source of the electrostatic field, current is the source of the magnetostatic field.

Since our interest in this appendix is to derive magnetic fields resulting from a current-carrying wire wound around a core of magnetic material, Maxwell's equations in this form are not useful. It is normally more convenient to use different forms of Maxwell's equations to solve for the magnetic field under these circumstances. It can be shown, by using Stokes's theorem, that Eq. (A.3) may be expressed as follows:

$$\oint_C \mathbf{H} \cdot dl = \int_S \mathbf{J} \cdot dS \tag{A.5}$$

The left-hand side gives the integration of the magnetic field along a closed contour C, and the right-hand side gives the total current density, $\mathbf{J}$ $(\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \cdots + \mathbf{J}_n)$, flowing through the surface area, S, enclosed by the contour C as shown in Fig. A.1. It is possible to simplify Ampere's law further by assuming the current is confined to a

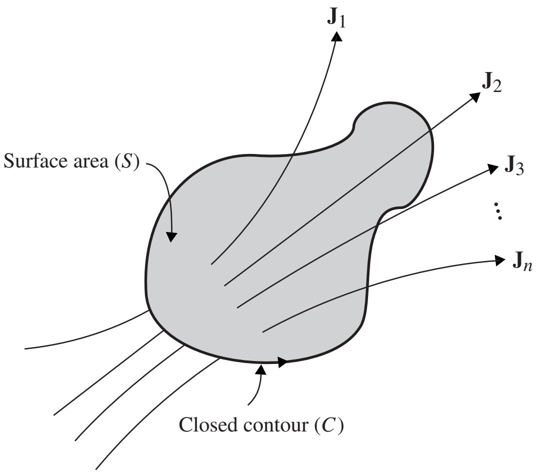


Figure A.1 Current density $(\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{J}_3 + \cdots + \mathbf{J}_n)$ passing through area S enclosed by contour C subject to the integral of Eq. (A.5).

[2]Maxwell's equation (A.3) is based on the electrostatic field; i.e., the electric field does not vary with time. In time-varying magnetic fields, an additional term, known as *displacement current*, is used in this equation, to form $\nabla \times \mathbf{H} = \mathbf{J} + \partial \mathbf{D}/\partial t$. Here, the displacement current will be assumed negligible since its contribution is significant only at extremely high frequencies.

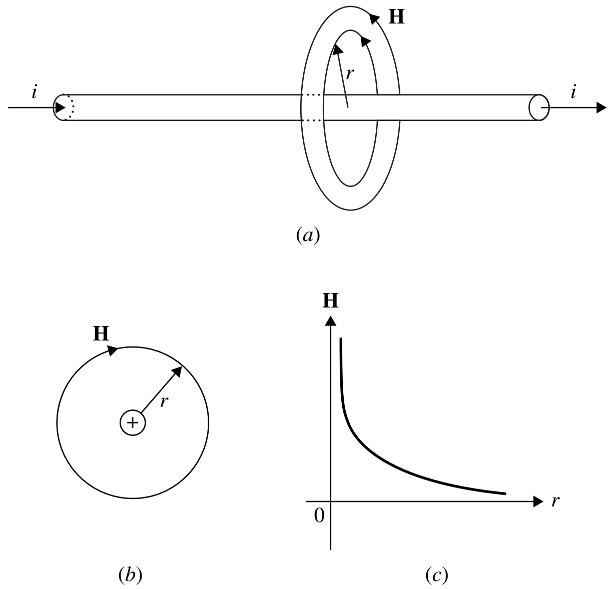


Figure A.2 Infinitely long wire conducting current i. (*a*) The magnetic field surrounding the wire. (*b*) The direction of the magnetic field at a distance of radius r. (*c*) The magnetic field versus distance from the wire.

wire rather than being distributed over a region in space. Consequently, the right-hand side gives i, where i is the total current flowing in the wire. Moreover, if we assume there are N wires each carrying the same current and encircled by the integral loop, then Ampere's law can be expressed as follows:

$$\oint_C \mathbf{H}dl = Ni \tag{A.6}$$

As an example, consider solving the above integral for an infinitely long wire carrying a current i as shown on Fig. A.2(*a*). The direction of the magnetic field is subject to the right-hand rule (the thumb is in the direction of the current and the fingers are in the direction of the magnetic field). The notations $\oplus$ and $\odot$ indicate that the current is flowing into and out of the paper, respectively.

To evaluate the integral, we apply Ampere's law to a circle with radius r and centered in the origin, as shown in Fig. A.2(*b*). A plot of the magnetic field versus the radius is shown in Fig. A.2(*c*). It is clear that the magnetic field due to the current decreases in the radial direction away from the conducting wire. The magnetic field lines are called *flux lines* or simply the *flux* of the magnetic field, denoted by ϕ, and the amount of the flux or the density of the flux is defined as follows:

$$\mathbf{B} = \frac{\boldsymbol{\phi}}{A} \tag{A.7}$$

where

$\boldsymbol{\phi}$ is the flux in webers (Wb)

$\mathbf{B}$ is the flux density in $\mathrm{Wb/m^2}$ or teslas (T)

A is the area through which the flux flows

A.2.1 Toroidal Structure

One of the important applications of magnetics to the field of power electronics is when a wire is wound around a doughnut-shaped magnetic material to form what is

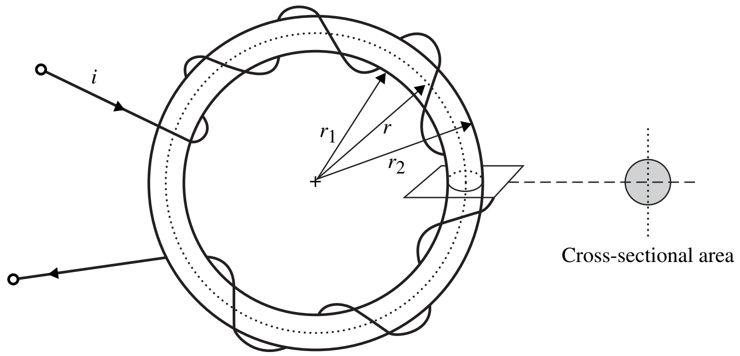


Figure A.3 Toroidal structure created by winding wire around doughnut-shaped magnetic material.

known as *toroid*, as shown in Fig. A.3. This is not only because a large class of inductors and transformers are manufactured with this structure, but also because the analysis of such a structure can be easily applied to other magnetics with various arrangements. We will apply Ampere's law to the magnetic structure and solve for its magnetic field.

When solving for **H**, we will assume that the permeability of the magnetic material is much higher than that of air; hence, the entire magnetic field is assumed to be confined within the toroid core. Also, it is accurate to assume that the flux density is constant throughout the structure. Notice that **H** can be determined by taking the integral contour at the inner radius r_1 or the outer radius r_2. It can be easily shown that if the radial dimension $(r_2 - r_1)$ is very small compared to r_1, then **H** can be assumed constant and to give a good approximation when evaluated over the mean radius $(r_1 + r_2)/2$. Applying Ampere's law to a line integral with radius r and the circle loop shown in the figure, we obtain

$$\oint_C \mathbf{H} \cdot dl = Ni$$

$$\mathbf{H}\int_0^{2\pi} d\theta = Ni$$

Solving for **H**, we have

$$\mathbf{H} = \frac{Ni}{2\pi r} = \frac{Ni}{l}$$

where r and l are the mean radius and length of the toroid, respectively.

The product Ni is normally referred to as the *magnetomotive force* (mmf), which is the magnetic field potential difference tending to force flux around the toroid. Moreover, by using the constitutive relation $\mathbf{B} = \mu\mathbf{H}$ and the definition of **B** given in Eq. (A.7), the flux in the magnetic material is given by

$$\phi = \frac{NiA\mu}{l} \tag{A.8}$$

where A is the cross-sectional area of the toroid.

EXAMPLE A.1

Consider a rectangular cross-sectional area of the toroidal magnetic core shown in Fig. A.4 with $r_1 = 3.5$ cm, $r_2 = 4$ cm, and a height of 1 cm. Assume $\mu = 2000\mu_o$ and $\mathbf{B}_{sat} = 0.3$ T. Determine:
(a) Maximum core flux before entering saturation
(b) The mmf required to produce this flux
(c) Number of turns that must be wound if the coil must carry 10 mA and still avoid saturation

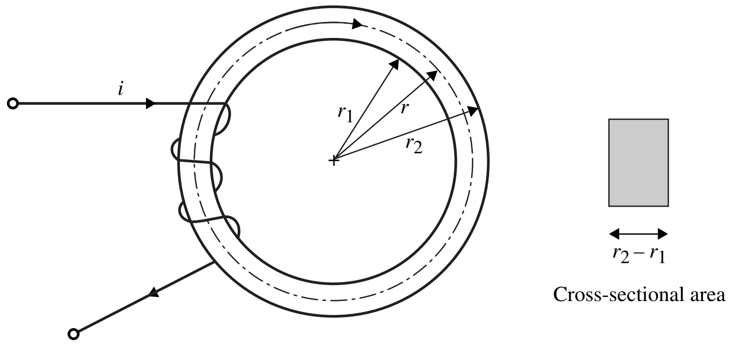


Figure A.4 Toroidal structure of Example A.1.

SOLUTION (**a**) The cross-sectional area of the core is given by

$$A = (r_2 - r_1)h$$
$$= 50 \times 10^{-6}\ \text{m}^2$$

The maximum flux is given by

$$\phi = \mathbf{B}A$$
$$= 15 \times 10^{-6}\ \text{Wb}$$

(**b**) To obtain the mmf, we must determine the mean length of the flux path, which is given by

$$l = 2\pi\frac{r_1 + r_2}{2}$$
$$= 23.56 \times 10^{-2}\ \text{m}$$

The mmf is given by

$$\text{mmf} = \frac{\phi l}{\mu A}$$
$$= \frac{(15 \times 10^{-6})(23.56 \times 10^{-2})}{(2000 \times 4\pi \times 10^{-7})(50 \times 10^{-6})}$$
$$= 28.125\ \text{A-turn}$$

(c) From the value of mmf, the number of turns, N, for $i = 10$ mA is 4.

A.3 MAGNETIC CIRCUITS

For many practical applications in power electronics, the design of special coil and core configurations are performed to produce various magnetic fields. As stated earlier, it is normally desirable to produce the maximum magnetic field with minimum magnetizing current in the coil. Because of the direct proportionality between current and magnetic field density, an attempt to increase the magnetic field intensity will result in an increase in the coil current. However, because of the relation $\mathbf{B} = \mu\mathbf{H}$, it is possible to increase the flux density by using material with high permeability compared to the air permeability. As discussed in the previous section, these materials are called *ferromagnetics*, from which almost all cores are made. Consequently, this choice is based on the assumption that most of the flux is confined to the magnetic material, with no flux flow in the surrounding medium, air or otherwise. This is because the flux will flow in a high μ path rather than in the low μ of the air. This phenomenon is analogous to the way in which current flows in wires, which have a conductivity much higher than the conductivity of the air. This observation, and others to be made later on in this section, have led to new approaches to analyze magnetic circuits. It has been customary to make this analogy between "electric circuits" and "magnetic circuits" in order to simplify the analysis and design of magnetic circuits, since electric circuits are well understood by electrical engineers.

Magnetic circuit analysis has proven useful in the design of inductors, transformers, and other special magnetic devices. In electric circuits, based on Kirchhoff's voltage and current laws (KVL and KCL), all branch currents and nodal voltages are normally determined. In magnetic circuits, the following questions arise: How much current is needed to magnetize or demagnetize a given core? What is the resultant flux? When do we obtain maximum flux?

The analysis of magnetic circuits will be based on the following two Maxwell's equations:

$$\oint_C \mathbf{H} \cdot dl = Ni \qquad \text{(Ampere's law)} \tag{A.9}$$

$$\oint_C \mathbf{B} \cdot dS = 0 \qquad \text{(Gauss's magnetic law)} \tag{A.10}$$

Equation (A.9) suggests that the total magnetic field in a closed path of length l must equal the total applied magnetomotive force. In other words, we may consider the right-hand side of this equation as a current source and the left-hand side as the resultant or induced mmf. This is analogous to state that the source voltage around a closed loop equals the total resultant voltage (drop) in the same loop. This is like dealing with KVL for mmf, rather than for emf.

Similarly, Eq. (A.10) suggests that the sum of the total magnetic flux, ϕ, into a closed region in space is equal to zero. This is analogous to KCL for ϕ rather than for i. In other words, we may state that the sum of all magnetic flux into a small region must be zero. It is obvious that the mechanism of the flow of flux in magnetic circuits is similar to the way current flows in electric circuits.

To further illustrate the similarities between magnetic and electric circuits, consider the equation for the flux derived in the previous section, namely:

$$\phi = \frac{Ni}{l} A\mu \tag{A.11}$$

If we let a parameter known as reluctance $\Re$ be defined as

$$\Re = \frac{1}{\mu A} \tag{A.12}$$

then the flux may be expressed as

$$\phi = \frac{Ni}{\Re} \tag{A.13}$$

or

$$\begin{aligned} \text{mmf} &= Ni \\ &= \Re\phi \end{aligned} \tag{A.14}$$

Equation (A.14) reminds us of the way we define the resistivity for a conductor of length l, cross-sectional area A, and conductivity σ:

$$R = \frac{l}{\sigma A} \tag{A.15}$$

Moreover, Eq. (A.14) is similar to Ohm's law relating voltage drop to the resistor and current. Just as the resistance opposes the current flow in the conductor, the reluctance opposes the flow of the flux in the magnetic circuit.

The flux linkage is normally defined by

$$\lambda = N\phi \tag{A.16}$$

and the inductance is defined as

$$L = \frac{\lambda}{i} \tag{A.17}$$

In terms of magnetic circuit geometry, L can be written as

$$L = \frac{\mu A N^2}{l} \tag{A.18}$$

It is also customary to express the flux using the following relation:

$$\phi = \mathcal{P} Ni \tag{A.19}$$

where $\mathcal{P}$ is known as the *permeance*. In terms of the reluctance, the permeance is given by

$$\mathcal{P} = 1/\Re \tag{A.20}$$

Table A.1 summarizes the analogy between the electric and magnetic circuit parameters.

Table A.1 Analogy between Magnetic and Electric Circuits

Electric circuits	Magnetic Circuits
Electromotive force (emf) (volts)	Magnetomotive force (mmf) (Ni)
Current (I)	Magnetic flux (ϕ)
Voltage drop (volts)	Magneto volts (Hl)
Resistance, R	Reluctance, $\Re$
Current density, $J = I/A$	Flux density, $B = \phi/A$
KVL for emf	KVL for mmf
KCL for currents	KCL for flux
Conductance, $G = 1/R$	Permeance, $\mathcal{P} = 1/\Re$
Conductivity, σ	Permeability, μ
Conductors	Ferromagnetics
Insulators	Diamagnetics

EXAMPLE A.2

Consider a rectangular core with a winding N that turns as shown in Fig. A.5. Assume the cross-sectional areas of the side and top/bottom segments of the core are A_1 and A_2, respectively. Develop the electric circuit equivalence and determine the reluctance, flux, and inductance of this configuration.

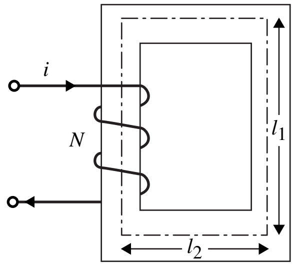


Figure A.5 Magnetic structure for Example A.2.

SOLUTION Since the permeability of the core is much higher than that of the air, we assume that the entire flux produced by Ni is confined to the core. There are four segments (legs) in this core: the side segments, of the same reluctance $\Re_1$, and the top/bottom segments, also of the same reluctance $\Re_2$. The coil and the excitation current are represented as an electromotive source, as shown in Fig. A.6.

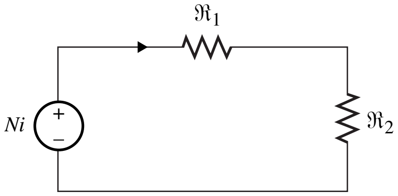


Figure A.6 Equivalent electric circuit of Fig. A.5.

The reluctances $\Re_1$ and $\Re_2$ are given by

$$\Re_1 = \frac{2l_1}{\mu A_1}$$

$$\Re_2 = \frac{2l_2}{\mu A_2}$$

where l_1 and l_2 are the mean lengths of the core as shown in Fig. A.5, and μ is the permeability of the core.

The flux produced in the winding is given by

$$\phi = \frac{Ni}{\Re_T}$$

where $\Re_T = \Re_1 + \Re_2$, and the inductance is given by

$$L = \frac{N^2}{\Re_T}$$

EXAMPLE A.3

Figure A.7 shows a three-legged magnetic structure made of material with $\mu = 3000\mu_o$. Winding of N turns is placed at the center leg and an air gap of width l_g is made in the right leg with the dimensions shown. Develop an equivalent electric circuit and determine the flux and the inductance of the winding.

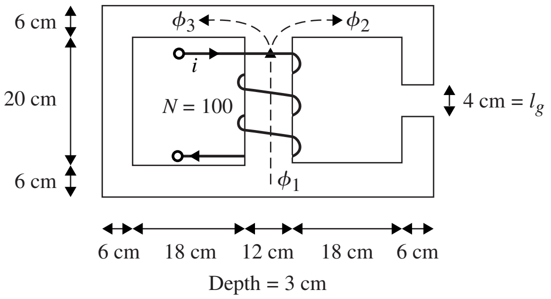


Figure A.7 Magnetic structure for Example A.3.

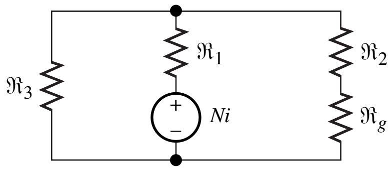


Figure A.8 Equivalent electric circuit of Fig. A.7.

SOLUTION As in Example A.2, it will be assumed $\mu >> \mu_o$ and the cross-sectional area of the air gap is the same as the cross-sectional area of the core. (Neglect fringing effects, to be discussed later.) The equivalent electric circuit is shown in Fig. A.8, where

$\Re_1$ = Reluctance of the center leg
$\Re_2$ = Reluctance of the right-leg core segment
$\Re_3$ = Reluctance of the left-leg core segment
$\Re_g$ = Reluctance of the free space (air)

From Fig. A.7, we obtain the mean length for these reluctances:

$$l_1 = 26 \text{ cm}$$

$$l_g = 4 \text{ cm}$$

$$l_2 = 2(27) + 22 = 76 \text{ cm}$$

$$l_3 = 2(27) + 26 = 80 \text{ cm}$$

The cross-sectional areas of the center leg, A_1, and the remaining segments, A_2 (including the air), are given by

$$A_1 = 36 \text{ cm}^2$$

$$A_2 = 18 \text{ cm}^2$$

The reluctances are calculated as follows:

$$\Re_1 = \frac{l_1}{\mu A_1}$$

$$= 6.89 \times 10^5$$

$$\Re_g = \frac{l_g}{\mu_o A_2} = 1.77 \times 10^7$$

$$\Re_2 = \frac{l_2}{\mu A_2} = 11.2 \times 10^4$$

$$\Re_3 = \frac{l_3}{\mu A_2} = 11.78 \times 10^4$$

The total flux in the center leg is calculated from

$$\phi_1 = \frac{Ni}{\Re_T} = 0.62 \text{ mWb}$$

where $\Re_T = (\Re_2 + \Re_g) \parallel \Re_3 + \Re_1 = 8.06 \times 10^5$.

The flux in $\Re_2$ and $\Re_3$ is given by

$$\phi_2 = \phi_1 \frac{\Re_3}{\Re_2 + \Re_3 + \Re_g} = 4.07 \text{ mWb}$$

$$\phi_3 = \phi_1 \frac{\Re_2 + \Re_g}{\Re_2 + \Re_3 + \Re_g} = 0.616 \text{ mWb}$$

The inductance of the winding is given by

$$L = N\frac{\phi_1}{i} = \frac{N^2}{\Re_T} = 12.41 \text{ mH}$$

As illustrated in the examples, the calculation of flux and inductance is straightforward and quite simple. Even though these calculations are approximate, they often give satisfactory results. Some sources of inaccuracy in calculating the flux in the core can be summarized as follows:

1. Leakage flux into the low-permeability surrounding air is not exactly zero. This is because the conductivity of a current-carrying wire in an electric circuit is higher than the conductivity of surrounding air by 12 to 15 orders of magnitude, whereas the conductivity difference between magnetic materials and the surrounding medium is around 2 to 3 orders of magnitude. This is a fundamental difference between magnetic and electric circuits.
2. In air-gapped cores, the cross-sectional area of the air is not quite that of the core. This is due to the additional flux that exists outside of the volume directly between the core segments, which results in an increase in the

effective cross-sectional area of the gap. This phenomenon is known as the *fringing effect*, and the flux is called *fringing flux*.

3. Due to the nonlinear nature of ferromagnetic material, its permeability is not always constant under any current excitation. As a result, reluctances and, consequently, inductances exhibit a nonlinear relation between the mmf and flux. This is another fundamental difference between magnetic and electric circuits.
4. To simplify the evaluation of the closed-loop integral to obtain the magnetic field density, it is assumed that the flux path is an *average* or mean length. Even though the difference is not significant, it contributes to the error in the calculations.
5. The magnetic field density and intensity are not uniform throughout the cross-sectional area of the core. Unlike the cross-sectional area of the conductivity of an electric wire, which is very small when used to calculate the resistivity of the conductive material, the cross-sectional area in a magnetic circuit is not necessarily small. Hence, magnetic fields are not really uniform.

A.4 THE MAGNETIZING CURVE

Thus far, based on the constitutive relation $\mathbf{B} = \mu\mathbf{H}$, it has been assumed that, regardless of the size of the applied mmf, the permeability of the magnetic material is constant. This assumption is only true in the free space, where μ_o is constant. The permeability is highly nonlinear for iron and ferromagnetic materials, as will be illustrated in this section.

To understand the behavior of magnetic material, we first consider the relationship between the flux ϕ and magnetizing force Ni. Consider the structure made of a ferromagnetic material shown in Fig. A.9(*a*), with a variable mmf source. We start at point A in Fig. A.9(*b*) with the material that has never been magnetized, i.e., $\phi = i = 0$. Now we apply magnetizing force Ni to the coil; this will result in a proportional increase in

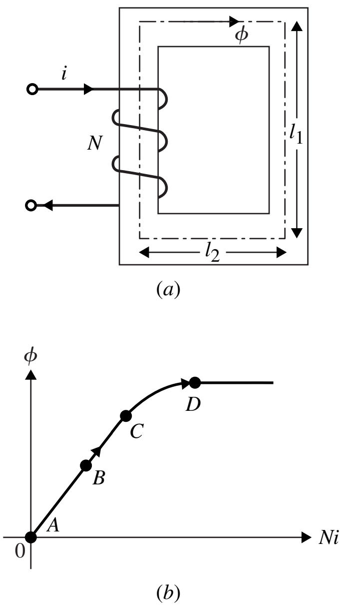


Figure A.9 Flux versus mmf in ferromagnetic material. (*a*) Magnetic structure excited by mmf force. (*b*) Typical ϕ vs. Ni plot for ferromagnetic material.

the magnetic flux to point *B* as shown on the curve. Notice that at this point, a small increase in mmf produces a large increase in the flux, ϕ. If we continue to increase the current, a point will be reached at which any additional increase in the mmf produces a relatively small increase in the flux, as shown at point *C*. At point *D*, any further increase in the current produces no change in the flux. At this point and beyond, the magnetic material is said to be in *saturation*. This curve is known as a *magnetizing curve*. The shape of the linear region and the magnitude of the saturation vary from one type of ferromagnetic material to another.

Another interesting property of a magnetic material is known as *hysteresis*. This phenomenon is illustrated in Fig. A.10, where the loop 1234561 is known as the *hysteresis loop*. To illustrate how this loop is created, let us assume that the applied magnetic force, **H**, is produced by a sinusoidally varying excitation current. First we assume that the material is magnetized and has reached its saturation point 1. Then when the current starts decreasing, the magnetizing current starts following a different path. At point 2, even though the magnetic force is zero, a magnetic flux remains in the material; B_R known as *residual flux* or *remnant magnetization*. This is how permanent magnets are made. At point 3, the material experiences a negative magnetic force that forces the flux to zero, i.e., demagnetization. The value that causes the material to demagnetize is known as *coercive mmf*, $\mathbf{H}_C$. The material reaches negative saturation at point 4. As **H** starts increasing again, the curve follows a different path until it reaches positive saturation once again at point 1. Points 5 and 6 correspond to remnant magnetization and coercive mmf, respectively. One important feature of the **B-H** curve is that **B** is not a single-valued function of **H**. In other words, the amount of flux at any given point in the **B-H** curve depends not only on the value of **H** at that point, but also on the previous value of **B** or on the history of **B**. As a result, ferromagnetic materials can be used as memory devices.

The physical explanation for the behavior of magnetic material can be clearly understood by studying the quantum mechanical forces between the atoms in the presence of a magnetic field. Only a brief qualitative description of this behavior is given in this section.

All magnetic materials, whether diamagnetic, paramagnetic, or ferromagnetic, possess net *magnetic moment* due to both the orbital and spinning motions of the electrons in their corresponding atoms. In the absence of an externally applied magnetic field, the degree to which the net magnetic moment exists varies between materials. In diamagnetic materials, the net magnetic moment is zero. In the presence of an external magnetic field, a net magnetic moment is created due to the combined interaction between the external magnetic field and spinning and orbital electrons, which results

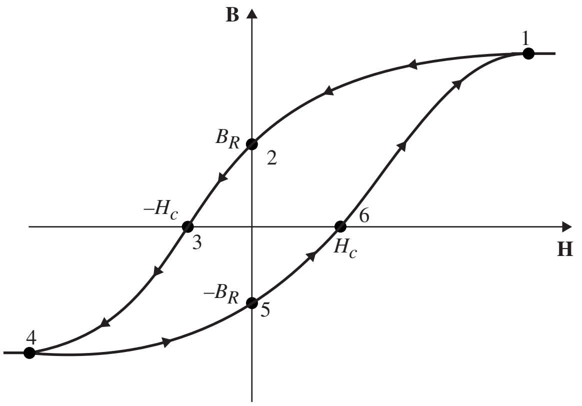


Figure A.10 B-H magnetizing curve.

in perturbation in their angular velocities. This effect of magnetization is very small in this type of material and too weak to be considered for any practical application. Moreover, diamagnetic material exhibits no permanent magnetization.

Unlike the case for diamagnetics, the magnetic moments due to orbiting and spinning electrons in paramagnetic material do not cancel completely. When an external magnetic field is applied, the alignment of magnetic moments in the direction of the magnetic field takes place. Consequently, an increase in the net flux density is obtained. Still, the resultant magnetic flux is quite small and the relative permeability of such material is close to 1.

Unlike diamagnetics and paramagnetics, ferromagnetic material shows a high degree of susceptibility to an externally applied magnetic field. In ferromagnetic materials, it has been experimentally shown that due to the strong forces exerted by the neighboring magnetic dipoles, which are produced from the spinning electrons in the atoms, all the magnetic dipoles are aligned in parallel, resulting in magnetized small regions known as *domains*. These domains range in linear dimension between 10^{-6} and 10^{-4} meters, and each contains about 10^{16} atoms.

Figure A.11 depicts a simplified symbolic domain model, with each domain containing a number of magnetic dipoles aligned in parallel to produce a magnetized domain, with the domains magnetized in different directions. The domains are clearly separated by regions of hundreds of atoms. Because of the random nature of the direction of magnetization, the net magnetization of the ferromagnetic material is zero.

Under an externally applied magnetic field, some domains align themselves in the direction of the applied field, increasing the magnetic flux density. The stronger the applied field, the more domains are aligned and the larger the magnetic flux density gets. Depending on the type of the ferromagnetic material, there exists a certain magnetic force beyond which the process of alignment is not reversible. In other words, once the external field is removed, the magnetized domains do not return to their original position. This can be seen in the **B-H** curve by noting that B_R does not become zero once the external field is removed. This phenomenon can be easily explained if we recognize that if energy is needed to orient or rotate the magnetic dipoles in the direction of the externally applied field, energy must also be applied to rotate these dipoles to their original random positions. This is why a *coercive force*, H_C, must be applied to demagnetize the material. Typical values of H_C for permanent magnets range from one hundred to thousands of amperes per meter. Another way to demagnetize a ferromagnetic material is to

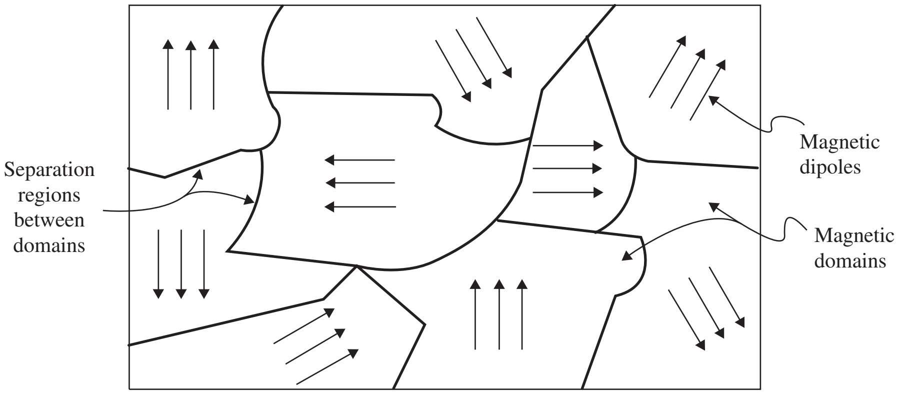


Figure A.11 Schematic representation of the domain model of a ferromagnetic material.

raise its temperature to a level that causes the domains to disorganize and return to their random orientation. The temperature at which this occurs is known as the *Curie temperature,* T_c. Typical values of T_c range from 200 to 1000°C. Another important point to note is that once all domains are aligned in one direction, the ferromagnetic material becomes saturated and starts behaving like paramagnetic material, i.e., magnetically transparent. At this point, its permeability decreases to μ_o.

A.5 INDUCTORS

Inductance, whether self- or mutual, results from the application of a third Maxwell's equation known as Faraday's law, which is given by

$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law)} \tag{A.21}$$

By integrating Eq. (A.21) over a specified surface area, applying Stokes's theorem to convert a surface integral to a line integral around a path that bounds this surface, and substituting for $\mathbf{B} = \boldsymbol{\phi}/A$, Eq. (A.21) can be rewritten as follows:

$$\oint_C \mathbf{E} \cdot dl = -\frac{d\boldsymbol{\phi}}{dt} \tag{A.22}$$

If we assume the conducting coil has N turns, then the total flux bound by the surface integral through which the magnetic field exists is given by $N\phi$, which is known as the *flux linkage*, as defined in Eq. (A.16). Recognizing the left-hand side of Eq. (A.22) as an induced voltage (we also assume the electric field exists only in the conducting wire), we may write Eq. (A.22) as

$$\begin{aligned} e &= -N\frac{d\phi}{dt} \\ &= -\frac{d\lambda}{dt} \end{aligned} \tag{A.23}$$

The minus sign is expression of Lenz's law, which indicates that the induced voltage opposes the increase in the flux flow.

EXAMPLE A.4

Consider the toroidal structures shown in Fig. A.12.

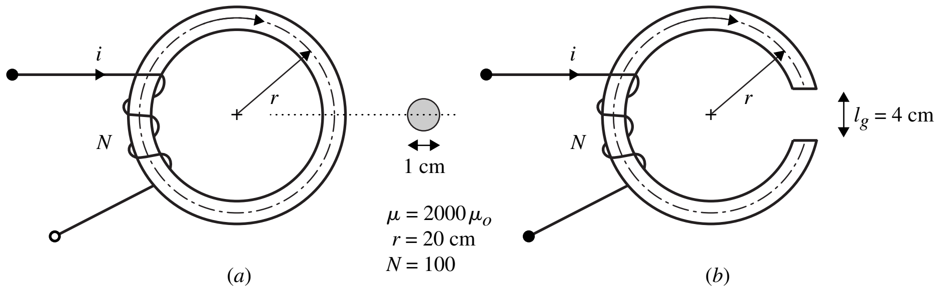


Figure A.12 Magnetic structures for Example A.4.

(**a**) Determine the inductances for both the ungapped and gapped magnetic structures made from the same material shown in Fig. A.12(*a*) and (*b*), respectively.
(**b**) If the permeability of the magnetic material is increased by 15%, determine the inductances of part (*a*).

SOLUTION (**a**) The cross-sectional area is given by

$$A = \pi r^2$$
$$= 0.7854 \times 10^{-4}$$

The total reluctance of the ungapped core is given by

$$\Re_T = \frac{2\pi r}{\mu A}$$
$$= 159154.6 \text{ A-turn/Wb}$$

The reluctance of the gapped core is given by

$$\Re_{T,\text{ gap}} = \frac{2\pi r - l_g}{\mu A} + \frac{l_g}{\mu_o A}$$
$$= 4.0524 \times 10^8 \text{ A-turns/Wb}$$

Hence, the inductances of the gapped and ungapped cores are given by

$$L_{\text{gap}} = \frac{N^2}{\Re_{T,\text{ gap}}}$$
$$= 24.68\ \mu\text{H}$$
$$L = \frac{N^2}{\Re_T}$$
$$= 62.8 \text{ mH}$$

(**b**) If μ is increased by 15%, then the new reluctances for $\mu = 23000\mu_o$ are

$$\Re_T = 1.384 \times 10^5 \text{ A-turn/Wb}$$
$$\Re_{T,\text{ gap}} = 3.524 \times 10^8 \text{ A-turn/Wb}$$

resulting in the following new gapped and ungapped core inductances:

$$L_{\text{gap}} = 28.38\ \mu\text{H}$$
$$L = 72.2 \text{ mH}$$

It is important to notice that an increase of 15% in μ results in an increase of 15% and 0.25% in the inductances of the ungapped and gapped core structures, respectively.

A.6 TRANSFORMERS

The typical structure of a transformer with two windings on a common core is shown in Fig. A.13. When one of the windings with N_1 turns (primary) is excited by an external source, an induced voltage is created in the other winding with N_2 turns (secondary). This is because both windings share the same magnetic core and, consequently, link the same flux. Depending on the power rating, operating frequencies, and intended

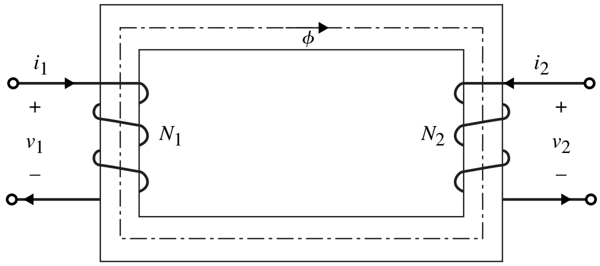


Figure A.13 Magnetic structure of a two-winding transformer.

application, various types and shapes of core structures are used. Line-frequency transformers (50 Hz, 60 Hz, or 400 Hz) are normally used for isolation and stepping down/up the primary voltage. Depending on whether they are used for distribution, transmission, or household applications, the power rating varies widely. In power electronics applications, high-frequency transformers are very common, whose functions are mainly to transfer voltage, provide isolation, and store energy. Special types of transformers are used in instrumentation for sensing purposes. For example, *potential* and *current* transformers are used to sample a high primary voltage and a high line current, respectively. Regardless of their application, transformers generally have four relevant ratings: voltage, current, apparent power, and frequency. The apparent power (the product of the rms current and the rms voltage) sets the limit on the maximum I^2R loss in the transformer windings in order to limit heating of the transformer coils, which could damage the insulation or drastically shorten the transformer's life. Transformer voltage and frequency ratings serve two purposes: to limit the core losses and to prevent the transformer from saturation, as explained later in this section. Depending on the number of windings on a shared magnetic circuit, transformers can be single- or multiple-phase types. Because of their widespread applications, we will study only the single-phase and three-phase transformers.

A.6.1 Ideal Transformers

The operation of an ideal transformer is easy to understand because it's assumed that both coil and core losses are negligible and the two windings (primary and secondary) are perfectly coupled; i.e., they are linked with the same flux. Since the majority of the applications of a transformer are for voltage stepping and voltage isolation, its terminal voltages are important parameters. Because of this, additional expressions are needed to relate the voltage with the magnetic field and flux. The key relation in understanding the transformer operation is Faraday's law, which is given by

$$\oint_C \mathbf{E} \cdot dl = -\frac{d\boldsymbol{\phi}}{dt} \qquad \text{(Faraday's law)} \tag{A.24}$$

The left-hand side of Eq. (A.24) is an expression of the induced voltage or emf resulting from the presence of the magnetic flux, ϕ. In other words, Faraday's law states that the induced voltage across a conducting wire is proportional to the rate of change of the flux through the wire with respect to time. If we assume the conducting wire has N turns, then Eq. (A.24) can be rewritten in terms of the total induced voltage as follows:

$$v(t) = -N\frac{d\phi}{dt} \tag{A.25}$$

Applying Eq. (A.24) to the transformer shown in Fig. A.13, we obtain

$$v_1(t) = -N_1 \frac{d\phi_1}{dt} \tag{A.26}$$

$$v_2(t) = -N_2 \frac{d\phi_2}{dt} \tag{A.27}$$

where ϕ_1 and ϕ_2 are the fluxes in the primary and secondary windings, respectively. Since perfect coupling is assumed, ϕ_1 and ϕ_2 must be the same; consequently, the terminal voltage relation for the transformer is given by

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \tag{A.28}$$

Since we also assume no losses, the total input and output powers must be equal; hence, the following terminal current relation can be obtained:

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \tag{A.29}$$

As stated earlier, one function of the transformer is impedance transformation. If an impedance Z_L is connected across the secondary winding, then it can be easily shown that the input impedance seen in the primary winding is given by

$$Z_{\text{in}} = Z_L \left(\frac{N_1}{N_2} \right)^2 \tag{A.30}$$

This equation is useful when the entire circuit is to be reflected from one side of the winding to the other side. This approach could simplify the analysis of complex circuits.

To identify the direction of the windings, whether clockwise or counterclockwise, a "dot" convention has been adopted. If the positive reference direction of either voltage is applied to one winding, then the dotted end of the other winding is positive. For example, in Fig. A.14(*a*) and (*b*) the windings for the primaries are wound in the counterclockwise direction and the secondaries are wound in the counterclockwise and clockwise directions, respectively.

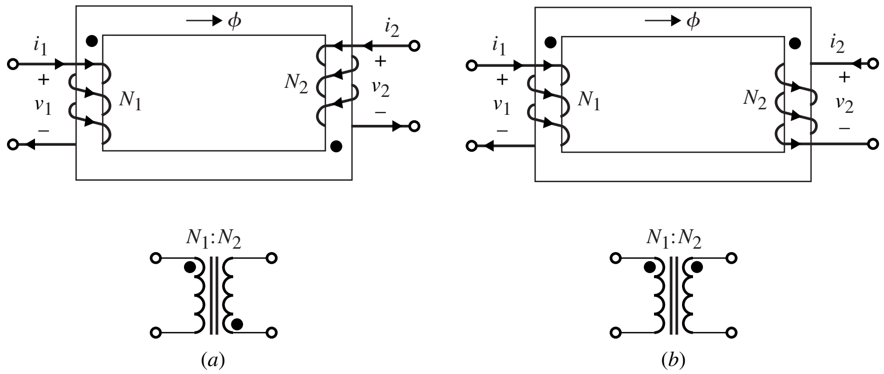


Figure A.14 Transformer "dot" notation. (*a*) The primary and secondary are wound in the counterclockwise direction. (*b*) The secondary is wound in the clockwise direction.

EXAMPLE A.5

Consider an *RLC* load connected to the secondary side of a transformer in a converter circuit as shown in Fig. A.15(*a*). Draw an equivalent circuit by reflecting the entire load circuit to the primary side.

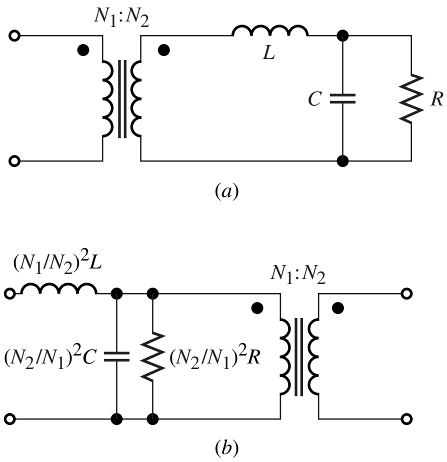


Figure A.15 (*a*) Transformer circuit with *LRC* used. (*b*) Equivalent circuit of (*a*) with the entire secondary side reflected to the primary side.

SOLUTION The resultant configuration is shown in Fig. A.15(*b*). The terminal voltage and current are scaled by N_1/N_2 and N_2/N_1, respectively.

A.6.2 Nonideal Transformers

The practical transformer is far from ideal due to the following nonidealities that are always present in the magnetic structure.

Nonperfect Coupling

Practical transformers are not perfectly coupled. This is because the flux linkage between the two windings is not the same; some flux leaks into the surrounding medium. As stated earlier, unlike in electric circuits, the air permeability is only a few orders of magnitude smaller than the core permeability, resulting in a small flux leaving the magnetic path and returning through the air as shown in Fig. A.16. This leaked flux is the origin of the

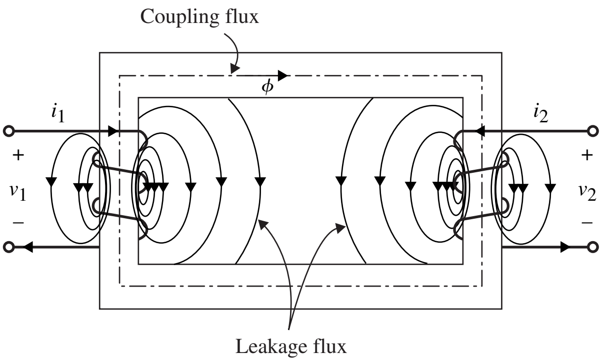


Figure A.16 Transformer magnetic structure illustrating coupling and leakage flux.

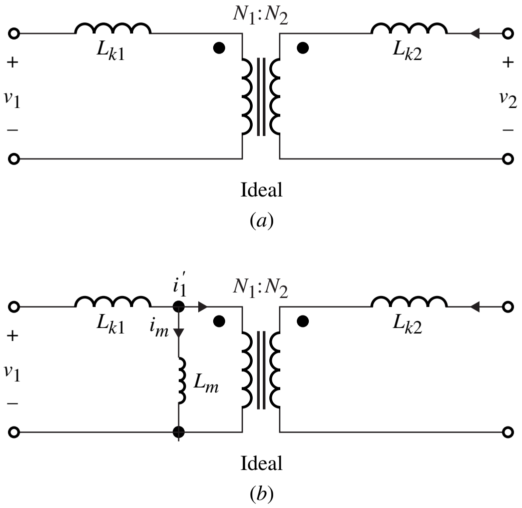


Figure A.17 Transformer model including (*a*) primary and secondary leakage inductances, and (*b*) magnetizing inductance reflected in the primary side.

imperfection in coupling. To simplify the analysis and design of transformer circuits, it is always useful to come up with models for any nonidealities. The leakage flux in the primary and secondary windings is normally modeled by two series inductive elements known as *leakage inductances*, L_{k1} and L_{k2}, respectively, as shown in Fig. A.17.

It is clear from Fig. A.17(*a*) that because of the presence of L_{k1} and L_{k2}, the terminal voltage relation given in Eq. (A.28) is no longer valid. Voltage drops are present across these inductances, resulting in energy loss and, therefore, less efficiency. The need to understand leakage effects and their origins is not due to concern for efficiency. In fact, when high-permeability cores are used with good coupling, it is common to design transformers with efficiency above 90%. The major concern is with the adverse effects these leakage fields have on the overall transformer performance. This concern is more serious in high-frequency power electronics applications, where the voltage drops across these leakage inductances become very large, causing regulation problems. An additional drawback in such applications is when the leakage inductors form resonating circuits with the distributed capacitances of the transformer windings and other diode and transistor junction capacitors, causing high voltage and current peaks that could damage the power switching devices and other circuit components.

Modeling the leakage effects in transformers as linear, lumped inductor elements in series with both windings with assumed constant values for a large range of operating frequencies is very accurate since the major portion of the closed leakage flux is through the air with constant permeability.

Finite Permeability

The permeability of the core is not infinite. Therefore, in order to produce a flux in the secondary winding, the primary winding must be excited by *Ni*. Even when the secondary winding is open-circuited, the primary current needed to magnetically couple the secondary winding will not be zero. This current is known as *magnetizing current* and flows through the *magnetizing inductance,* L_m. Depending on the winding at which L_m is measured, magnetizing inductance can be modeled at both windings. Normally, the magnetizing inductance is modeled as a shunt finite element in the primary side as shown in Fig. A.17.

The magnetizing current, i_m, shown in Fig. A.17(*b*) is the current required to create the flux in the primary winding. Theoretically speaking, if μ of the core is infinite, then it is possible to produce the flux in the primary winding, consequently coupling the secondary winding without magnetizing current. This represents the case for an ideal transformer with infinite L_m. Note that under dc excitation, the input current to the transformer, i'_1, is zero since the magnetizing inductance becomes short-circuited.

Core Losses

Nonideal transformers experience two types of core losses: hysteresis and eddy current losses, to be discussed in Section A.6.6. Because of these losses, additional current to i_m will flow in the primary winding, even when the secondary winding is open-circuited. This current is known as *core current*, i_{core}. Figure A.18 shows a modified equivalent transformer model including a shunt resistor, R_{core}, across L_m to model the core losses.

Copper Loss

Due to the current conduction in both winding coils of the transformer, usually additional equivalent resistors, R_{Cu1} and R_{Cu2}, are placed in series with L_{k1} and L_{k2} to model the copper losses in the primary and secondary windings, respectively, as shown in Fig. A.19.

A more simplified model, known as a transformer π model, can be obtained by reflecting the secondary-side circuit to the primary side as shown in Fig. A.20. Furthermore, if we assume that the magnetizing current is smaller than the terminal current, i_1, it is possible to further simplify the transformer model of Fig. A.20 as shown in Fig. A.21. The reflected equivalent resistance and inductance are given by

$$R_{\text{eq}} = R_{\text{Cu}1} + \left(\frac{N_1}{N_2}\right)^2 R_{\text{Cu}2}$$

$$L_{k,\text{eq}} = L_{k1} + \left(\frac{N_1}{N_2}\right)^2 L_{k2}$$

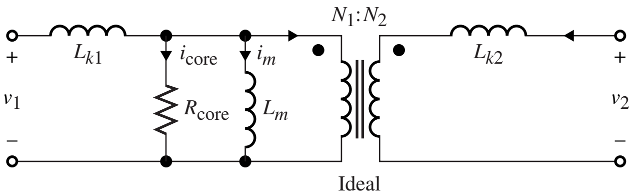


Figure A.18 Transformer model including the core losses and the leakage and magnetizing inductances.

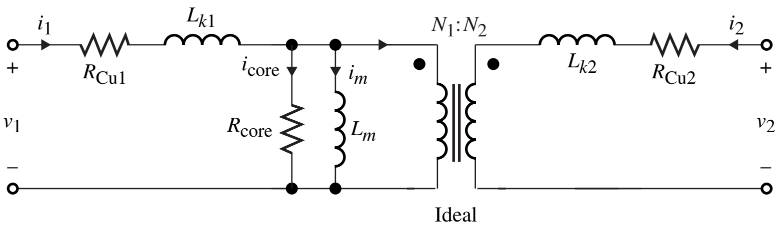


Figure A.19 Transformer model including the core losses, leakage and magnetizing inductances, and copper losses.

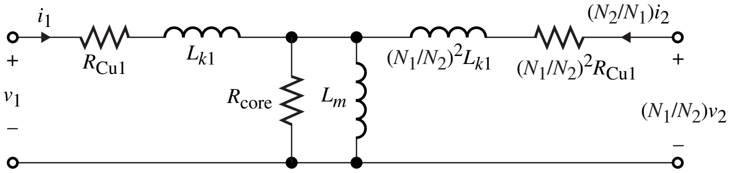


Figure A.20 Transformer π model.

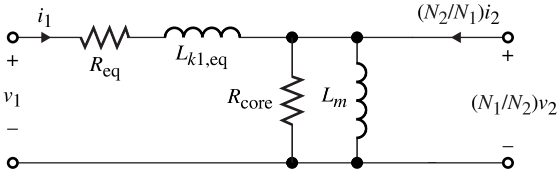


Figure A.21 Simplified transformer model of Fig. A.20.

We should point out that the parasitic capacitance has not been included in the model since its effect becomes dominant only at extremely high frequencies.

A.6.3 Transformer Equations

In the previous section, we described the model of a nonideal transformer using its leakage and magnetizing inductances as well as its coil and core losses. However, a more useful way to represent a nonideal transformer is by describing its terminal voltages and currents as linear time-invariant coupled-circuit differential equations in terms of self- and mutual inductances.

Consider a two-winding transformer as shown in Fig. A.22 including the effect of the primary and secondary leakage flux N_1i_1 and N_2i_2, respectively. The mutual fluxes ϕ_{m1} and ϕ_{m2} produced by N_1i_1 and N_2i_2 are also shown in the figure. The total fluxes produced at the primary and secondary windings are given in Eqs. (A.31) and (A.32), respectively.

$$\phi_1 = \phi_{l1} + \phi_{m1} + \phi_{m2} \tag{A.31}$$

$$\phi_2 = \phi_{l2} + \phi_{m1} + \phi_{m2} \tag{A.32}$$

Applying Faraday's law to Eqs. (A.31) and (A.32), we obtain

$$\begin{aligned} v_1 &= N_1\frac{d\phi_1}{dt} \\ &= N_1\frac{d}{dt}(\phi_{l1} + \phi_{m1}) + N_1\frac{d\phi_{m2}}{dt} \end{aligned} \tag{A.33}$$

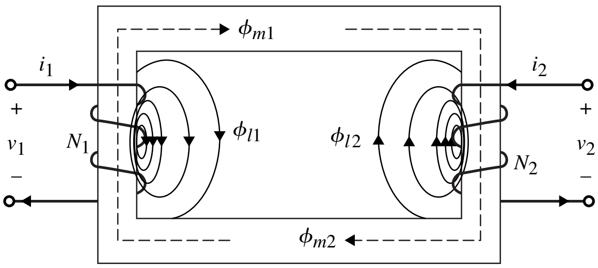


Figure A.22 Transformer magnetic core with mutual and leakage fluxes.

$$v_1 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d}{dt}(\phi_{l2} + \phi_{m2}) + N_2 \frac{d\phi_{m1}}{dt} \tag{A.34}$$

The self-inductances of the primary and secondary are defined as follows:

$$L_1 = \frac{N_1(\phi_{l1} + \phi_{m1})}{i_1} \tag{A.35}$$

$$L_2 = \frac{N_2(\phi_{l2} + \phi_{m2})}{i_2} \tag{A.36}$$

The mutual inductances at the primary and secondary windings are defined as follows:

$$L_{21} = \frac{N_1 \phi_{m2}}{i_2} \tag{A.37}$$

$$L_{12} = N_2 \frac{\phi_{m1}}{i_1} \tag{A.38}$$

If the magnetic structure is reciprocal, then $L_{12} = L_{21} = M$; hence, Eqs. (A.37) and (A.38) give

$$L_1 = L_{k1} + \frac{N_1}{N_2} M \tag{A.39}$$

$$L_2 = L_{k2} + \frac{N_2}{N_1} M \tag{A.40}$$

From Eqs. (A.33), (A.34), (A.39), and (A.40), the transformer equations in terms of terminal voltages and currents are given by

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{A.41}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \tag{A.42}$$

A.6.4 Leakage Inductances

Calculating the leakage inductance of a given magnetic geometry for an inductor or transformer is not a straightforward problem. Detailed dimensions of the windings and their physical location in the magnetic structure must be known; only then can an estimate of the leakage inductance be obtained. To minimize the transformer leakage inductances, normally the two windings are placed one on top of the other to enhance the flux linkage. It can be shown that if the number of turns, winding thickness (build-up), and insulation thickness (space between windings) are decreased and the winding length is increased, a smaller leakage inductance can be obtained (see Problem A.21). Furthermore, it is possible to experimentally determine the total leakage inductance of a transformer by

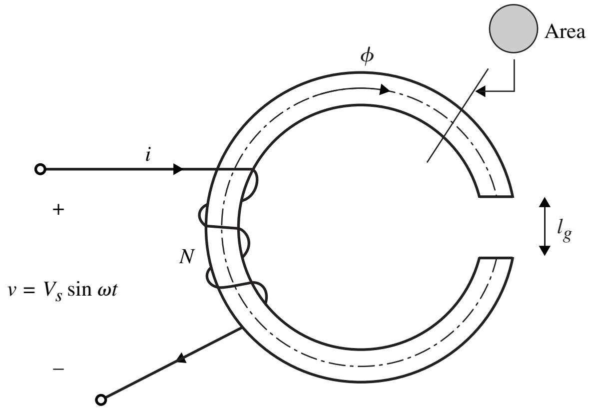


Figure A.23 Toroidal inductor with air gap.

performing two measurements known as open- and short-circuit tests, as illustrated in Problem A.15.

A.6.5 Saturation

As stated earlier, when the magnetic material saturates, a large change in the excitation current in the primary of the transformer results in no change in the coupling flux. The core becomes magnetically transparent. In limited applications, such as magnetic amplifiers and fluorescent lamp ballasts, the saturation property of the magnetic material is essential. Otherwise, saturation is avoided in most applications. Voltage, current, and frequency may cause the core to saturate as illustrated in the air-gapped toroidal inductor shown in Fig. A.23.

The inductance of Fig. A.23 can be shown to be independent of the permeability of the material if we assume $l_g >> l\mu_o/\mu$. Since at saturation the permeability of the core reduces to that of the air, this relation no longer holds. Consequently, the inductance becomes a function of both permeability and material type, and therefore unstable. The question normally asked is, when does the core saturate? It is obvious that the core saturates when its magnetic density, **B**, exceeds B_{sat} of the material. Since the external terminals of any transformer and inductor are voltages and currents, it is more useful to address the question of saturation in terms of these parameters. If we assume the excitation voltage in Fig. A.23 is a sinusoid given by $v_s(t) = V_s \sin\omega t$, then it can be shown that the following relation must hold to avoid saturation:

$$\frac{V_s}{N} \le 2\pi B_{\text{sat}} A f \tag{A.43}$$

where A is the core cross-sectional area, V_s is the maximum voltage, and f is the frequency of the applied signal. The left-hand side of Eq. (A.43) gives the maximum volts per turn allowed to prevent saturation. If the peak voltage is fixed, it is possible to set a limit on the minimum frequency necessary to avoid saturation, which is given by

$$f_{\min} = \frac{V_s}{2\pi N A B_{\text{sat}}} \tag{A.44}$$

A.6.6 Magnetic Losses

Two types of losses exist in magnetic structures: *core losses* and *copper losses*. The copper losses exist in the winding coils due to the current conduction and are given by

$$P_{\text{Cu}} = I_{\text{rms}}^2 R_{\text{Cu}} \qquad \text{(A.45)}$$

The core losses are attributed to both hysteresis losses and eddy current losses. The former are the result of unrecovered energy spent in the constant rotation or realignment of magnetic domains within the magnetic material. The larger the area enclosed by the hysteresis loop, the higher the loss. The loss due to the hysteresis loop is proportional to the frequency and magnetic field:

$$P_{\text{hys}} \propto f B_R^n \qquad 2 \geq n \geq 1.5 \qquad \text{(A.46)}$$

The energy lost due to eddy currents is proportional to the square of the frequency and the square of the magnetic field:

$$P_{\text{eddy}} \propto f^2 B^2 \qquad \text{(A.47)}$$

The larger the size of eddy current loops, the higher the losses within the core. To break these loops into smaller ones, it is customary to use small stripes on lamination cores.

A.6.7 Core Material and Types

The choice of the appropriate type and shape of core material has become an extremely important consideration in designing inductors and transformers for today's power circuits. This is in large part due to a substantial increase in the operating frequencies of power converters, which have reached 5–10 MHz. This push for higher and higher frequencies is mainly driven by the ever-increasing demand for smaller and lighter power converters. At these frequencies, smaller magnetic components can be used; also, the effect of the leakage inductances, self-capacitance of the winding, and inter-winding capacitance could become more significant. As a result, the choices of magnetic material, core geometry, conducting wires, and fabrication techniques are challenging considerations. The following four types of material are currently in wide use in the design of magnetic circuits.

1. *Air-core*. This type of core is used when magnetic field distortion due to the nonlinearity of the magnetic structure is not desirable. Air-core structures produce stable inductor and transformer values, since μ_o is constant. The winding is normally wound around a prefabricated form to sustain it.
2. *Laminated iron*. This material is used to minimize the effect of eddy current losses. This is achieved by coating lamination with some sort of electric insulation. Iron lamination cores are used mostly in line- and low-frequency applications. Lamination thickness and number, as well as the size of the air gap, are some design parameters.
3. *Powdered iron*. Depending on the core geometry, this type is normally used for frequencies up to 1 MHz and in high-Q inductors. The core losses and permeability are normally controlled by the size and composition of magnetic particles.

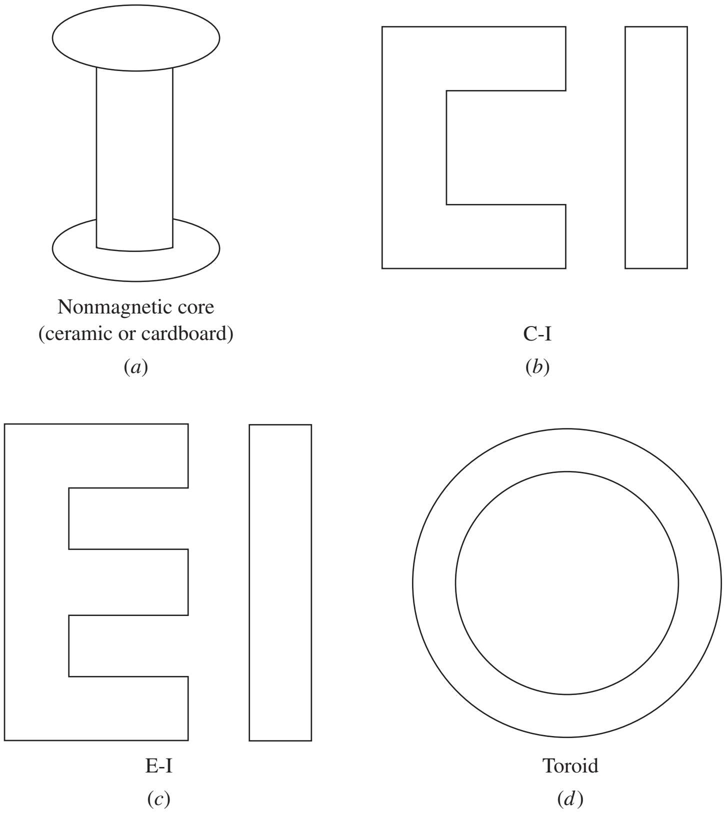


Figure A.24 Different types of cores. (*a*) Nonmagnetic. (*b*), (*c*), (*d*) Magnetic.

4. *Ferrite*. Because it has high permeability and high resistivity to eddy currents, ferrite core material is used in high-frequency applications up to 100 MHz. However, its B_{sat} is much lower than that of laminated or powdered iron core materials. As a result, ferrite cores are limited to lower-current applications. They also have a relatively low Curie temperature. Nevertheless, the ferrite core type is the most popular in the design of high-frequency power electronics.

Figure A.24 shows different shapes of cores widely used in today's power electronics applications. Generally speaking, the shape and geometry of magnetic cores are dictated by various electrical and electromechanical requirements. Examples of core arrangements are pot, toroid, E-I, and C-I.

A.7 THREE-PHASE TRANSFORMERS

Three-phase systems are the most popular ac electrical systems in the world, whether generated, transmitted, or distributed. This is in large part due to the economic advantages in the distribution and generation of three-phase systems using rotating machines. Moreover, unlike the power in single-phase systems, the total instantaneous power in a three-phase system is constant. This interesting feature of three-phase

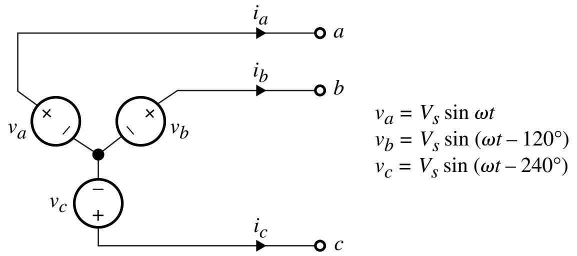


Figure A.25 Y-connected three-phase system.

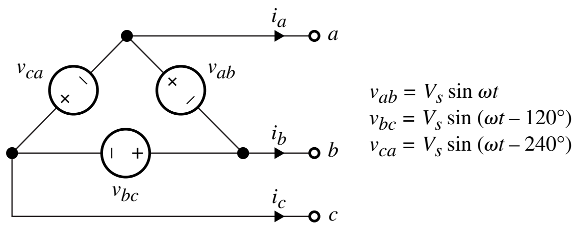


Figure A.26 Δ-connected three-phase system.

systems makes them attractive for power rotating machinery since it causes less vibration compared to a single-phase system, whose instantaneous power frequency is twice the line frequency.

Because of their importance in the implementation of three-phase systems, three-phase transformers are important devices in modern power electronic systems. Hence, a brief discussion of three-phase transformers is presented in this section.

Depending on whether the voltage source has a common reference point, known as the *neutral point*, voltages in three-phase systems can be represented as *wye* (Y) or *delta* (Δ) connections. In both system configurations, the voltage sources are equal in amplitude and frequency and have a 120° phase shift between each other. The Y-connected three-phase system is shown in Fig. A.25, where currents i_a, i_b, and i_c are line currents and voltages v_{ab}, v_{bc}, and v_{ca} are line voltages. It is apparent from Fig. A.25 that the line currents are the same as the phase currents, and it can be easily shown that the amplitudes of the line voltages are increased by $\sqrt{3}$ and phase-shifted by 30° from the phase voltages, v_a, v_b, and v_c. Figure A.26 shows the alternative way of connecting three-phase voltages, the Δ connection. Notice the line voltages and the phase voltages are the same, whereas the line currents i_a, i_b, and i_c have magnitudes $\sqrt{3}$ times the magnitude of the phase currents i_{ba}, i_{cb}, and i_{ac} and are shifted by $30°$.

One easy and straightforward way to make a three-phase transformer is to take three single-phase transformers and connect them in various Δ and Y configurations. An alternative way is to use a common core to create a set of three windings as shown in Fig. A.27. For economy and to increase efficiency and power density, this approach is more common these days. Like the voltage source connections in a three-phase system, each of the windings in the three-phase transformer can also be connected in Δ and Y configurations. As a result, there are four possible three-phase transformer connections: Δ-Δ, Δ-Y, Y-Δ, and Y-Y. Examples of a Y-Δ connection created from three single-phase transformers and a common core structure are shown in Fig. A.28(*a*) and (*b*), respectively, with the equivalent transformer schematic shown in Fig. A.28(*c*).

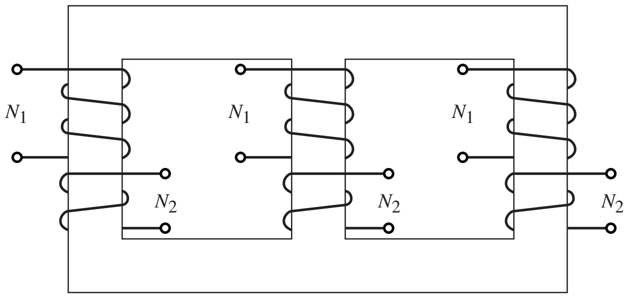


Figure A.27 Three-set of windings on a common core.

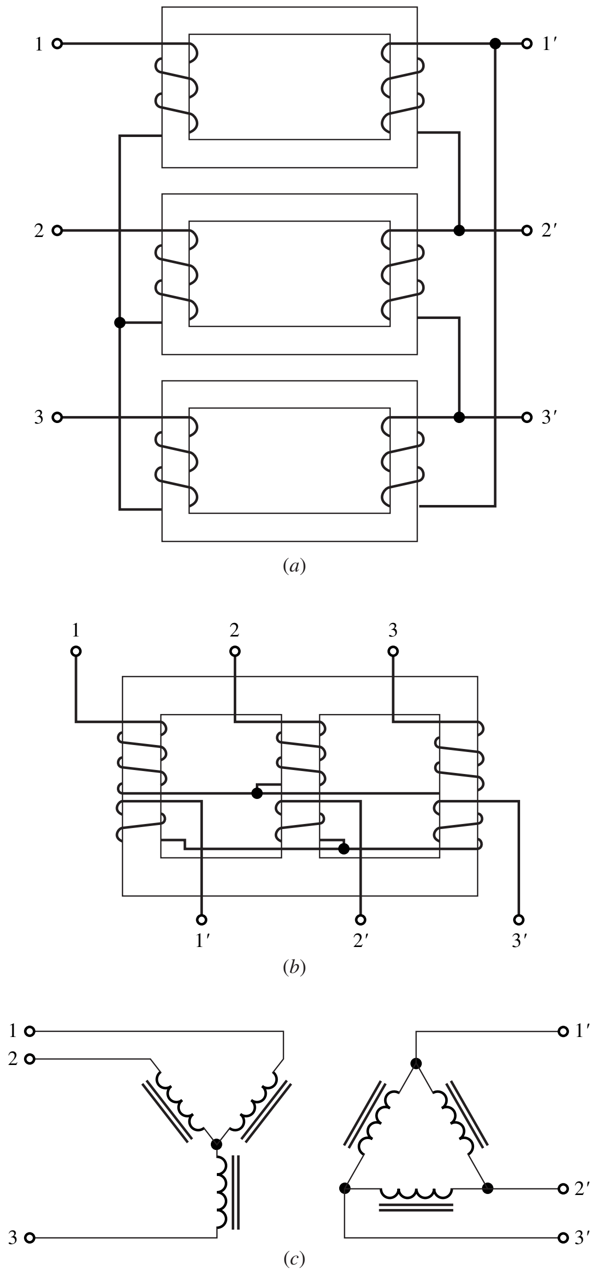


Figure A.28 Three-phase transformer in the Y-Δ configuration. (*a*) Three single two-winding transformers. (*b*) One single three-winding transformer. (*c*) Schematic representation.

PROBLEMS

A.1 Consider a coaxial line consisting of a solid inner and outer conductors of thickness a and $(c-b)$, respectively, each carrying current I, as shown in Fig. PA.1. Show that the magnetic field is expressed in the following function:

$$\mathbf{H} = \begin{cases} \dfrac{Ir}{2\pi a^2} & 0 < r \le a \\ \dfrac{I}{2\pi r} & a < r \le b \\ \dfrac{I}{2\pi r}\dfrac{(c^2-r^2)}{(c^2-b^2)} & b < r \le c \\ 0 & c < t \le \infty \end{cases}$$

Assume the currents are uniformly distributed and the current into the solid inner conductor equals the current in the outer shell.

A.2 By using the magnetic circuit approach, determine the inductances of the magnetic structures shown in Fig. PA.2. Assume $\mu = 2000\,\mu_o$ and the depth of the core cross-section is 4 cm.

A.3 Repeat Problem A.2 for Figure PA.2(a) and (d) by assuming the permeability of the core is decreased by 20%. Determine the percentage change in the new inductance values.

A.4 Consider Fig. PA.2(e) with i = 4 A flowing in the 250-turn coil. Determine the flux, flux density, and magnetic field intensity in each leg segment.

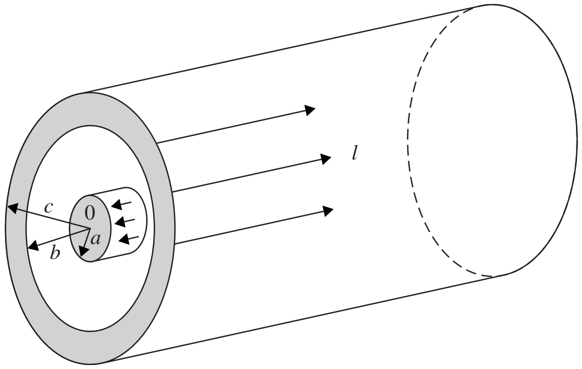


Figure PA.1

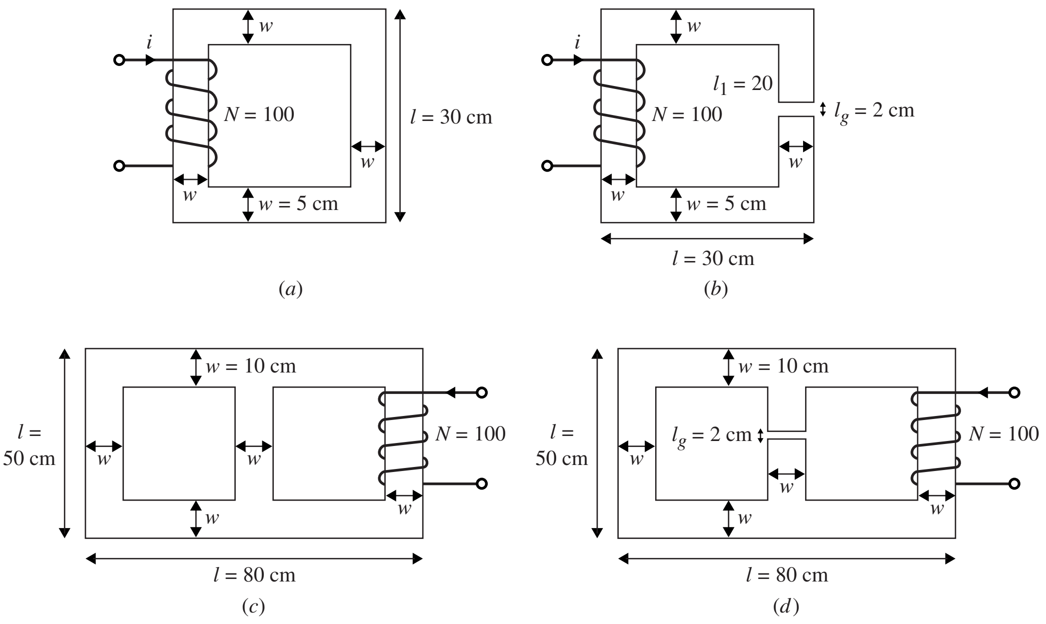
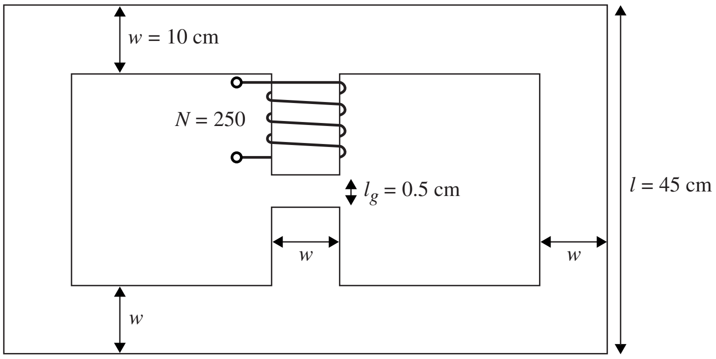


Figure PA.2



(*e*)

Figure PA.2 (*continued*)

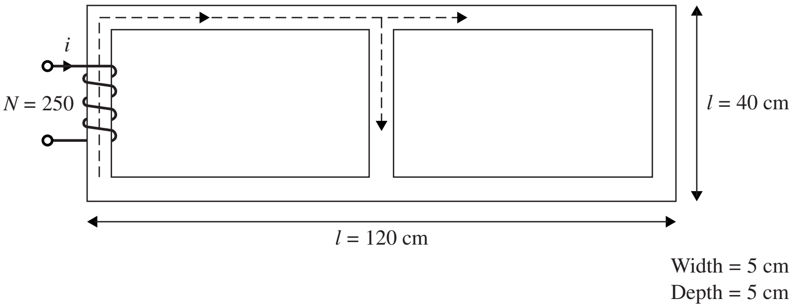


Figure PA.5

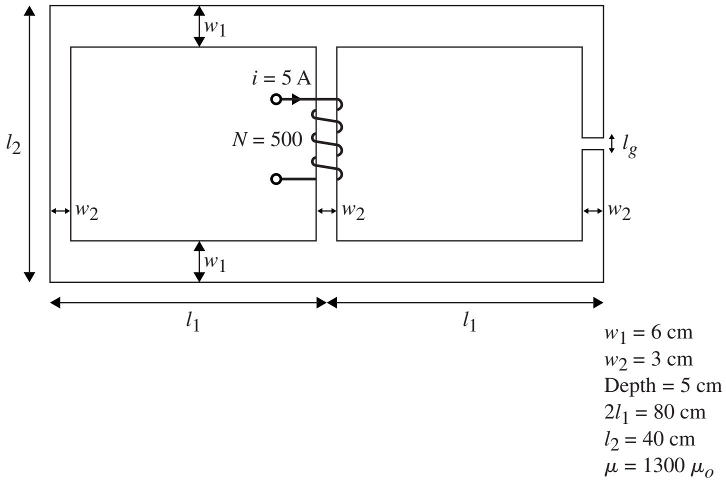


Figure PA.6

A.5 Consider the magnetic structure shown in Fig. PA.5 with $\mu = 3000\,\mu_o$. Determine the coil current that will produce 0.4 T in the center leg.

A.6 It is desired to produce a flux of 1.5 mWb in the air gap of the core shown in Fig. PA.6. Determine the air gap that will establish this flux.

A.7 Determine the inductance of a coil wound in air with the dimensions shown in Fig. PA.7 for $l_g = 0$, 0.5, and 1 cm.

A.8 Consider a toroidal coil structure with N turns wound around a circular cross-sectional core as

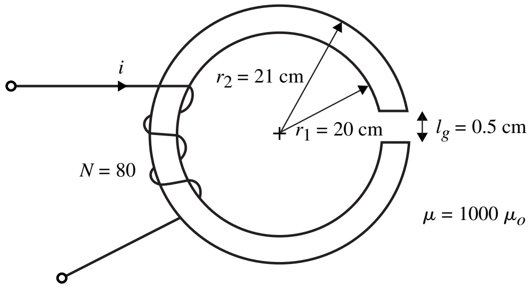
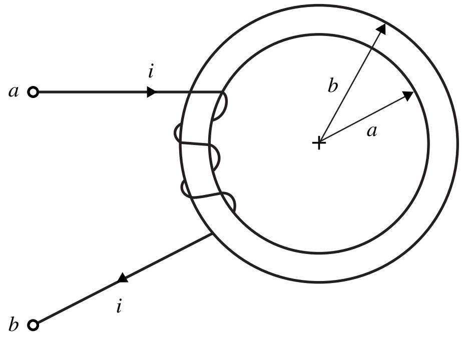


Figure PA.7

shown in Fig. PA.8(a). Assume the resistance of the copper coil is modeled by R_{Cu}, resulting in a possible equivalent circuit as shown in Fig. PA.8(b).



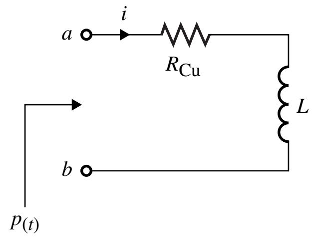


Figure PA.8

(**a**) If the excitation current is sinusoidal, $i(t) = I_p \sin \omega t$, show that the instantaneous power at the terminals ab is given by

$$p(t) = R_{Cu} I_p^2 \sin^2 \omega t + \frac{N^2 \mu A I_p^2 \omega}{\pi(b+a)} \cos \omega t \sin \omega t$$

(**b**) Show that the average input power between the terminals ab is given by

$$P_{ave} = R_{Cu} \frac{I_p^2}{2}$$

(**c**) Show that the total instant energy stored in the magnetic field is given by

$$W = \frac{\pi^2 (b-a)^2 (b+a)}{8 \mu}$$

A.9 Consider the air-gapped toroidal core with the dimensions shown in Fig. PA.9.
(**a**) Determine the inductance L.
(**b**) Repeat part (a) for $\mu = 500 \mu_o$.
(**c**) Determine the minimum length of the air gap, l_g, so that the inductances obtained in parts (a) and (b) are within ±2%.

A.10 Figure PA.10 shows a square core with air gap, l_g. Assume the applied voltage is sinusoidal with peak voltage 3000 V and frequency 60 Hz, and $\mu = 4500 \mu_o$.
(**a**) Determine the maximum flux ϕ.
(**b**) If B_{sat} = 2.2 T, determine the minimum frequency that will keep the core out of saturation.

A.11 The fringing effect and leakage flux could affect the value of the inductance. Consider the magnetic circuit with air gap given in Fig. PA.11.
(**a**) Calculate L with leakage flux and fringing effect negligible.
(**b**) Repeat part (a) by assuming that the air-gapped effective area is increased by 5%.
(**c**) Repeat part (b) by further assuming that 5% of the produced flux leaks to the surrounding air.
(**d**) What is the percentage change of the calculated inductances of parts (b) and (c) with respect to part (a)?

A.12 A group of electromechanical devices known as actuators consist of fixed and movable sections of ferromagnetic material. When excited by injecting current, i, the resultant magnetic field magnetizes the movable part and consequently lifts a mechanical load. Figure PA.12 shows one possible ferromagnetic structure for an actuator. Assume B_{sat} = 1.2 T and $\mu = 2000 \mu_o$, and ignore leakage flux.

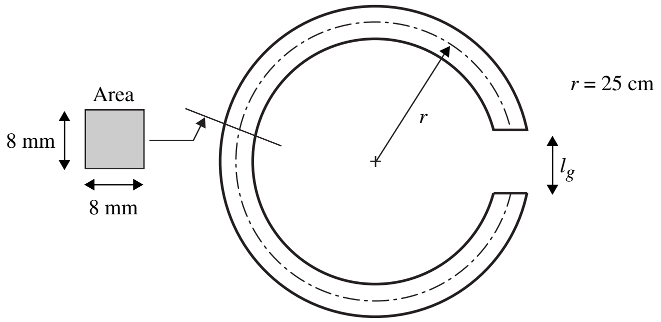


Figure PA.9

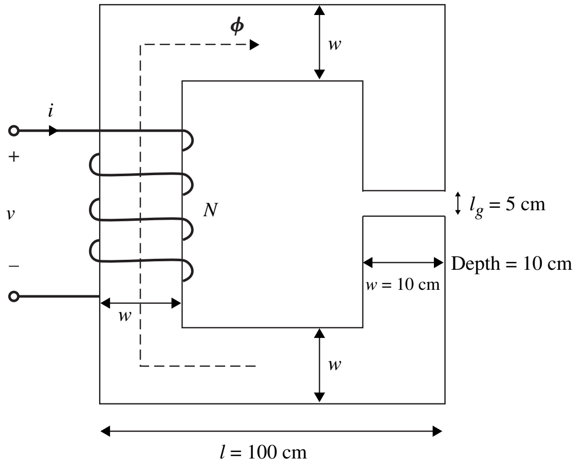


Figure PA.10

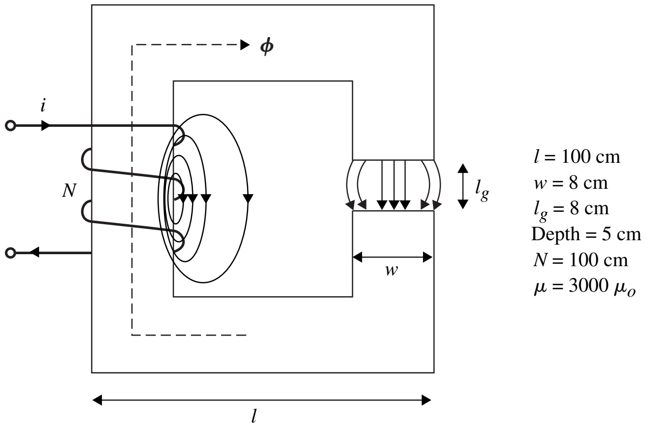


Figure PA.11

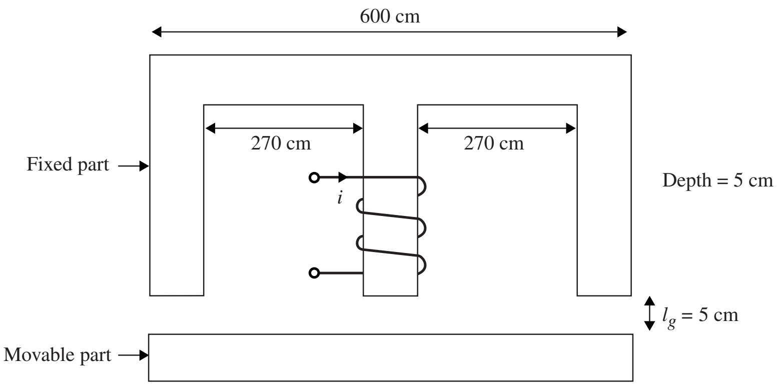


Figure PA.12

(**a**) Determine the maximum air-gap flux produced in the center leg.

(**b**) Determine the current i that would produce a flux density equal to $0.5B_{sat}$ in each of the two side air gaps.

A.13 Determine the maximum air gap, l_g, needed to establish $0.8B_{sat}$ in the center leg for $i = 10$ A in Problem A.12.

A.14 Consider a transformer with a turn ratio $N_1/N_2 = 200$ and rated at 240 kVA (44 kV/220 V) used to step down a 60 Hz voltage in a distributed system.

(**a**) Determine the rated primary and secondary currents.

(**b**) Determine the load impedance seen between the primary terminals when the load is fully loaded.

A.15 (**a**) A 60 Hz, 110 VA rms voltage is applied to the primary of a transformer with the secondary winding left open-circuited. The following two measurements are obtained:

(*i*) The primary current is found to be 5 A rms.

(*ii*) The average (real) power between the primary terminals is found to be 80 W.

This test is known as the open-circuit test. Use the transformer equivalent circuit shown in Fig. A.20 to determine the core resistance R_{core} and the magnetizing inductance L_m. (*Hint:* Assume the voltage drops across R_{Cu1} and L_{k1} are negligible.)

(**b**) With the same applied voltage as in part (*a*), the output voltage is shorted and the short-circuit current is measured and found to be 150 mA rms, and new measurements are taken as follows:

(*i*) The primary current is found to be 40 mA rms.

(*ii*) The average power at the primary terminal is found to be 25 W.

This test is known as the *short-circuit test.* Determine the total copper resistance, R_{Cu}, given by

$$R_{Cu} = R_{Cu1} + \left(\frac{N_1}{N_2}\right)^2 R_{Cu2}$$

(*Hint:* Assume the current in the magnetizing circuit, R_{core} and L_m, is negligible.)

A.16 Because of the series impedance in the primary side of the transformer, the output voltage will change under various load conditions, even if the input voltage remains constant. The degree to which the output voltage change between low-load and full-load conditions is known as *load regulation*, LR, defined as

$$\mathrm{LR} = \left|\frac{v_{o,l} - v_{o,f}}{v_{o,f}}\right| 100\%$$

where $v_{o,l}$ and $v_{o,f}$ are the low- and full-load output voltages, respectively. This parameter is very important to the design of transformers.

(**a**) Using the simplified transformer equivalent circuit given in Fig. PA.16 with load impedance Z_l, show that LR is given by

$$\mathrm{LR} = \frac{Z_s Z_m (Z_{l,l} - Z_{l,f})}{(Z_m Z_{l,l} + Z_s Z_m + Z_s Z_{l,l}) Z_{l,f}}$$

where $Z_{l,f}$ and $Z_{l,l}$ indicate the load impedance at full- and low-load conditions, respectively.

(**b**) If the low-load condition occurs at $Z_{l,l} = \infty$ and assuming $R_{core} = \infty$ and $R_{Cu} = 0\ \Omega$, show that LR is given by

$$\mathrm{LR} = \frac{\omega\left(L_{k1} + \left(\frac{N_1}{N_2}\right)^2 L_{k2}\right)}{|Z_{l,f}|}$$

(**c**) If a 250 kHz input voltage source with $V_s = 200$ V is applied to the primary winding, determine LR for the following values: $R_{eq} = 0.2\ \Omega$, $L_{eq} = 125\ \mu$H, $R_{core} = 125\ \Omega$, $L_m = 450\ \mu$H, $Z_{l,l} = 57\angle 6°\ \Omega$, and $Z_{l,l} = 0.5\angle 78°\ \Omega$.

A.17 It is customary to represent transformers by their primary and secondary coupling coefficients k_1 and k_2, respectively. If k_1 and k_2 are defined as

$$k_1 = \frac{L_{m1}}{L_{m1} + L_{k1}}$$

$$k_2 = \frac{L_{m2}}{L_{m2} + L_{k2}}$$

$$M = \sqrt{k_1 k_2}$$

show that the transformer model given in Fig. A.20 can be presented as shown in Fig. PA.17.

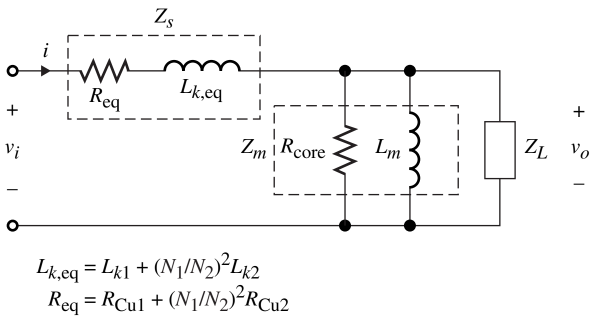


Figure PA.16

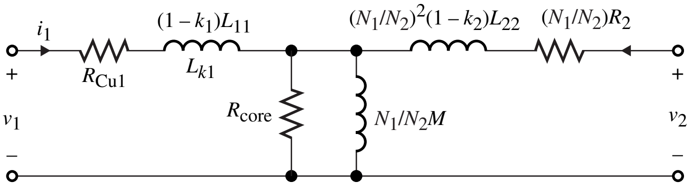


Figure PA.17

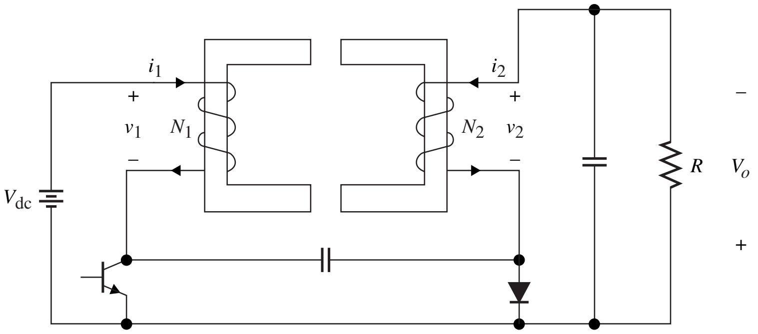


Figure PA.18

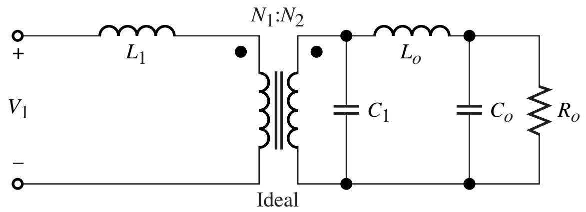


Figure PA.19

A.18 In some power electronic circuits where more than one inductor is used, it is desirable to implement two or more inductors on a single magnetic structure. Such inductors are known as *coupled inductors*. The Cuk converter shown in Fig. PA.18 is one example of a coupled inductor circuit.

If it is assumed v_1 and v_2 are identical ac voltages and the leakage inductances of the N_1 and N_2 windings are modeled by $\Re_{l,1}$ and $\Re_{l,2}$ respectively, show that:

(**a**) The current $i_1(t) = 0$ when

$$\frac{N_2}{N_1} = \frac{\Re_{l2}}{\Re_{l1} + \Re_m}$$

(**b**) The current $i_2(t) = 0$ when

$$\frac{N_1}{N_2} = \frac{\Re_{l1}}{\Re_{l2} + \Re_m}$$

where $\Re_m$ is the total reluctance of the magnetic circuit, including the reluctance of the air.

A.19 Derive the equivalent circuit for Fig. PA.19 by reflecting the entire impedance to the primary side. Assume ideal transformers.

A.20 Determine the inductance of a coil wound in the air as shown in Fig. PA.20.

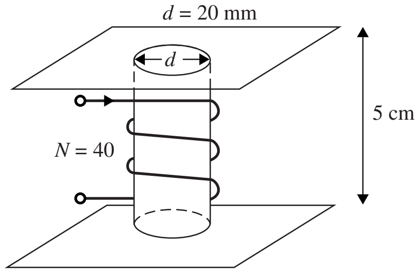


Figure PA.20

A.21 To reduce the transformer leakage inductances, normally the primary and secondary windings are placed one on top of the other as shown in Fig. PA.21(*a*). By assuming that the magnetic field is distributed as shown in Fig. PA.21(*b*), derive an expression for the leakage inductance in terms of the given parameters. Assume the cross-sectional area of the core is A and the number of turns is N.

A.22 It is common to find transformers with more than two windings on a single core. Figure PA.22(*a*) and (*b*) shows a transformer structure with two and three windings respectively. Assume that the cross-sectional area of the center leg is A_1 and that of the remaining parts is A_2.

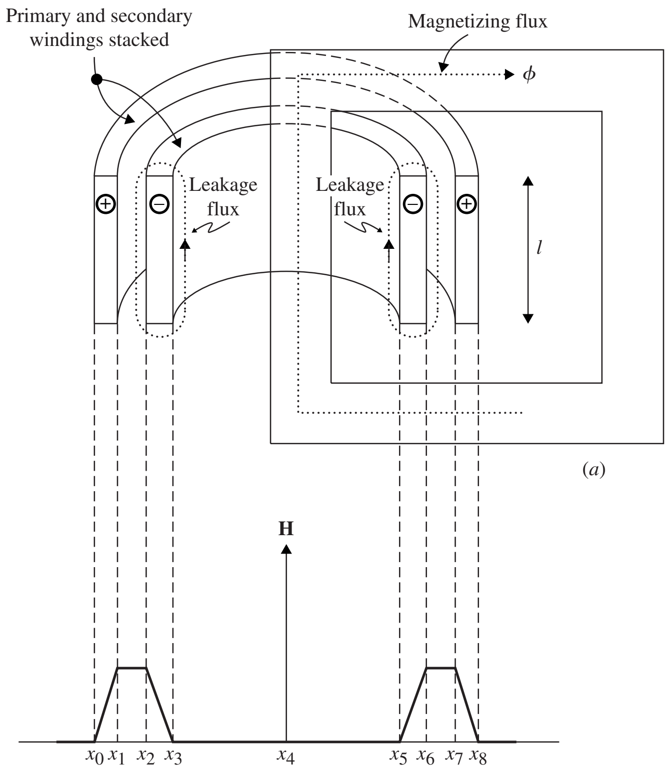
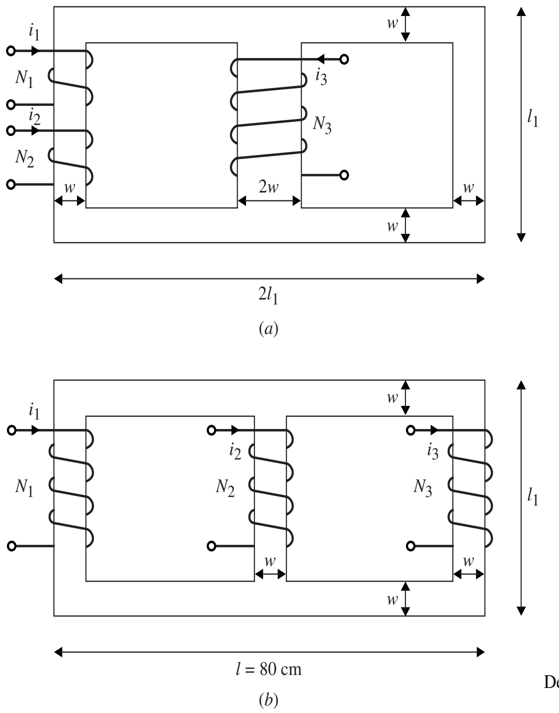


Figure PA.21



Depth = 5 cm
w = 5 cm

Figure PA.22

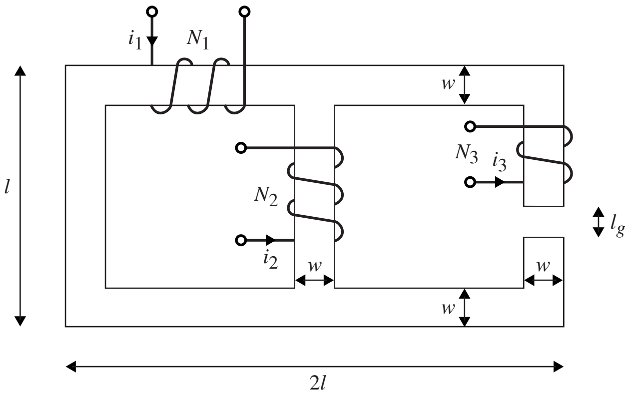


Figure PA.23

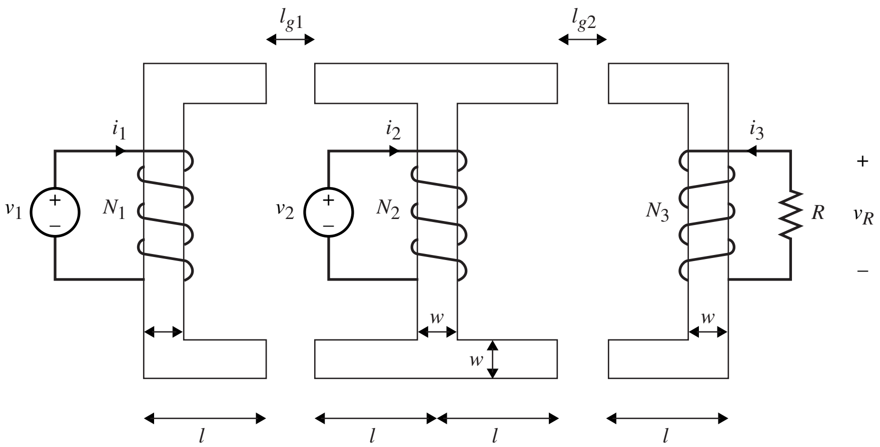


Figure PA.24

(**a**) Develop the magnetic circuit equivalents of these structures
(**b**) Determine the flux in the center leg in terms of the shown parameters.

A.23 Derive the expression for the flux, ϕ, in the center leg of the magnetic circuit of Fig. PA.23. Assume uniform cross-sectional area A and width W. The flux will be expressed in terms of N_1, N_2, N_3, i_1, i_2, i_3, l_g, l, w, A, and μ.

A.24 Determine the voltage, v_R, in terms of the given parameters in the three-winding magnetic circuit shown in Fig. PA.24. Assume the cross sectional area is A and the permeability is μ.

A.25 (**a**) Determine the flux in the center leg of the three-winding magnetic structure shown in Fig. PA.22(b). Assume $N_1 i_1$, $N_2 i_2$, and $N_3 i_3$ are 5, 10, and 8 A-turn, respectively, and $\mu = 3000\,\mu_o$.
(**b**) Determine the inductance for winding N_1 with $i_2 = i_3 = 0$.

A.26 Show how a set of three single-phase transformers can be connected to create a three-phase configuration for each of the following connections:
(**a**) Y-Δ
(**b**) Δ-Y
(**c**) Δ-Δ

A.27 Show that in a balanced Y-connected three-phase system, the line currents and the line voltages are also balanced three-phase systems.