The Buck Resonant Converter

- Replacing the switch by the resonant-type switch, to obtain a quasi-resonant PWM buck converter
- It can be shown that there are four modes of operation under the steady-state condition

![Fig 6.8](image-url) (a) Conventional buck converter with L-type resonant switch. (b) Simplified equivalent circuit.
Buck Converter: Equivalent Modes

Fig 6.9 (a) Equivalent circuit for Mode I. (b) Equivalent circuit for mode II. (c) Equivalent circuit for mode III. (d) Equivalent circuit for mode IV.
Buck Converter: Steady-State Analysis

Mode I \([ 0 \leq t < t_1] \)

- Mode I starts at \(t = 0\) when \(S\) is turned \(ON\)
- Assume for \(t > 0\), both \(S\) and \(D\) are \(ON\)
- The capacitor voltage, \(v_c\), is zero and the input voltage is equal to the inductor voltage
  \[
  V_{\text{in}} = L \frac{di_L}{dt}
  \]
- The inductor current, \(i_L\), is given by
  \[
  i_L(t) = \frac{V_{\text{in}} t}{L}
  \]
- As long as the inductor current is less than \(I_o\), the diode will continue conducting and the capacitor voltage remains at zero.
  \[
  I_o = \frac{V_{\text{in}} t_1}{L}
  \]
- Hence, the time interval \(= t_1\) is given by
  \[
  \Delta t_1 = t_1 = \frac{LI_o}{V_{\text{in}}}
  \]
- This is the inductor current charging state.
Steady-State Analysis (cont’d)

Mode II \([t_1 \leq t < t_2]\)

- Mode II starts at \(t_1\), diode is open resonant stage between \(L\) and \(C\)
- The first-order differential equations that represent this mode are

\[
C \frac{dv_c}{dt} = i_L - I_o \tag{6.5a}
\]

\[
L \frac{di_L}{dt} = V_{in} - v_c \tag{6.5b}
\]

- Inductor current is given by

\[
\frac{d^2 i_L}{dt^2} + \frac{1}{LC} i_L = \frac{I_o}{LC} \tag{6.6}
\]

- The general solution for \(i_L(t)\) is given by

\[
i_L(t) = A_1 \sin \omega_o (t - t_1) + A_2 \cos \omega_o (t - t_1) + A_3 \tag{6.7}
\]

- Resonant angular frequency and constants:

\[
\omega_o = \sqrt{\frac{1}{LC}} \quad A_1 = \frac{V_{in}}{L \omega_o}
\]

- \(A_2 = 0\), \(A_3 = I_o\)
Steady-State Analysis (cont’d)

Equations \( i_L \) and \( v_c \) are given by

\[
i_L(t) = I_o + \frac{V_{in}}{Z_o} \sin \omega_o (t - t_1)
\]

(6.8)

\[
v_c(t) = V_{in} [1 - \cos \omega_o (t - t_1)]
\]

(6.9)

\[Z_o = \sqrt{\frac{L}{C}}\] is known as the characteristic impedance

The time interval in this mode can be derived at \( t = t_2 \) by setting \( i_L(t_2) = 0\),

\[
i_L(t_2) = I_o + \frac{V_{in}}{Z_o} \sin \omega_o (t_2 - t_1)
\]

(6.10)

\[= 0\]

therefore,

\[
\Delta t_2 = t_2 - t_1 = \frac{1}{\omega_o} \sin^{-1} \left( -\frac{Z_o I_o}{V_{in}} \right) = \frac{1}{\omega_o} \left( \pi + \sin^{-1} \frac{Z_o I_o}{V_{in}} \right)
\]

(6.11)

Mode III starts at \( t = t_2 \), when the switch is turned \( OFF \).
Mode III \([t_2 \leq t < t_3]\): 

At \(t_2\), the inductor current becomes zero, and the capacitor linearly discharges from \(V_c(t_2)\) to zero during \(t_2\) to \(t_3\).

The capacitor current equals to \(I_o\) as given by,

\[
i_c = C \frac{dv_c}{dt} = -I_o \tag{6.12}
\]

The capacitor voltage \(v_c(t)\) is obtained from Eq. (6.12) from \(t_2\) to \(t_3\) with \(V_c(t_2)\) as the initial value,

\[
v_c(t) = \frac{-I_o}{C}(t-t_2) + V_c(t_2) \tag{6.13}
\]

The initial value \(V_c(t_2)\) is obtained from previous mode,

\[
V_c(t_2) = V_{in} \left[1 - \cos \omega_o(t_2 - t_1)\right] \tag{6.14}
\]

Substituting Eq. (6.14) into Eq. (6.13),

\[
v_c(t) = \frac{-I_o}{C}(t-t_2) + V_{in} \left[1 - \cos \omega_o(t_2 - t_1)\right] \tag{6.15}
\]

At \(t = t_3\), the capacitor voltage becomes zero,

\[
\Delta t_3 = t_3 - t_2 = \frac{C}{I_o} V_{in} \left[1 - \cos \omega_o(t_2 - t_1)\right] \tag{6.16}
\]

At this point, the diode turns \(ON\) and the circuit enters Mode IV.
Mode IV \( [t_3 \leq t < t_4] \): 

At this mode the switch remains *OFF*, but the diode starts conducting at \( t = t_3 \). Mode IV continues as long as the switch is *OFF*, and the output current starts the free-wheeling stage through the diode.

Initial conditions
\[
i_l(t) = 0 \\
v_c(t) = 0
\]

By turning *ON* the switch at \( t = T_s \), the cycle repeats these four modes. The time \( \Delta t_4 \) is given by,
\[
\Delta t_4 = T_s - \Delta t_1 - \Delta t_2 - \Delta t_3 \tag{6.17}
\]
Typical Steady-State Waveforms

Voltage Gain

The expression for the voltage gain, \( M = \frac{V_o}{V_{in}} \)

Figure 6.10 shows the steady state waveforms for \( V_c \) and \( i_L \) for the buck converter with L-type switch.

![Steady-state current and voltage waveforms of buck L-type converter](image)

**Fig 6.10** Steady-state current and voltage waveforms of buck L-type
Voltage Gain – $1^{st}$ method

Substitute for $v_c(t)$ from intervals $(t_2-t_1)$ and $(t_3-t_2)$, to yield,

$$V_o = \frac{1}{T_s} \left[ \int_{t_1}^{t_2} V_{in} (1 - \cos \omega_o (t - t_1)) \, dt + \int_{t_2}^{t_3} \left( \frac{-I_o}{C} (t - t_2) + V_c(t_2) \right) \, dt \right]$$  \hspace{1cm} (6.18)

The voltage gain ratio is given by,

$$\frac{V_o}{V_{in}} = \frac{1}{T_s} \left[ (t_2 - t_1) - \frac{\sin \omega_o (t_2 - t_1)}{\omega_o} - \frac{I_o}{V_{in} C} \left( \frac{(t_3 - t_2)^2}{2} + \frac{V_c(t_2)}{V_{in}} (t_3 - t_2) \right) \right] \hspace{1cm} \text{Book correction}$$  \hspace{1cm} (6.19)

Substitute for $(t_2-t_1)$, $(t_3-t_2)$ and $V_C(t_2)$ from the above modes, a closed form expression for $M$ in terms of the circuit parameters can be obtained.
Voltage Gain – 2nd method

The total input energy over one switching cycle,

\[ E_{in} = \int_{0}^{T_s} i_{in} V_{in} \, dt \]  \hspace{1cm} (6.20)

Since \( i_{in} \) is equal to \( i_L(t) \), Eq. (6.20) is rewritten as,

\[ E_{in} = \int_{0}^{t_1} i_L(t) V_{in} \, dt + \int_{t_1}^{t_2} i_L(t) V_{in} \, dt \]  \hspace{1cm} (6.21)

Substituting for \( i_L(t) \) from Eqs. (6.2) and (6.8) into the above integrals, Eq. (6.21) becomes,

\[ E_{in} = V_{in} \left\{ \frac{V_{in}}{2L} t_1^2 + I_o (t_2 - t_1) - \frac{V_{in}}{Z_o \omega_o} \left[ \cos \omega_o (t_2 - t_1) - 1 \right] \right\} \]  \hspace{1cm} (6.22) Book correction

Substituting for \( \cos \omega_o (t_2 - t_1) = 1 - \frac{I_o (t_3 - t_2)}{CV_{in}} \) from (6.16)

\[ E_{in} = V_{in} \left\{ \frac{t_1}{2} I_o + I_o (t_2 - t_1) + \frac{V_{in}}{Z_o \omega_o} \left[ \frac{I_o (t_3 - t_2)}{CV_{in}} \right] \right\} \]  \hspace{1cm} (6.23)

with, \( Z_o \omega_o = \frac{1}{C} \), Eq. (6.23) becomes,

\[ E_{in} = V_{in} I_o \left[ \frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right] \]  \hspace{1cm} (6.24)
Voltage Gain (cont’d)

The output energy over one switching cycle is:

$$E_o = \int_{0}^{T_s} I_o V_o \, dt = I_o V_o T_s$$  \hspace{1cm} (6.25)

Equating the input and output energy expressions

$$I_o V_o T_s = V_{in} I_o \left[ \frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right]$$  \hspace{1cm} (6.26)

From Eq. (6.26) the voltage gain is expressed by,

$$\frac{V_o}{V_{in}} = \frac{1}{T_s} \left[ \frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right]$$  \hspace{1cm} (6.27)

Substituting for $t_1$, $(t_2-t_1)$ and $(t_3-t_2)$ from previous equations, the voltage gain becomes

$$\frac{V_o}{V_{in}} = \frac{1}{T_s} \left\{ \frac{L I_o}{2 V_{in}} + \frac{1}{\omega_o} \sin^{-1} \frac{Z_o I_o}{V_{in}} + CV_{in} \left[ 1 - \cos \omega_o (t_2 - t_1) \right] \right\}$$  \hspace{1cm} (6.28)
Normalization

To simplify and generalize the gain equation, the following normalized parameters are defined:

\[ M = \frac{V_o}{V_{in}} \] \hspace{1cm} \text{normalized output voltage} \hspace{1cm} (6.29a)

\[ Q = \frac{R_o}{Z_o} \] \hspace{1cm} \text{normalized load} \hspace{1cm} (6.29b)

\[ I_o = \frac{V_o}{R_o} \] \hspace{1cm} \text{average output current} \hspace{1cm} (6.29c)

\[ f_{ns} = \frac{f_s}{f_o} \] \hspace{1cm} \text{normalized switching frequency} \hspace{1cm} (6.29d)

By substituting Eq. (6.29) into Eq. (6.28), the final voltage gain is simplified into

\[ M = \frac{f_{ns}}{2\pi} \left[ \frac{M}{2Q} + \alpha + \frac{Q}{M} (1 - \cos \alpha) \right] \hspace{1cm} (6.30) \]

where,

\[ \alpha = \sin^{-1} \left( -\frac{M}{Q} \right) \hspace{1cm} (6.31) \]
Buck-Control Characteristic Curve

A plot of the control characteristic curve of $M$ vs. $f_{ns}$ under various normalized loads is given in Fig. 6.11

![Control characteristic curve of M vs. f_{ns} for the ZCS buck converter.](image)

**Fig 6.11** Control characteristic curve of $M$ vs. $f_{ns}$ for the ZCS buck converter.
Example 6.1

Consider the following specifications for a ZCS buck converter of Fig. 6.8(a). Assume the parameters are: $V_{in} = 25V$, $V_o = 12V$, $I_o = 1A$, $f_s = 250kHz$

Design for the resonant tank parameters $L$ and $C$ and calculate the peak inductor current, and peak capacitor voltage. Determine the time interval for each mode.

Solution:

The voltage gain is $M = \frac{V_o}{V_{in}} = \frac{12}{25} = 0.48$. Select $f_{ns} = 0.4$. Determine $Q$ from either the control characteristic curve of Fig. 6.11 or from the gain equation of Eq. (6.30). This results in $Q$ approximately equaling 1. Since $R_o = \frac{V_o}{I_o}$, the characteristic impedance is given by,

$$Z_o = \frac{R_o}{Q} = 12\Omega$$

$$= \sqrt{\frac{1}{LC}} = 12\Omega$$  \hspace{1cm} (6.32)

The second equation in terms of $L$ and $C$ is obtained from $f_o$. From the normalized switching frequency, $f_o$ may be given by,

$$f_o = \frac{f_s}{f_{ns}}$$

$$f_o = \frac{625kHz}{0.4} = 625kHz$$

In terms of the angular frequency, $\omega_o$,

$$\omega_o = 2\pi f_o = \sqrt{\frac{1}{LC}}$$  \hspace{1cm} (6.33)

Solving Eqs. (6.32) and (6.33) for $L$ and $C$, we obtain,

$$L = \frac{Z_o}{\omega_o} = \frac{12\Omega}{2\pi \times 625 \times 10^3 \text{ rad} / \text{sec}}$$

$$= 3.06 \times 10^{-6} \approx 3\mu H$$

$$C = \frac{1}{Z_o \omega_o} = \frac{1}{12 \times 2 \times \pi \times 625 \times 10^3}$$

$$\approx 0.02 \mu F$$
Example 6.1 (cont’d)

The peak inductor current, is given by,

\[ I_{\text{peak}} = I_o + \frac{V_{\text{in}}}{Z_o} \]

\[ \approx 3 \text{ A} \]

The peak capacitor voltage is:

\[ V_{c,\text{peak}} = 2 V_{\text{in}} \]

\[ = 50 \text{ V} \]

The time intervals are calculated from the following expressions:

\[ t_1 = \frac{I_o L}{V_{\text{in}}} = \frac{(1 \text{ A}) \times (3 \times 10^{-6} \text{ H})}{25 \text{ V}} \approx 0.122 \mu\text{s} \]

\[ t_2 = t_1 + \frac{1}{\omega_o} \sin^{-1} \left( -\frac{Z_o I_o}{V_{\text{in}}} \right) \approx 0.122 + \frac{1}{2\pi f_o} \sin^{-1} \left( -\frac{12 \times 1}{25} \right) \approx 0.795 \mu\text{s} \]

\[ t_3 = t_2 + \frac{CV_{\text{in}} (1 - \cos \omega_o (t_2 - t_1))}{I_o} \approx 1.79 \mu\text{s} \]

For \( t_{\text{max}} \) we have

\[ \omega_o (t'_{\text{max}} - t_1) = \frac{\pi}{2} \rightarrow t'_{\text{max}} = \frac{\pi}{2} + t_1 \]

\[ = 0.4 \mu\text{s} + 0.122 \mu\text{s} = 0.522 \mu\text{s} \]

\[ t_4 = 4 \mu\text{s} = T_s \]

Exercise 6.2 for practice